Engineering Tripos Part IIA 2006-7

Module 3D1 Solutions

1. a) $G_s = 2650/1000 = 2.65$

area of tube $= \pi\, 0.9^2/4 = 0.636\ m^2$

loose volume $= 0.636 \times 0.2 = 0.127\ m^3$

loose dry density $= 182/0.127 = 1433\ kg/m^3$

but $\rho_{d,min} = 2650/V_{max}$

$\therefore V_{max} = 1.849$; $e_{max} = 0.849$

dense volume $= 0.636 \times 0.162 = 0.103\ m^3$

dense dry density $= 182/0.103 = 1766\ kg/m^3$

$\therefore V_{min} = 1.500$; $e_{min} = 0.500$

$\rho_{damp,\ compact} = 216/0.127 = 1701\ kg/m^3$

$\therefore \rho_{dry\ comp} = 1701/1.04 = 1636\ kg/m^3$

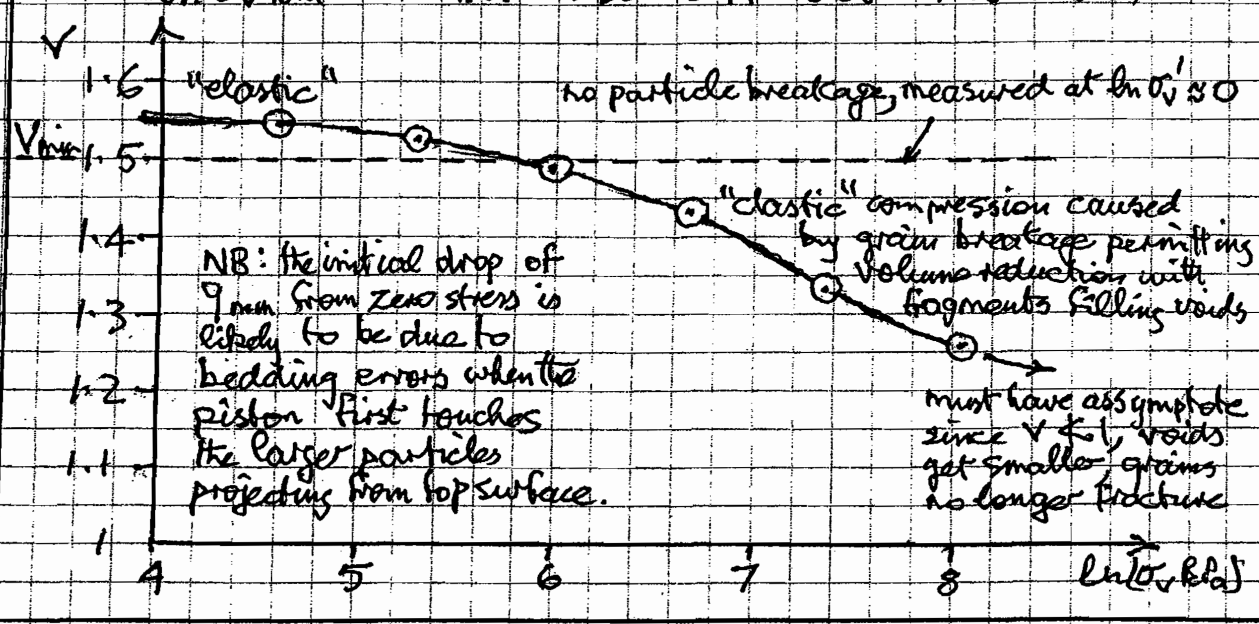
$\therefore V_{compact} = 1.620$; $e_{comp.} = 0.620$

relative density $I_D = (e_{max} - e_{compact})/(e_{max} - e_{min})$
$= 0.229/0.349 = 66\%$

A severe earthquake could compact this material further to get I_D closer to 1.0. If drained this could lead to significant subsidence; if undrained it could lead to "liquefaction" – i.e. loss of effective stress, stiffness and strength temporarily.

