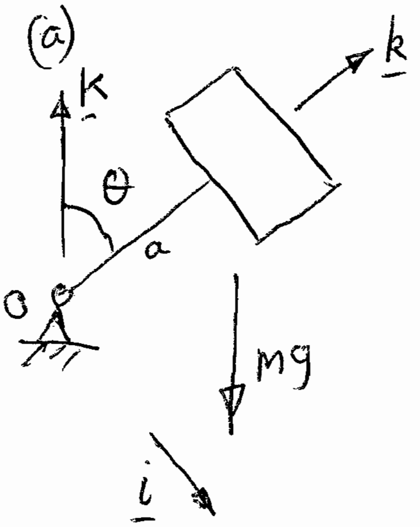
Part IIA 3C5 2007

PART IIA 2007
3C5 Dynamics
Dr H Hunt

1. (a)



The only couple acting is due to mg

$$\underline{Q} = mga\sin\theta\ \underline{j} \qquad (1)$$

Gyro equation #3

$$Q_3 = C\dot{\omega}_3 \qquad \text{but } (1) \rightarrow Q_3 = 0$$

$$\therefore \dot{\omega}_3 = 0 \qquad \therefore \omega_3 = \text{const}$$

(b) $\underline{Q} = \dot{\underline{h}} \qquad \therefore \underline{Q}\cdot\underline{K} = \dot{\underline{h}}\cdot\underline{K}$

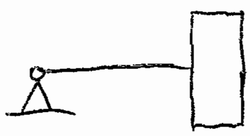
and from (1) $\underline{Q}\cdot\underline{K} = 0 \qquad \therefore \dot{\underline{h}}\cdot\underline{K} = 0$

but $\underline{K}$ is a constant vector $\therefore \frac{d}{dt}(\underline{h}\cdot\underline{K}) = 0$

$$\therefore \underline{h}\cdot\underline{K} = \text{const}$$

Initial value $= C\omega$

(c)



Gyro equation #2

$$Q_2 = A\dot{\Omega}_2 + (A\Omega_3 - C\omega_3)\Omega_1$$

where $\Omega_1 = -\dot{\phi}\sin\theta$

$\Omega_2 = \dot{\theta}$

$\Omega_3 = \dot{\phi}\cos\theta$

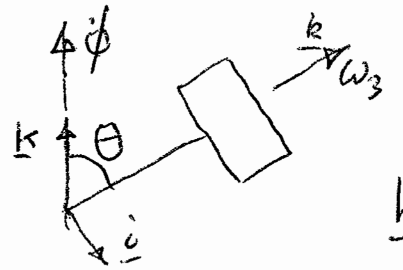
Steady state precession $\therefore \dot{\theta} = 0$

$$\therefore mga\sin\theta = -(A\dot{\phi}\cos\theta - C\omega_3)\dot{\phi}\sin\theta \qquad (2)$$

and put $\theta = \frac{\pi}{2} \qquad \therefore mga = C\omega_3\dot{\phi}$

$$\therefore \dot{\phi}_{final} = \frac{mga}{C\omega_3} \qquad \underline{\text{ans}}$$

1(c) cont. Now prove $\theta = \frac{\pi}{2}$ by conservation of $\underline{h}\cdot\underline{k}$



$$\underline{h} = A\Omega_1 \underline{i} + A\Omega_2 \underline{j} + C\omega_3 \underline{k}$$

$$\underline{h}\cdot\underline{K} = A\Omega_1 (\underline{i}\cdot\underline{K}) + C\omega_3 (\underline{k}\cdot\underline{K})$$

$\underline{i}\cdot\underline{K} = -\sin\theta$

$\underline{k}\cdot\underline{K} = \cos\theta$

$$\therefore C\omega_3 = +A\dot\phi\sin\theta\sin\theta + C\omega_3\cos\theta$$

$$\therefore C\omega_3(1-\cos\theta) = A\dot\phi\sin^2\theta$$
$$= A\dot\phi^{*}(1-\cos^2\theta)$$
$$= A\dot\phi^{*}(1-\cos\theta)(1+\cos\theta)$$

$$\therefore C\omega_3 = A\dot\phi^{*}(1+\cos\theta) \qquad (3)$$

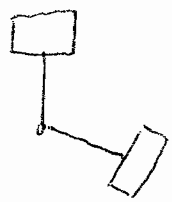
and with $\dot\phi_{final} = \dfrac{mga}{C\omega_3}$, $\theta_{final} = \frac{\pi}{2}$

$$\therefore C\omega_3 = \frac{A\,mga}{C\omega_3} \qquad *$$

but $(C\omega_3)^2 = (C\omega)^2 = \left(C\dfrac{\omega_{crit}}{2}\right)^2 = mgaA \qquad (4)$

So $*$ is satisfied.

