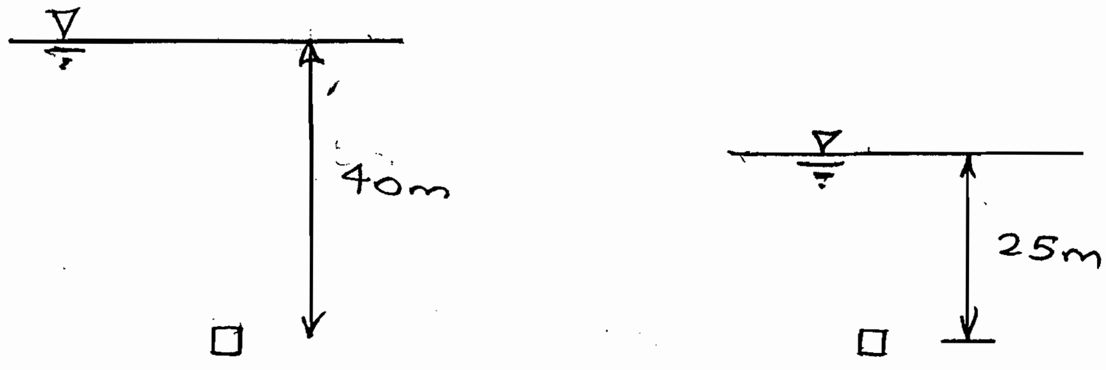
PART IIA 2007
3D2 Geotechnical engineering
Prof M Bolton

Question 1

(a)



(i) (ii)

From the Data Book

$$K_{o,nc} = 1 - \sin\phi_{crit}$$

$$\phi_{crit} = 25° \Rightarrow K_o = 1 - \sin 25° = \underline{\underline{0.58}}$$

(i): normal consolidation

$$\sigma_v = 40 \times 18 = 720 \text{ kN/m}^2$$

$$u = 40 \times 10 = 400 \text{ kN/m}^2$$

$$\therefore \sigma_v' = \sigma_v - u = 720 - 400 = \underline{320 \text{ kN/m}^2}$$

$$\sigma_h' = K_{o,nc} \cdot \sigma_v' = 0.58 \times 320 = 186 \text{ kN/m}^2$$

$$q = \sigma_v - \sigma_h = \sigma_v' - \sigma_h' = 320 - 186 = \underline{\underline{134 \text{ kN/m}}}$$

$$p' = \frac{1}{3}(\sigma_v + 2\sigma_h') = \frac{1}{3}(320 + 2 \times 186) = \underline{\underline{231 \text{ kN/m}}}$$

(ii): overconsolidated, due to unloading

$$\sigma_v = 25 \times 18 = 450 \text{ kN/m}^2$$

$$u = 25 \times 10 = 250 \text{ kN/m}^3$$

$$\therefore \sigma_v' = \sigma_v - u = 450 - 250 = \underline{\underline{200 \text{ kN/m}^2}}$$

Overconsolidation ratio $n = \dfrac{\sigma_v'{}_{max}}{\sigma_v'}$

$$\therefore n = \frac{320}{200} = 1.6$$

From Data Book

$$K_o = K_{o,nc}'\left[1 + \frac{\cancel{(n-1)}(n_{max}^{\alpha} - 1)}{\cancel{(n_{max}-1)}}\right]$$

$n = n_{max} = 1.6$ $\qquad \alpha = 1.2 \sin \phi_{crit}$
$= 1.2 \sin 25° = 0.51$

$$\therefore K_o = 0.58\left[1 + 1.6^{0.51} - 1\right] = \underline{\underline{0.74}}$$

[8]

(b) For stage (i) q and p' values are as shown earlier. Plotted as point A' on graph.

For stage (ii),

$q = \sigma_v - \sigma_h = \sigma_v' - \sigma_h'$ $\qquad \sigma_v' = 200 \text{ kN/m}^2$

$\sigma_h' = K_o \sigma_v' = 0.74 \times 200 = 148 \text{ kN/m}^2$

$\therefore q = 200 - 148 = \underline{\underline{52 \text{ kN/m}^2}}$

$p' = \frac{1}{3}(\sigma_v' + 2\sigma_h') = \frac{1}{3}(200 + 2 \times 148)$

$= \underline{\underline{165 \text{ kN/m}^2}}$

(165, 52) plotted as point B' on graph.