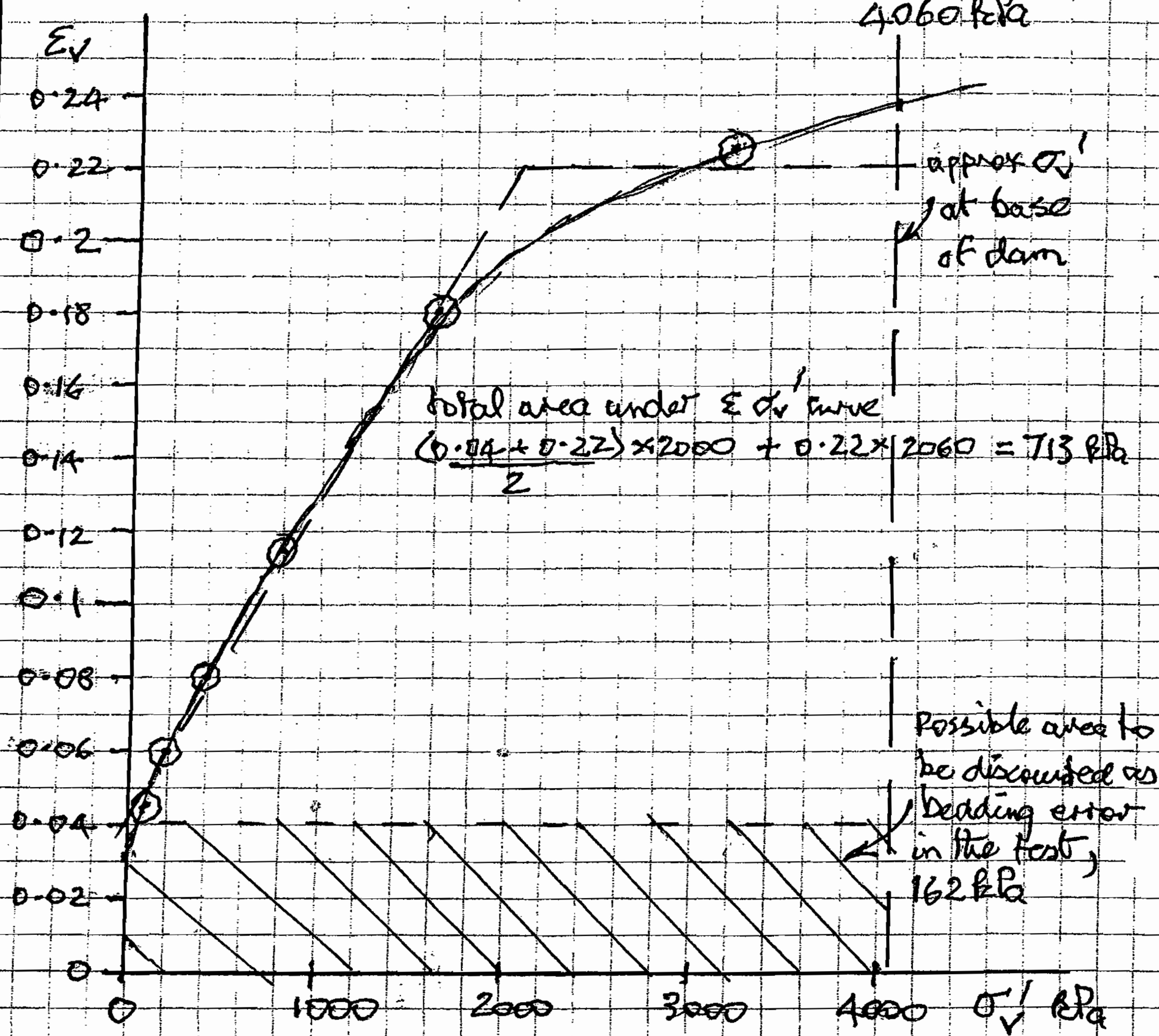
b) i)

σ_v kPa	0	100	200	400	800	1600	3200
Δh mm	0	9	12	16	23	36	45
h mm	200	191	188	184	177	164	155
$V = V_0\, h/h_0$	1.620	1.547	1.523	1.490	1.434	1.328	1.256
$\ln \sigma_v$ kPa	$-\infty$	4.61	5.30	5.99	6.68	7.38	8.07



ii) From zero stress, $\varepsilon_v = \Delta h/200$:

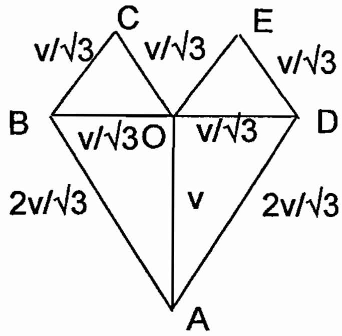
σ_v kPa	0	100	200	400	800	1600	3200 kPa
ε_v	0	0.045	0.060	0.080	0.115	0.180	0.225