(d) $(KE + PE)_{initial} = (KE + PE)_{final}$



$$\therefore \tfrac{1}{2}C\omega_3^2 + mga = \tfrac{1}{2}A\omega_1^2 + \tfrac{1}{2}A\omega_2^2 + \tfrac{1}{2}C\omega_3^2 + mga\cos\theta$$

but at θ_{max} $\omega_2 = 0$ and $\omega_1^2 = \dot\phi^2\sin^2\theta$

$$\therefore mga(1-\cos\theta) = \tfrac{1}{2}A\dot\phi^2\sin^2\theta$$
$$= \tfrac{1}{2}A\dot\phi^2(1-\cos^2\theta)$$

$$\therefore mga = \tfrac{1}{2}A\dot\phi^2(1+\cos\theta)$$

but ③ $\rightarrow$ $\dot{\phi}^2 = \left(\frac{C\omega_3}{A(1+\cos\theta)}\right)^2$

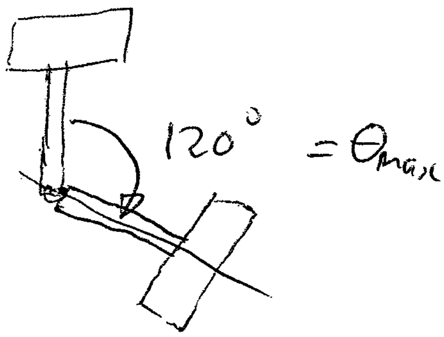
$$\therefore \quad mga = \frac{1}{2} A \frac{(C\omega_3)^2}{A^2(1+\cos\theta)}$$

and from ④ $$= \frac{1}{2A} \quad \frac{mgaA}{1+\cos\theta}$$

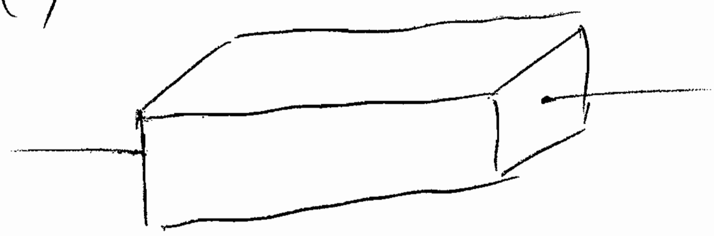
$$\therefore \quad 1+\cos\theta = \frac{1}{2}$$

$$\therefore \quad \cos\theta = -\frac{1}{2}$$

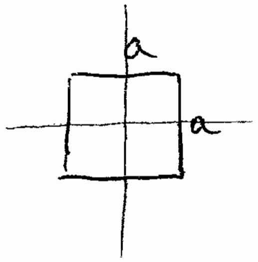
$$\therefore \quad \theta = \frac{2\pi}{3}$$



2. (a)



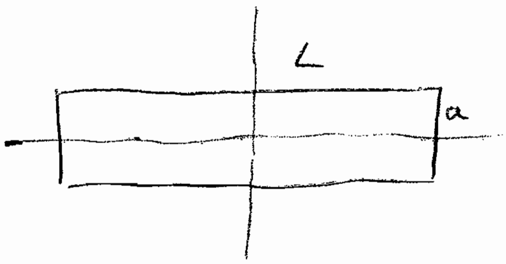
End view



$\therefore C = \frac{1}{12}Ma^2 + \frac{1}{12}Ma^2 = \underline{\underline{\frac{1}{6}Ma}}$