[4]

(c) Both specimens set up in triaxial test with same total and effective stresses as the in-situ stresses in the ground immediately prior to sampling.

i.e. $u_0 = 25 \times 10 = 250 \text{ kN/m}^2$

$\therefore \quad p = p' + u_0 = 165 + 250 = 415 \text{ kN/m}^2$

$q = 52 \text{ kN/m}^2$

total stresses plotted as point B (415, 52)

<u>Same</u> effective stress path for each test.

Critical state shear strength at failure

$(c_u) = 70 \text{ kN/m}^2$. $\quad \therefore q_f = 2c_u = \underline{\underline{140 \text{ kN/m}^2}}$

$q_f = M p'_f \qquad M = 1.0 \Rightarrow p'_f = \underline{\underline{140 \text{ kN/m}^2}}$

<u>Test 1</u> Increasing σ_a, σ_r = constant

$q = \sigma_a - \sigma_r \qquad \Delta q = \Delta\sigma_a - \Delta\sigma_r$

$p = \frac{1}{3}\sigma_a + \frac{2}{3}\sigma_r \qquad \Delta q = \frac{1}{3}\Delta\sigma_a + \frac{2}{3}\Delta\sigma_r$

$\Delta\sigma_r = 0 \qquad \therefore \frac{\Delta q}{\Delta p} = \frac{\Delta\sigma_a}{\frac{1}{3}\Delta\sigma_a} = 3$

$\Delta q = 140 - 52 = 88 \text{ kN/m}^2$

point C_1: (444, 140)

$\therefore \Delta p = \frac{1}{3}\Delta q = 88/3 = 29 \text{ kN/m}^2$

final total stress $p = 415 + 29 = 444$ kN/

final pore pressure $u_1 = p - p_f = 444 - 140$

$= \underline{\underline{304 \text{ kN/m}^2}}$

<u>Test 2</u> Decreasing σ_r, σ_a = constant $\Rightarrow \frac{dq}{dp} = -3/2$

$\Delta q = 88 \text{ kN/m}^2$ (as for Test 1)

point C_2: (356, 140)