Compacted density $\rho = 1701$ kg/m³, so initially $\gamma = 16.7$ kN/m³
So at mid-height σ_v estimated as $100 \times 16.7 = 1670$ kPa
At nearby point, $\sigma_v = 1600$ kPa, $v = 1.328$ so denser
$\rho = 216/(0.636 \times 0.164) = 2071$ kg/m³, so $\gamma = 20.3$ kN/m³

Take this value as representative of the whole dam. (approx.)
Then $\sigma_v' = 20.3\,z$
Consider element dz at z, compression $dy = \varepsilon_v\, dz$
So total self-weight compression $= \int_0^{200} \varepsilon_v\, dz$
But $dz \simeq d\sigma_v'/20.3$, so total comp. $= \frac{1}{20.3}\int_0^{200\times 20.3} \varepsilon_v\, d\sigma_v'$
Crude estimate $= 713/20.3 = 35$ m
Allowing for bedding error in the test, $(713-162)/20.3 = 27$ m
Main problem is ~15% extra fill to be placed by contractor.

2 (a) (i)



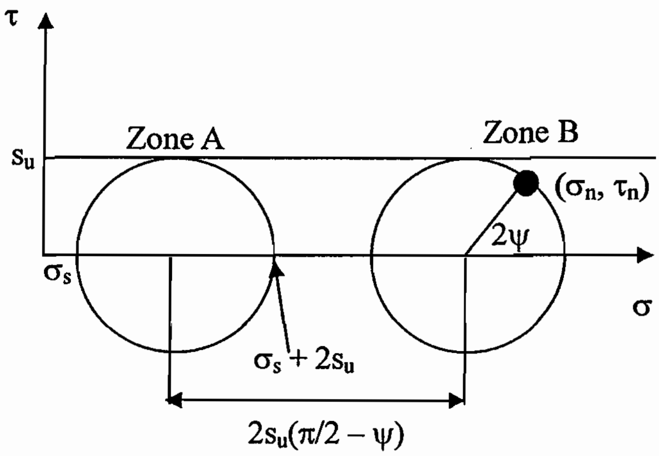
(ii) The work in = $q_f Bv - \sigma_s Bv$

The work dissipated = $s_u B(2v/\sqrt{3}) \times 2 + s_u B(v/\sqrt{3}) \times 6$

Equating these two equations give

$V = q_f B = [(10\sqrt{3}/3)s_u + \sigma_s]B$

(b) (i), (ii)



(iii) Using the stress fan concept, the distance between the two circles is $2s_u(\pi/2 - \psi)$. Hence, the two circles can be related by the following equation.

$$\sigma_n = V/B = \sigma_s + s_u + 2s_u(\pi/2 - \psi) + s_u \cos 2\psi$$

Also,

$$\tau_n = H/B = s_u \sin 2\psi$$

The above two equations can be combined into one.

$$V/B = \sigma_s + (\pi+1)s_u - s_u\sin^{-1}(H/Bs_u) + s_u\sqrt{1-\left(\frac{H}{Bs_u}\right)^2}$$

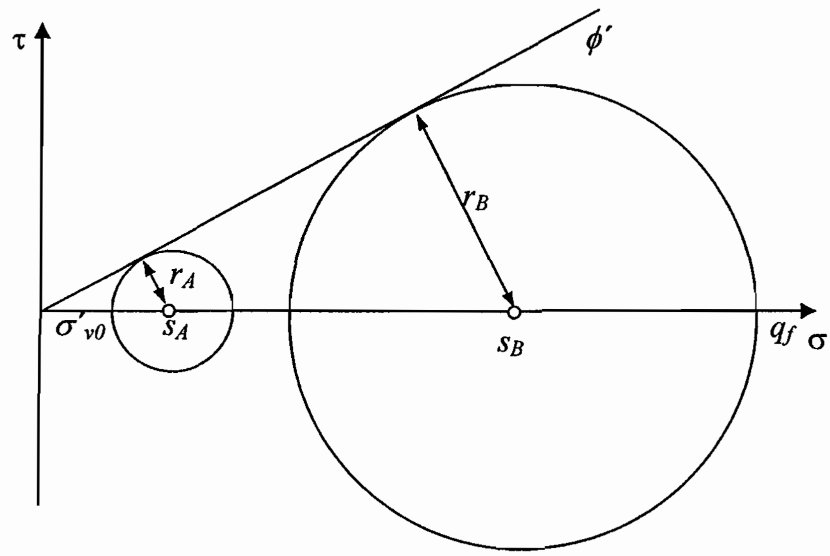
Hence,

$$\frac{V}{Bs_u} = 1+\pi+\frac{\sigma_s}{s_u} - \sin^{-1}\left(\frac{H}{Bs_u}\right) + \sqrt{1-\left(\frac{H}{Bs_u}\right)^2}$$