by $\perp$ axis theorem & data book

Side view

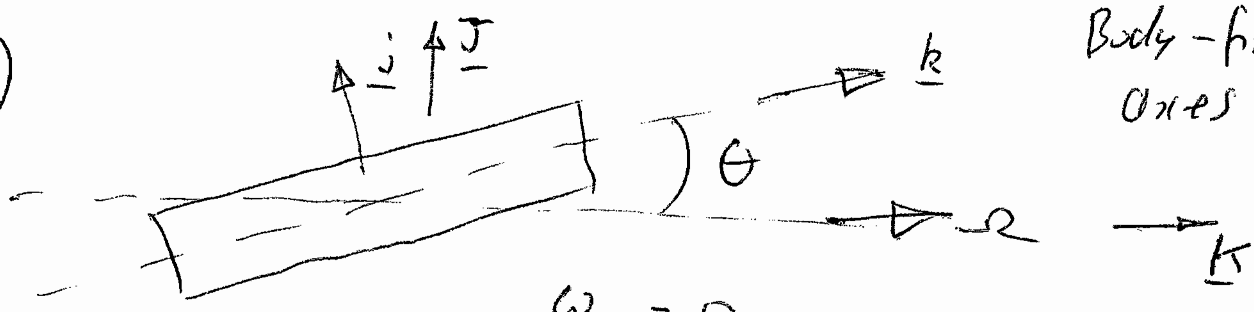


$A = \frac{1}{12}ma^2 + \frac{1}{12}mL^2$

$= \underline{\underline{\frac{1}{12}m(a^2+L^2)}}$

The body is an AAC body so behaves like a cylinder.

(b)



Body-fixed axes

$\omega_1 = 0$

$\omega_2 = -\Omega\sin\theta$

$\omega_3 = \Omega\cos\theta$

i/

$\therefore \underline{h} = A\omega_1\underline{i} + A\omega_2\underline{j} + C\omega_3\underline{k}$

$= -A\Omega\sin\theta\,\underline{j} + C\Omega\cos\theta\,\underline{k}$

and for ii/ resolve $\underline{h}$ into $\underline{I}\ \underline{J}\ \underline{K}$

2(b) ii cont.

$$\underline{h} \cdot \underline{K} = -A\Omega \sin\theta (\underline{j} \cdot \underline{K}) + C\Omega \cos\theta (\underline{k} \cdot \underline{K})$$

$$= -A\Omega \sin\theta (-\sin\theta) + C\Omega \cos\theta (\cos\theta)$$

$$= A\Omega \sin^2\theta + C\Omega \cos^2\theta$$

$$\underline{h} \cdot \underline{J} = -A\Omega \sin\theta (\underline{j} \cdot \underline{J}) + C\Omega \cos\theta (\underline{k} \cdot \underline{J})$$

$$= -A\Omega \sin\theta (\cos\theta) + C\Omega \cos\theta (\sin\theta)$$

$$= (C - A)\Omega \cos\theta \sin\theta$$

$$\underline{h} \cdot \underline{I} = 0$$

$$\therefore \underline{h} = \underline{\underline{(C-A)\Omega \cos\theta \sin\theta \, \underline{J} + (A\Omega \sin^2\theta + C\Omega \cos^2\theta)\underline{K}}} \quad (1)$$

ans

Now, $\underline{h} = \begin{bmatrix} \text{Inertia} \\ \text{Matrix} \end{bmatrix} \Omega \underline{k}$

$$= \begin{bmatrix} \cdot & \cdot & -e \\ \cdot & \cdot & -f \\ \cdot & \cdot & g \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix}$$

$$= e\Omega \underline{I} \mp f\Omega \underline{J} \mp g\Omega \underline{K}$$

Compare with (1)

$$\therefore f = -(C-A)\Omega \cos\theta \sin\theta$$

which is one of the products of inertia

(c) Use Euler's equations in body-fixed axes $\underline{i}\ \underline{j}\ \underline{k}$