$\Delta p = -\frac{2}{3} \times 88 = -59 \text{ kN/m}^2$

$\therefore$ final $p = 415 - 59 = 356 \text{ kN/m}^2$

$\therefore u_1 = 356 - 140 = \underline{216 \text{ kN/m}^2}$

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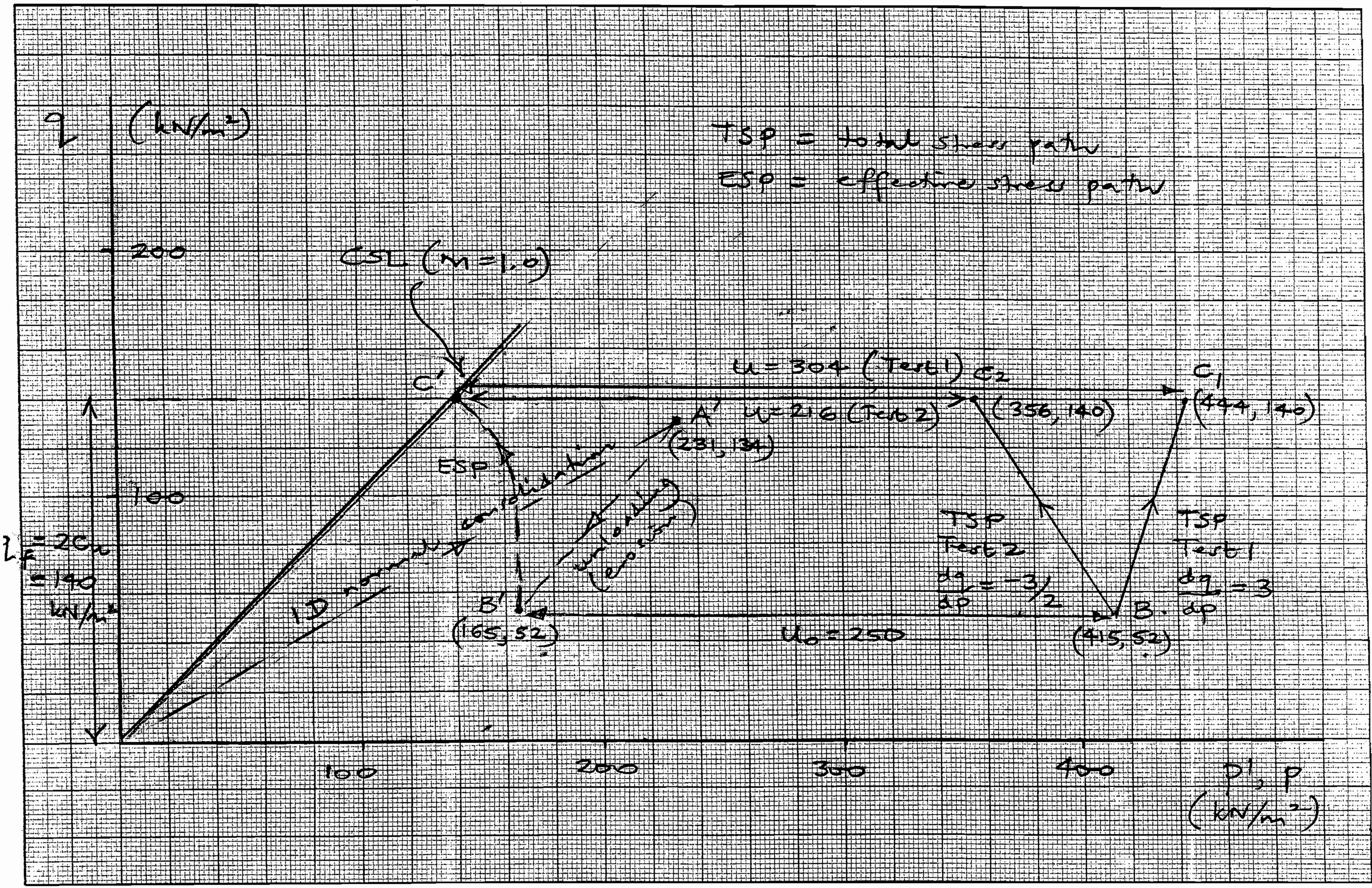
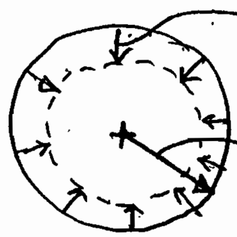

TSP = total stress path
ESP = effective stress path
q (kN/m²)
200
100
$q_f = 2c_u = 140$ kN/m²
CSL (M = 1.0)
C'
ESP
1D normal consolidation
unloading (erosion)
A' (231, 134)
B' (165, 52)
u = 304 (Test 1)
u = 216 (Test 2)
C₂ (356, 140)
C₁ (444, 140)
TSP Test 2 dq/dp = −3/2
TSP Test 1 dq/dp = 3
B (415, 52)
u₀ = 250
100
200
300
400
p', p (kN/m²)

Question 2 Solutions

(a) Assuming axisymmetric conditions, tunnel construction can be viewed as a contraction of a cylindrical cavity of tunnel radius r_c

- uniform radial contraction $= \rho_c$
- tunnel lining radial stress $= \sigma_c$
- initial total stress in ground $= \sigma_o$



$$\delta\sigma_c = \sigma_o - \sigma_c$$

From Data Book:

$$\delta\sigma_c = c_u\left(1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right)$$

For small strains $\quad \dfrac{\delta A}{A} \simeq \dfrac{2\pi r_c \rho_c}{\pi r_c^2} = \dfrac{2\rho_c}{r_c}$

$c_u = 100 \text{ kN/m}^2$

$G = 10 \text{ MN/m}^2$ $\quad \therefore \dfrac{G}{c_u} = \dfrac{10\times 10^3}{10^2} = 100$

$\rho_c = 20\text{mm}$

$r_c = 3\text{m}$ $\quad \therefore \dfrac{\delta A}{A} = \dfrac{2\rho_c}{r_c} = \dfrac{2\times 20}{3000} = 0.133$

$$\therefore \delta\sigma_c = 100\,(1 + \ln 100 + \ln 0.133)$$
$$= 100\,(1 + 4.61 - 2.01) = 360 \text{ kN/m}^2$$

Tunnel axis at depth 25m, $\gamma = 20 \text{ kN/m}^3$

$$\therefore \sigma_o = 25 \times 20 = 500 \text{ kN/m}^2$$

$$\therefore \sigma_c = \sigma_o - \delta\sigma_c = 500 - 360 = \underline{\underline{140 \text{ kN/m}^2}}$$