(iv) The foundation slips when $H = Bs_u$. Substituting this into the above equation,

$$V = (1+\frac{\pi}{2}+\frac{\sigma_s}{s_u})Bs_u$$

3. (a) (i) and (ii)



(iii) For Zone A $s_A - r_A = \sigma'_{v0}$ and $r_A/s_A = \sin\phi'$

For Zone B $s_B + r_B = q_f$ and $r_B/s_B = \sin\phi'$

Using the stress fan concept,

$$s_B/s_A = \exp\,(2 \times (\pi/2) \times \tan\phi')$$

Combining the above equations,

$$\frac{q_f}{\sigma'_{v0}} = N_q = \frac{(1+\sin\phi')}{(1-\sin\phi')}\exp(\pi\tan\phi')$$

(b) When $\phi' = 30$ degrees, $N_q = 18.4$ and $N_\gamma = 2(18.40 - 1)\tan 30 = 20.1$.

The pressure applied by the weight of the concrete block is 24 x 3 = 72 kPa. This needs to be reduced from the ultimate bearing capacity pressure to obtain the vertical load applied on to the block.

(i) The equivalent surcharge load $\sigma'_{v0} = 16 \times 2 = 32$ kN/m.
$q_f = (1/2) \times 20.1 \times 16 \times 3 + 18.4 \times 32 = 1071.2$ kN/m.
The effective unit weight is 16 kNm^{-3}.
The vertical load that can be applied on top of the block is
V = (1071.2 – 72) x 3 = <u>3.00 MN.</u>

(ii) The equivalent surcharge load $\sigma'_{v0} = 16 \times 2 = 32$ kN/m.

The effective unit weight is 19 – 10 = 9 kNm^{-3}.
q_f = (1/2) x 20.1 x 9 x 3 + 18.4 x 32 = 860.2 kN/m.
The vertical load that can be applied on top of the block is
V = (860.2 – 72) x 3 = <u>2.36 MN.</u>

(iii) The equivalent surcharge load σ'_{v0} = (19-10) x 2 = 18 kN/m.
The effective unit weight is 19 – 10 = 9 kNm^{-3}.
q_f = (1/2) x 20.1 x 9 x 3 + 18.4 x 18 = 602.6 kN/m.
The vertical load that can be applied on top of the block (considering that there is uplift water pressure at the base = 30 kPa) is
V = (602.6 – 72 + 30) x 3 = <u>1.68 MN.</u>

4 a) "Normal compression" involves breakage of clay agglomerates and collapse of the granular skeleton, leading to blocking of pore channels with fragments, and to severe drop in permeability k. Likewise, fragments will prop flexible clay platelets and E will increase strongly. On the other hand, bulk density $\rho = \rho_w (G_s + e)/(1+e)$ so even if e changes from 3 to 1, ρ only changes from $1.4\,\rho_w$ to $1.85\,\rho_w$ — not so dramatic.

$$c_v = \frac{kE_o}{\gamma_w} = \frac{k}{\gamma_w} \cdot \frac{v\sigma'}{\lambda} = \frac{k}{\gamma_w} \cdot \frac{v}{\lambda} \cdot \gamma' z_{\text{average}}$$

Because γ' does not change much, use γ' at z rather than γ' at $z/2$ or some even better average.

Now $\gamma' = \gamma_w (G_s - 1)/(1+e) = \gamma_w (G_s - 1)/v$

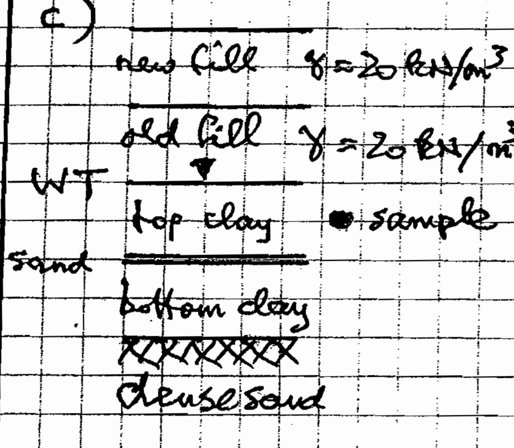
$$\therefore\ c_v = \frac{k}{\gamma_w} \cdot \frac{v}{\lambda} \cdot \frac{\gamma_w (G_s - 1)}{v} \cdot z = (G_s - 1)\frac{kz}{\lambda}$$

b) If $k = k_1/z$ in m, s units then:

$c_v = (G_s - 1)\, k_1/\lambda$ which for London Clay gives:

$c_v = 1.75\, k_1/0.161 = 10.9\, k_1\ \text{m}^2/\text{s}$ = constant!

c)



For the sample, at the centre of the top clay:

$\sigma'_{vi} = 20 \times 2 + \gamma'_{clay} \times 1$

But for London Clay

$v = 2.759 + 0.099 - 0.161 \ln \sigma'_{vi}$

$v = 2.858 - 0.161 \ln \sigma'_{vi}$

So if $\gamma'_{clay} = 10\ \text{kN/m}^3$:

$\sigma'_{v,i} = 50$ kPa, $v = 2.228$ so better $\gamma' = 9.8 \times 1.75/2.28$

i.e. better $\gamma' = 7.7\ \text{kN/m}^3$ and $\sigma'_{vi} \doteqdot 48$ kPa

New $\Delta\sigma$ due to the new fill is 40 kPa.
So the oedometer test with σ' going 50 kPa → 90 kPa was almost perfectly arranged.

Average stress is 70 kPa, so we expect $E_o \doteqdot \frac{2.28 \times 70}{0.161}$ or $E_o \simeq 991$ kPa.

Now in the test, we saw $\varepsilon_v = \Delta h_{ult}/h = 0{\cdot}85/20$

So $E_o = \frac{\Delta\sigma'}{\varepsilon_v} = \frac{40}{0{\cdot}85} \times 20 = \underline{941 \text{ kPa}}$

This is near enough, confirming similar to London Clay.

Now at 45 minutes = 2700 s, $R_v = \frac{\rho}{\rho_{ult}} = \frac{0{\cdot}5}{0{\cdot}85}$

So $R_v = 0{\cdot}59$ implying $T_v \simeq 0{\cdot}27$

i.e. $\frac{c_v \cdot 2700}{0{\cdot}01^2} = 0{\cdot}27$ with double drainage

$\therefore \underline{c_v = 10^{-8} \text{ m}^2/\text{s}}$

Now consider the lower clay.

$\sigma'_{v,i} \simeq 48 + 2 \times 8 \simeq 64 \text{ kPa}$

$\Delta\sigma$ remains 40 kPa so $\sigma'_{v,\text{average}} \simeq 84$ kPa.

Since $E_o \propto \sigma'$, we choose $E_o \simeq 941 \times 84/70$ kPa

So for lower clay, $E_o \approx 1129$ kPa.

However, c_v will remain the same, from (6)

For both layers:

$\rho_{ult} = 2\text{m} \times \frac{40}{941} + 2\text{m} \times \frac{40}{1129} = 0.085 + 0.071 \text{ m}$

So $\underline{\rho_{ult} = 156 \text{ mm}}$

Now 3 months $\simeq 3 \times \frac{365}{12} \times 24 \times 3600 \text{ s} \simeq 7.9 \times 10^6 \text{ s}$

at which $T_v \simeq \frac{10^{-8} \times 7.9 \times 10^6}{1^2}$ assuming the central sand layer is an effective drain. i.e. $T_v \simeq 0.079$

Then $R_v \simeq 0{\cdot}32$ and $\rho \doteqdot \underline{50 \text{ mm @ 3 months}}$

And at 12 months, $T_v \simeq 4 \times 0{\cdot}079 \simeq 0{\cdot}316$ so $R_v \simeq 0{\cdot}63$

and $\underline{\rho \doteqdot 98 \text{ mm @ 12 months}}$

3D1 SOIL MECHANICS
ANSWERS

1. (a) $e_{max} = 0.849$, $e_{min} = 0.500$, $e_{comp.} = 0.620$, $I_D = 66\%$
   (b)(i) –
   (ii) 27 m

2. (a) (i), (ii) –
   (b) (i), (ii), (iii) –
   (iv) $H = Bs_u$, $V = (1+\pi/2+2\sigma_s/s_u)Bs_u$

3. (a) (i), (ii), (iii) –
   (b) (i) 3.00 MN
   (ii) 2.36 MN
   (iii) 1.68 MN

4. (a) $(G_s - 1)kz/\lambda$
   (b) $10.9k_1$ m^2/s
   (c) Settlement at 3 months = 50 mm
   Settlement at 12 months = 98 mm
   Ultimate Settlement = 156 mm