$$Q_1 = A\dot{\omega}_1 - (A-C)\,\omega_2\,\omega_3$$

$$= 0 - (A-C)\,(-\Omega\sin\theta)(\Omega\cos\theta)$$

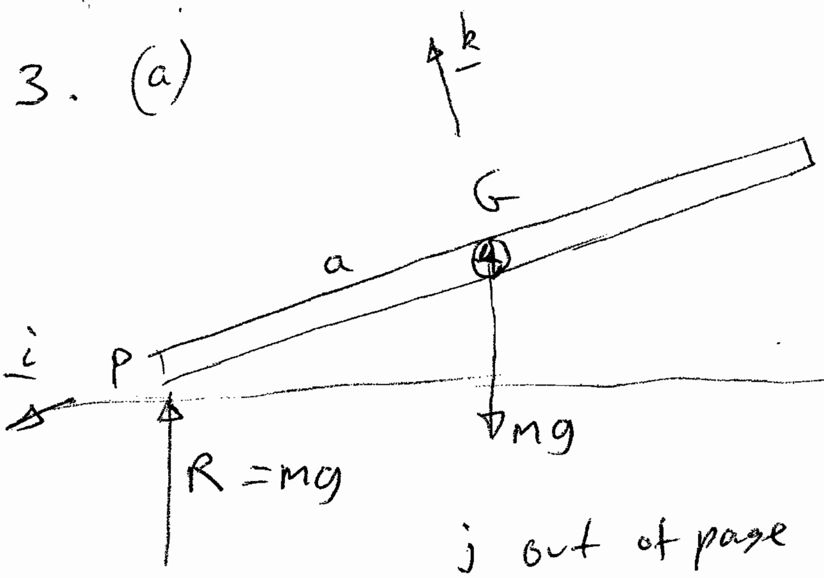
$$\underline{\underline{Q_1 = (A-C)\,\Omega^2 \sin\theta\cos\theta}}$$ ans

(d) $Q_1 = 0$ if $A = C$

$\therefore\ a = L$

This is when the rotor is a cube which is an AAA body, identical to a sphere which can spin about any axis with no dynamic moment.

3. (a)



$\underline{j}$ out of page

G is at rest

$\therefore \; R = mg$

(no net force on the coin)

$$\underline{Q} = -mga\cos\theta\,\underline{j}$$ taking moments about G

(b) Gyro equation "2"

$$Q_2 = A\dot{\Omega}_2 + (A\Omega_3 - C\omega_3)\Omega_1$$

with
$$\Omega_1 = -\dot{\phi}\sin\theta$$
$$\Omega_2 = \dot{\theta} = 0 \quad \text{in steady state}$$
$$\Omega_3 = \dot{\phi}\cos\theta$$

$$\therefore \quad -mga\cos\theta = (A\dot{\phi}\cos\theta - C\omega_3)(-\dot{\phi}\sin\theta) \quad ①$$

No slip at P :

$$\underline{\omega} \times a\underline{i} = 0 \qquad \therefore \left(\Omega_1\underline{i} + \Omega_2\underline{j} + \omega_3\underline{k}\right) \times a\underline{i} = 0$$

$$\therefore \quad \Omega_2 = 0$$

$$\text{and} \quad \omega_3 = 0$$

3 (b) cont. so (i) $\rightarrow$ $mga\cos\theta = A\dot\phi\sin\theta$

$$\therefore \dot\phi^2 = \frac{mga}{A}\cot\theta$$

but for a thin coin $C = \frac{1}{2}ma^2$

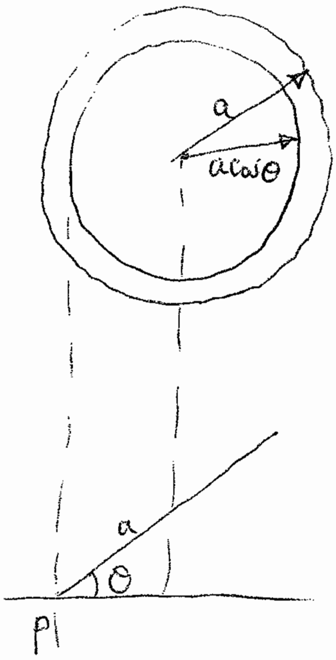
$A = \frac{1}{4}ma^2$

$$\therefore \dot\phi^2 = \frac{4mga}{ma^2}\cot\theta = 4\frac{g}{a}\cot\theta$$

$$\therefore \dot\phi = 2\sqrt{\frac{g}{a}\cot\theta} \quad \text{ans}$$

This is the rate of turning of the refeence frame ie the wobbling rate

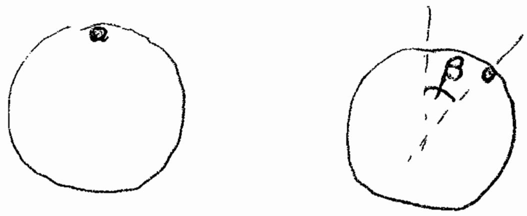
3(c) For the turning rate of the head of the coin consider the circle on which point P rolls.