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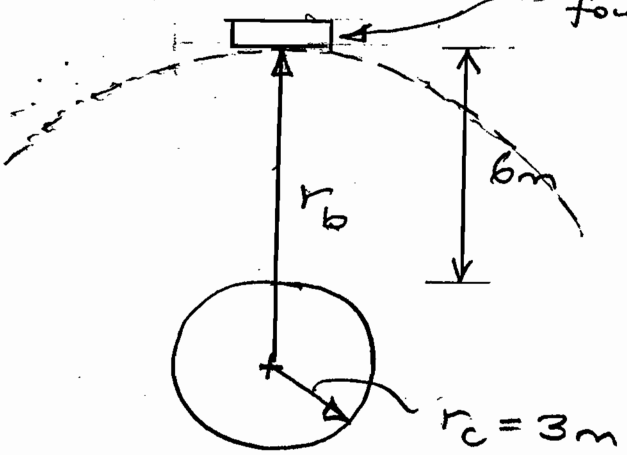
(b) For undrained conditions, constant volume is maintained

and for axisymmetric conditions

$$2\pi r \rho = \text{constant}$$

ρ = radial displacement at radius r



$$r_c \rho_c = r_b \rho_b$$

$$\therefore \quad \rho_b = \frac{r_c}{r_b} \cdot \rho_c$$

$$= \frac{3}{9} \cdot 20$$

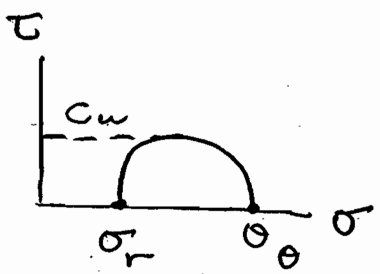
$r_b = 3 + 6 = 9m$ $\qquad = \underline{\underline{6.7 \text{ mm}}}$

[4]

(c) In elastic zone

$$\left.\begin{aligned} \sigma_r &= \sigma_0 - \frac{G\,\delta A}{\pi r^2} \\ \sigma_\theta &= \sigma_0 + \frac{G\,\delta A}{\pi r^2} \end{aligned}\right\} \therefore \ \sigma_\theta - \sigma_r = 2G\frac{\delta A}{\pi r^2}$$

In plastic zone, soil has failed at shear stress $\tau = c_u = \dfrac{\sigma_\theta - \sigma_r}{2}$



$\therefore$ at boundary of elastic and plastic zones $(r = r_p)$

$$2c_u = \frac{2G\,\delta A}{\pi r_p^2}$$

$$\therefore \quad r_p = \left(\frac{G\,\delta A}{\pi c_u}\right)^{1/2}$$

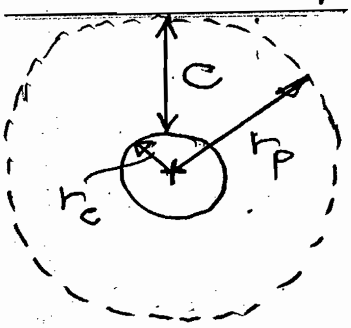
$$\frac{\delta A}{A} = 0.133 \quad (\text{from before})$$

for small strains $A \simeq \pi r_c^2$

$$\therefore \; r_p = \left(\frac{10 \times 10^3 \times 0.133 \times \cancel{\pi} \times 3^2}{\cancel{\pi} \times 100} \right)^{1/2}$$

$$= 10.94\,\text{m}, \quad \underline{\underline{\text{say } 11\,\text{m}}}$$

level above plastic zone



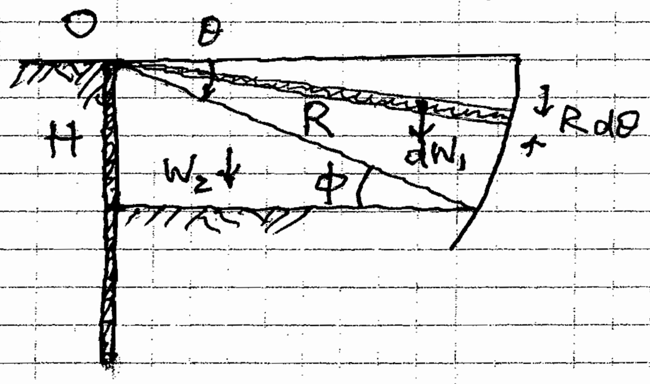
$\therefore$ to be above plastic zone

$$C = r_p - r_c \simeq 11 - 3 = \underline{\underline{8\,\text{m}}}$$

[8]

3. a) We will take moments of weight balanced by the moment of soil resistance.

The unbalanced moment is due to the soil that has been removed. Consider the sector weight W_1 and the triangle W_2.



$$\text{Missing moment} = \int dW_1 \cdot \frac{2}{3}R\cos\theta + W_2 \frac{R}{3}\cos\phi$$

$$= \rho g \int_0^{\phi} \frac{1}{2}R^2 \cdot \frac{2}{3}R\cos\theta \, d\theta + \rho g \frac{1}{2} H \frac{R\cos\phi}{3} R\cos\phi$$

$$= \frac{1}{3}\rho g R^2 \cdot R\sin\phi + \frac{1}{6}\rho g H R^2 (1-\sin^2\phi)$$

$$= \underline{\frac{1}{6}\rho g H R^2 \left(3 - \frac{H^2}{R^2}\right)}$$

If τ is the shear stress on the circle radius R, the resisting moment $= \tau R(\pi - \phi) \cdot R$

$$= \underline{\tau R^2 (\pi - \sin^{-1} H/R)}$$

Equating:

$$\tau = \frac{H\rho g}{6} \frac{\left(3 - \frac{H^2}{R^2}\right)}{\left(\pi - \sin^{-1}\frac{H}{R}\right)}$$

When $H \ll R$, H^2/R^2 and $\sin^{-1} H/R$ both $\to 0$, so

$$\tau \to \frac{H\rho g}{2\pi} = \frac{H\rho g}{6} \times 1.05$$

When $H/R = 0.5$ we get $\tau = \frac{H\rho g}{6} \times 1.05$

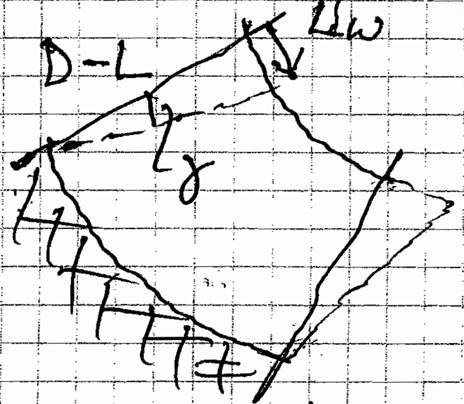
Hence $\tau = \frac{H\rho g}{6} \pm 5\%$ for H/R 0 to 0.5.

b) $\left(\frac{\tau}{c_u}\right) = \left(\frac{\gamma}{\gamma_u}\right)^b$ prior to failure at $\tau = c_u$.

Then $\gamma = \gamma_u \left(\frac{\tau}{c_u}\right)^{\frac{1}{b}}$

So $\gamma \simeq \gamma_u \left[\frac{H\rho g}{6c_u}\right]^{\frac{1}{b}}$

This no longer varies with R. So there is constant shear strain for the circular shear zone from R=D (fixed) to R=L at which the rotation will be $L\delta w$.



$\therefore \delta w = \frac{(D-L)}{L}\gamma$

So $\delta w = \gamma_u \left(\frac{D-L}{L}\right)\left[\frac{H\rho g}{6c_u}\right]^{\frac{1}{b}}$

c) Substituting values:

$$0.002 = 0.1 \cdot \frac{20}{20}\left[\frac{H\, 1.65 \times 9.81}{6 \times 40}\right]^2$$

$$\therefore 0.02 = (0.0674H)^2$$

$$\therefore H \ngtr 2.1\text{ m}$$

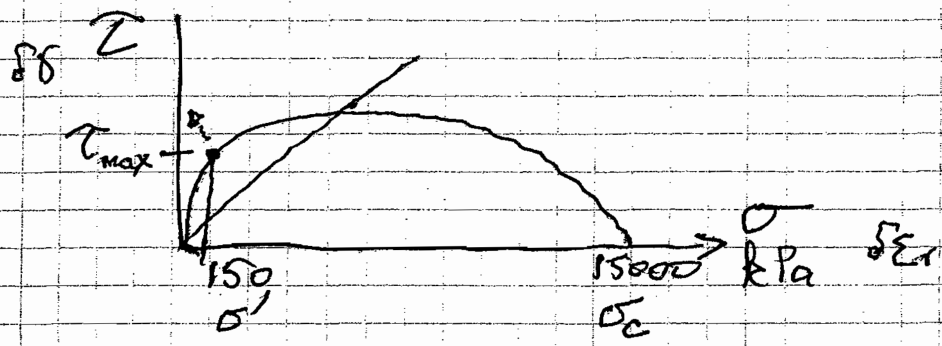
d) The soil would completely fail when $\gamma = \gamma_u$.