In one wobble the circumfeence of length $2\pi a$ rolls on a circle of circumfeence $2\pi a\cos\theta$

So a spot on the edge of the coin has advanced a distance

$2\pi a - 2\pi a\cos\theta$



after one wobble

So the apparent angle β through which the coin has turned is $\dfrac{2\pi a(1-\cos\theta)}{a}$

The denominator could equally be $a\cos\theta$, but at this stage assume small θ

$\therefore \quad \cos\theta \simeq 1 - \frac{1}{2}\theta^2$

Ignore the θ^2 bit in the denominator, but not in the numerator

$$\therefore \quad \beta \simeq 2\pi \tfrac{1}{2}\theta^2 = \pi\theta^2$$

The time taken for one wobble of the coin

$$T = \frac{2\pi}{\dot\phi} = \frac{2\pi}{2\sqrt{\frac{g}{a\theta}}}$$

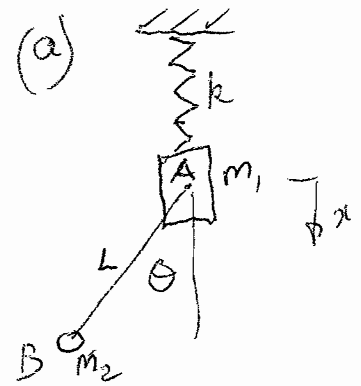
again for small θ
$\cot\theta \simeq \frac{1}{\theta}$

$$= \pi\sqrt{\frac{a\theta}{g}}$$

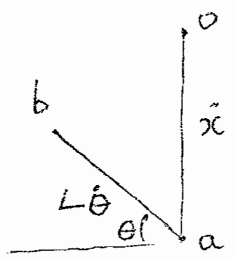
So $\dot\beta = \dfrac{\beta}{T} = \dfrac{\pi\theta^2}{\pi\sqrt{\frac{a\theta}{g}}} = \sqrt{\dfrac{g\theta^3}{a}}$ ans

This is the rate of turning of the head on the coin. $\dot\beta \to 0$ as $\theta \to 0$ as observed.

4. (a)



Velocity diagram



$$T = \tfrac{1}{2} m_1 \dot{x}^2 + \tfrac{1}{2} m_2 \left[\left(\dot{x} - L\dot{\theta}\sin\theta \right)^2 + \left(L\dot{\theta}\cos\theta \right)^2 \right]$$

$$= \tfrac{1}{2}(m_1+m_2)\dot{x}^2 - \tfrac{1}{2} m_2 (2\dot{x}L\dot{\theta}\sin\theta) + \tfrac{1}{2} m_2 L^2 \dot{\theta}^2 (\sin^2\theta + \cos^2\theta)$$

$$T = \tfrac{1}{2}\left[(m_1+m_2)\dot{x}^2 + m_2\left(L^2\dot{\theta}^2 - 2L\dot{x}\dot{\theta}\sin\theta \right) \right] \quad \text{ans}$$

$$V = \tfrac{1}{2} k x^2 - m_1 g x - m_2 g\left(x - L(1-\cos\theta)\right) \quad \text{ans}$$

(b) x - equation

$$\boxed{\text{Lagrange}: \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_j}\right) - \frac{\partial T}{\partial x_j} + \frac{\partial V}{\partial x_j} = Q_j}$$

$$\frac{\partial T}{\partial \dot{x}} = (m_1+m_2)\dot{x} - m_2 L\dot{\theta}\sin\theta$$

$$\frac{\partial T}{\partial x} = 0$$

$$\frac{\partial V}{\partial x} = kx - (m_1+m_2)g$$

$$\therefore (m_1+m_2)\ddot{x} - m_2 L\ddot{\theta}\sin\theta - m_2 L\dot{\theta}^2 \sin\theta\cos\theta + kx - (m_1+m_2)g = 0$$

ans.

check: put $\theta = 0$

$$\therefore (m_1+m_2)\ddot{x} + kx = (m_1+m_2)g$$

usual equation for mass on spring ✓

4(b) cont. θ equation

$$\frac{\partial T}{\partial \dot\theta} = m_2 L^2 \dot\theta - m_2 L \dot x \sin\theta$$

$$\frac{\partial T}{\partial \theta} = - m_2 L \dot x \dot\theta \cos\theta$$

$$\frac{\partial V}{\partial \theta} = + m_2 g L \sin\theta$$

$$\therefore \; m_2 L^2 \ddot\theta - m_2 L \ddot x \sin\theta - m_2 L \dot x \dot\theta \cos\theta + m_2 L \dot x \dot\theta \cos\theta + m_2 g L \sin\theta = 0$$

$$\therefore \; \underline{L \ddot\theta - \ddot x \sin\theta + g \sin\theta} = 0$$ ans.

Check : put $x = 0$ $\therefore$ $\ddot\theta + \frac{g}{L}\sin\theta = 0$, usual pendulum equation ✓

4(c) put $\sin\theta \sim \theta$
$\cos\theta \sim 1$ and ignore small terms

$$\left.\begin{aligned} &\therefore (m_1 + m_2)\ddot x + kx = (m_1 + m_2) g \\ &\text{and} \quad L\ddot\theta + g\theta = 0 \end{aligned}\right\} \text{ans}$$

4(d) Put $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1 - \frac{\theta^2}{2}$

into T & V, keeping up to order^2

$$T \simeq \frac{1}{2}\left[(m_1+m_2)\dot{x}^2 + m_2 L^2 \dot{\theta}^2\right]$$

$$V \simeq \frac{1}{2} k x^2 - (m_1+m_2) g x + \frac{1}{2} m_2 g L \theta^2$$

Make a substitution $x = y + x_0$

$x^2 = y^2 + 2x_0 y + x_0^2$

$$\therefore V = \frac{1}{2} k\left(y^2 + 2x_0 y + x_0^2\right) - (m_1+m_2) g (y + x_0) + \frac{1}{2} m_2 g L \theta^2$$

Choose $x_0 = \dfrac{(m_1+m_2) g}{k}$

and arbitrarily shift datum for V to $\frac{1}{2} k x_0^2 - (m_1+m_2) x_0$

$$\therefore V \simeq \frac{1}{2} k y^2 + \frac{1}{2} m_2 g L \theta^2$$

and since $\dot{x} = \dot{y}$

$$T = \frac{1}{2}\left[(m_1+m_2)\dot{y}^2 + m_2 L^2 \dot{\theta}^2\right]$$

T & V give diagonal mass and stiffness matrices

$$[M] = \begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2 L^2 \end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix} k & 0 \\ 0 & m_2 g L \end{bmatrix}$$

4(d) Put $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1 - \frac{\theta^2}{2}$ into T & V, keeping terms up to order^2

$$T \simeq \tfrac{1}{2}\left((m_1+m_2)\dot{x}^2 + m_2 L^2 \dot{\theta}^2\right)$$

$$V \simeq \tfrac{1}{2} k x^2 - (m_1+m_2) g x + \tfrac{1}{2} m_2 g L \theta^2$$

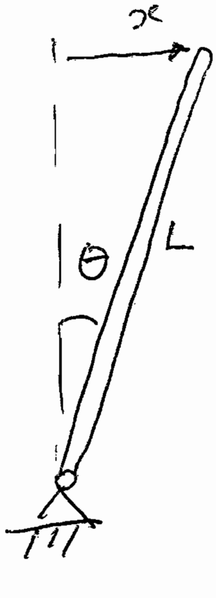
Only quadratic terms count in the mass & stiffness matrices so

$$[M] = \begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2 L^2 \end{bmatrix} \qquad [K] = \begin{bmatrix} k & 0 \\ 0 & m_2 g L \end{bmatrix}$$

and $$[M]\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + [K]\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} (m_1+m_2) g \\ 0 \end{bmatrix}$$

this comes from $\frac{\partial V}{\partial x} = -(m_1+m_2) g$ which is a constant throughout the motion.