So $FoS = \frac{6c_u}{H\rho g}$

and $\delta w = \gamma_u \frac{(D-L)}{L}\left[FoS\right]^{-\frac{1}{b}}$

So FoS is useless <u>on its own</u> in controlling movement.

4. a) A: For dense sand take the initial plastic yield stress $\sigma_c = 15000$ kPa.



The equation of the yield surface gives:

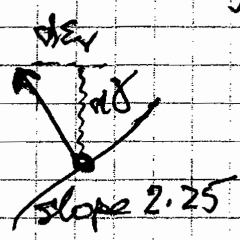
$$\frac{\tau_{max}}{\sigma'} = \tan 32° \ln\frac{15000}{150} = 2.878$$

So $\underline{\tau_{max} = 432 \text{ kPa}}$ and $\phi_{max} = 70.8°$

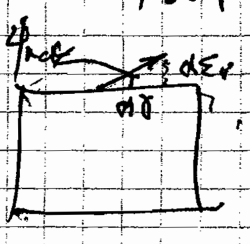
At that yield point

$$\frac{d\tau}{d\sigma} = -\tan\phi_{crit} + \ln 100 \tan\phi_{crit} = 2.25$$

Using normality



$$\frac{d\varepsilon_v}{d\gamma} = 2.25$$



So $\tan\psi_{max} = 2.25$

so $\underline{\psi_{max} = 66.1°}$

B: For minimum voids ratio, $I_D = 1$

Then $I_R = I_D I_C - 1 = \ln 100 - 1 = 3.6$

So in plane strain, $\phi_{max} - 32° = 0.8\,\psi_{max} = 18°$

$\therefore$ $\underline{\phi_{max} = 50°,\ \psi_{max} = 22.5° \text{ and } \tau_{max} = 179 \text{ kPa}}$

4. cont.

A is a theory based on the normality rule for dilatant soils, which is found to be inaccurate.
B is based on actual data from many sands.

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b) The plastic work equation for SSA is:

$$\sigma \, \delta\varepsilon_v^p + \tau \, \delta\gamma^p = \mu_{crit} \, \sigma \, \delta\gamma^p$$

$$\therefore \quad \frac{\tau}{\sigma} = \mu_{crit} - \frac{\delta\varepsilon_v^p}{\delta\gamma^p}$$

So if there is any $\delta\varepsilon_v^p, \delta\gamma^p$ due to cycling <u>above</u> the critical stress ratio, $\tau/\sigma > \mu_{crit}$, then

$$\frac{\delta\varepsilon_v^p}{\delta\gamma^p} < 0$$ indicating dilation & softening

But if there is plastic strain due to cycling <u>below</u> the critical stress ratio, $\tau/\sigma < \mu_{crit}$, then

$$\frac{\delta\varepsilon_v^p}{\delta\gamma^p} > 0$$ indicating compaction + hardening

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c) (i) Very dense sand — see (a) B.
If $\phi_{max} = 50°$, $\phi_{mob} = 0.8 \times 50 = 40°$
This will be mobilised cyclically every time a storm wave hits. But $40° > 32°$ so each cycle will result in irreversible softening and the foundation will eventually fail when ϕ_{max} has itself reduced to $40°$. Need to restrict ϕ_{mob} to less than ϕ_{crit}, eg $30°$.

4cat c) (ii) Medium to loose sand with piles carrying the vertical load of the bridge. Even strong earthquakes only shake laterally at about 0·3g (say) so on horizontal planes we might get $\tan\phi_{mob} = 0{\cdot}3$, $\phi_{mob} \doteq 17°$.

Then $\phi_{mob} < \phi_{crit}$ so if there is any plastic strain due to cycling, the sand will try to contract. But it is saturated. The short term response will be to strongly reduce effective stresses by the generation of excess pore water pressures. This is known as cyclic liquefaction: all stiffness is temporarily lost. The sand will then be unable to prevent the slender piles from buckling, and the piers will collapse. Need to specify fewer piles of higher EI sufficient to prevent buckling when the sand liquefies.