5 (a)



Think in terms of θ

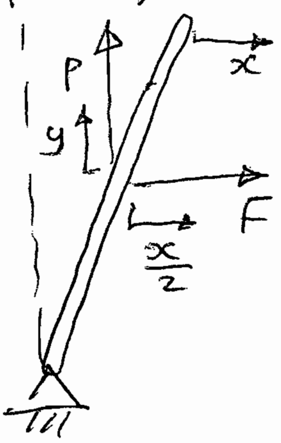
$T = \frac{1}{2} I \dot{\theta}^2$

Now do a coordinate transformation

$$x = L \sin\theta$$
$$\dot{x} = L\dot{\theta}\cos\theta$$
$$\dot{x}^2 = L^2\dot{\theta}^2\cos^2\theta = \dot{\theta}^2(L^2 - L^2\sin^2\theta)$$
$$= \dot{\theta}^2(L^2 - x^2)$$

$$\therefore \quad T = \frac{1}{2} I \frac{\dot{x}^2}{L^2 - x^2} \qquad \underline{\underline{\text{ans}}}$$

For generalized forces use a work argument



$$dW = F\frac{dx}{2} + P\,dy$$

easy by inspection

needs thought

$$y = \frac{L}{2}(\cos\theta - 1)$$
$$dy = \frac{L}{2}(-\sin\theta\, d\theta) = -\frac{x}{2}\,d\theta$$

and $\quad dx = L\cos\theta\, d\theta = L\sqrt{1-\sin\theta}\; d\theta$

$$\therefore d\theta = \frac{dx}{\sqrt{L^2 - x^2}}$$

$$\therefore \quad dW = Q\,dx = \frac{F}{2}\,dx - \frac{Px}{2\sqrt{L^2-x^2}}\,dx$$

$$\therefore \quad Q = \frac{F}{2} - \frac{Px}{2\sqrt{L^2-x^2}} \qquad \underline{\underline{\text{ans}}}$$

5 (b) $\quad \dfrac{\partial T}{\partial \dot{x}} = \dfrac{I\dot{x}}{L^2 - x^2} \qquad \dfrac{\partial T}{\partial x} = \dfrac{I\dot{x}^2 x}{(L^2 - x^2)^2}$

$V = 0$ because all forces are included in Q

so $\dfrac{\partial V}{\partial x} = 0$

5(b) cont'd   Lagrange : $\frac{d}{dt}\left(\frac{I\dot{x}}{L^2-x^2}\right) - \frac{I\dot{x}^2x}{(L^2-x^2)^2} = Q$

$$\therefore \frac{I\ddot{x}(L^2-x^2) - I\dot{x}(-2x)}{(L^2-x^2)^2} - \frac{I\dot{x}^2x}{(L^2-x^2)^2} = Q$$

$$\therefore \frac{I\ddot{x}}{L^2-x^2} + \frac{I\dot{x}^2x}{(L^2-x^2)^2} = \frac{F}{2} - \frac{Px}{2\sqrt{L^2-x^2}} \quad \underline{\underline{\text{ans}}}$$

Small vibration $\therefore x \ll L$ and ignore $\dot{x}^2x$ terms

$$\therefore \frac{I\ddot{x}}{L^2} + \frac{Px}{2L} = \frac{F}{2} \qquad \therefore \ddot{x} + \underbrace{\frac{PL}{2I}}_{\omega_n^2}x = \frac{FL^2}{2I}$$

$$\therefore \omega_n = \sqrt{\frac{PL}{2I}} \quad \underline{\underline{\text{ans}}}$$

5(c)   Use θ $\therefore$ $T = \frac{1}{2}I\dot{\theta}^2$
$V = 0$ as before

$$dW = Q\,d\theta = F\frac{dx}{2} + P\,dy$$

$$\left.\begin{aligned} dx &= L\cos\theta\,d\theta \\ dy &= -\tfrac{L}{2}\sin\theta\,d\theta \end{aligned}\right\} \text{ as before}$$

$$\therefore Q = F\frac{L}{2}\cos\theta - P\frac{L}{2}\sin\theta$$

$$\frac{\partial T}{\partial\dot{\theta}} = I\dot{\theta} \qquad \frac{\partial T}{\partial\theta} = 0 \qquad \frac{\partial V}{\partial\theta} = 0$$

$$\therefore I\ddot{\theta} = F\frac{L}{2}\cos\theta - P\frac{L}{2}\sin\theta \quad \underline{\underline{\text{ans}}}$$

(Check, small θ $\therefore$ $I\ddot{\theta} + \frac{PL}{2}\theta = \frac{FL}{2}$
as before with $x = L\theta$)

5(c) cont'd substitute $x = L\sin\theta$

$$\dot{x} = L\dot{\theta}\cos\theta$$

$$\ddot{x} = L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta$$

into the x- equation from 5(b)

$$\frac{I}{L^2-x^2}\left(L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta\right) + \frac{I(L\dot{\theta}\cos\theta)^2 L\sin\theta}{(L^2-x^2)^2} =$$

$$= \frac{F}{2} - \frac{PL\sin\theta}{2\sqrt{L^2-(L\sin\theta)^2}}$$

$$\therefore \frac{I}{L\cos^2\theta}\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) + \frac{I\dot{\theta}^2\sin\theta}{L\cos^2\theta} = \frac{F}{2} - \frac{P\sin\theta}{2\cos\theta}$$

$$\therefore I\ddot{\theta} = \frac{FL}{2}\cos\theta - \frac{PL}{2}\sin\theta$$ as required

Notes
- Several candidates obtained full marks, simply by not making mistakes.
- Fudging the "show thats" was obvious
- "$V = 0$ because buoyancy forces are equal and opposite to gravity earned" no marks

Engineering Tripos Part IIA 2007

Answers

1. (c) $\dot{\phi} = \dfrac{mga}{C\omega} = \sqrt{\dfrac{mga}{A}} = \dfrac{C\omega}{A}$ (d) 120°

2. (a) $m(a^2+L^2)/12$, $m(a^2+L^2)/12$, $ma^2/6$
   (b) $\omega = 0\,\boldsymbol{i} - \Omega\sin\theta\,\boldsymbol{j} + \Omega\cos\theta\,\boldsymbol{k}$
   (i) $\boldsymbol{h} = -\mathrm{A}\Omega\sin\theta\,\boldsymbol{j} + \mathrm{C}\Omega\cos\theta\,\boldsymbol{k}$
   (ii) $\boldsymbol{h} = (C-A)\Omega\cos\theta\sin\theta\,\boldsymbol{J} + (A\Omega\sin^2\theta + C\Omega\cos^2\theta)\,\boldsymbol{K}$
   product of inertia $= -(C-A)\cos\theta\sin\theta$
   (c) $Q_1 = (A-C)\Omega^2\cos\theta\sin\theta$
   (d) $L = a$

3. (a) $\boldsymbol{Q} = -\,mga\cos\theta\,\boldsymbol{j}$ (b) $2\sqrt{\dfrac{g}{a\tan\theta}}$ (c) $\sqrt{\dfrac{g\theta^3}{a}}$

4. (a) $V = \frac{1}{2}k\theta^2 - m_1gx - m_2g(x - L(1-\cos\theta))$
   (b) $(m_1+m_2)\ddot{x} - m_2L\ddot{\theta}\sin\theta - m_2L\dot{\theta}^2\sin\theta\cos\theta + kx - (m_1+m_2)g = 0$
   $L\ddot{\theta} - \ddot{x}\sin\theta + g\sin\theta = 0$
   (c) $(m_1+m_2)\ddot{x} + kx = (m_1+m_2)g$
   $L\ddot{\theta} + g\theta = 0$
   (d) $M = \begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2L^2 \end{bmatrix}$ $K = \begin{bmatrix} k & 0 \\ 0 & m_2gL \end{bmatrix}$

5. (b) $\dfrac{I\ddot{x}}{L^2-x^2} + \dfrac{I\dot{x}^2x}{(L^2-x^2)^2} = \dfrac{F}{2} - \dfrac{Px}{2\sqrt{L^2-x^2}}$
   (c) $T = \frac{1}{2}I\dot{\theta}^2$ $Q = \dfrac{FL\cos\theta}{2} - \dfrac{PL\sin\theta}{2}$

HEMH May 2007