3D3 STRUCTURAL MATERIALS AND D[ESIGN] PT IIA 2007

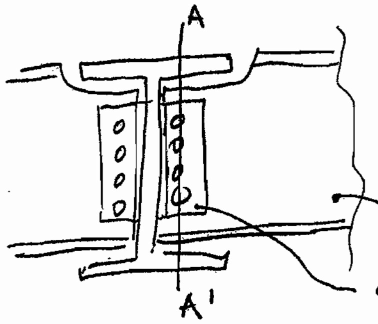
PART IIA 2007
3D3 Structural materials and design
Dr J Lees

07 Q1 1/3

1 (a) bookwork

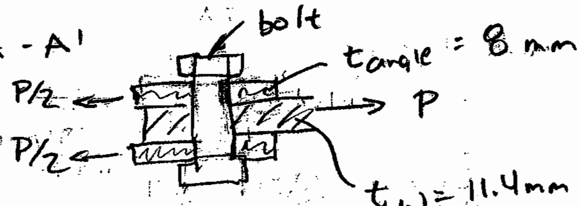
- use of lower bound theory of plasticity
- load path must be in equilibrium with the applied loads and nowhere exceed yield
- an example would be slab → secondary beam → primary beam → column
- need ductile materials, no local failures
- a good load path tries to make good use of materials and is typically a) short b) simple to understand and c) does not put awkward stress resultants on to materials (so no torque in beams unless unavoidable)

b)(i)



20mm ϕ bolts in 22mm ϕ holes
ULS shear = 375 MPa
ULS bearing = 550 MPa
457×191×98 UB t_w = 11.4 mm
90×90×8 angle

- Check shear - section at A - A'
  2 shear planes



$$V_{max} = \pi \frac{20^2}{4} \times \underset{\text{shear planes}}{\underset{\uparrow}{2}} \times \underset{\text{bolts at A-A'}}{\underset{\uparrow}{4}} \times 375 = 942\text{kN}$$

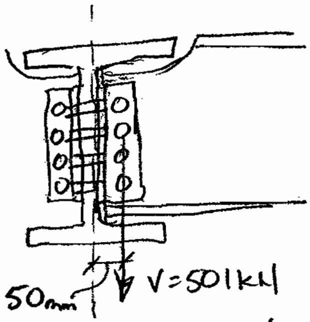
- check bearing - web of secondary beam most critical

$$V_{max} = \underset{\text{bolt}\,\phi}{\underset{\uparrow}{20}} \times \underset{t_w}{\underset{\uparrow}{11.4}} \times \underset{\text{bolts}}{\underset{\uparrow}{4}} \times 550 = 501\text{ kN}$$

∴ bearing controls, maximum force = 501 kN

1b(i) continued

find design forces on bolts through primary web

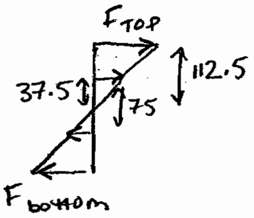


consider one side of beam

ULS tension = 500 MPa

M along line of primary bolts $= 501 \times 50\text{mm} = 25.1\text{ kNm}$, $V_{applied} = 501\text{kN}$

· consider linear distribution of forces



$$F_i = C \times \frac{d_i}{\Sigma d_i^2}$$

$$F_{TOP} = 25.1\text{ kNm} \times 1000 \times \frac{112.5}{2(112.5)^2 + 2(37.5)^2}$$

$$= 100.4\text{ kN}$$

maximum tension in top bolts (1 either side)

$$\therefore \sigma_{top\ applied} = \frac{100.4\text{ kN} \times 10^3}{\underset{\text{bolt on either side}}{2}} \Big/ \pi 20^2/4 = 159.8\text{ MPa}$$

shear in each bolt $= 501\text{kN}/8 = 62.6\text{kN}$

$$\therefore \tau_{applied} = 62.6\text{kN} \times 10^3 / \pi 20^2/4 = 199.3\text{ MPa}$$

check interaction

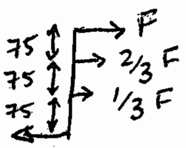
$$\left(\frac{\sigma}{\sigma_{max}}\right) + \left(\frac{\tau}{\tau_{max}}\right) \leq 1$$

$$\left(\frac{159.8}{500}\right) + \left(\frac{199.3}{375}\right) = 0.32 + 0.53 = 0.85 \leq 1$$

connection is adequate

Note - could also assume different force distribution.

e.g.



would give $F \times 75\left(3 + \tfrac{2}{3} \times 2 + \tfrac{1}{3} \times 1\right) = 25.1 \times 1000$

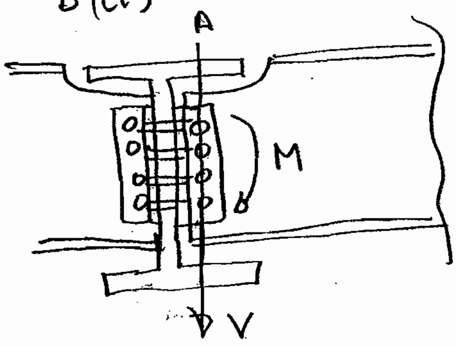
$\therefore F = 71.7\text{ kN}$

which leads to $\sigma_{top} = \dfrac{71.7 \times 10^3}{2} \Big/ \pi \dfrac{20^2}{4} = 114.1\text{ MPa}$

$$\frac{114.1}{500} + \frac{199.3}{375} = 0.76 \leq 1$$

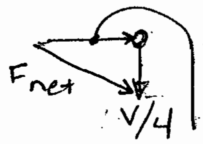
1b(i) continued
other checks - need to check angle OK for shear, bending, prying, also check edge & end distances

b(ii)



$V = 350\,kN$
$M = ?$

forces in secondary beam at top bolt level



reaction from moment $= F_{top}$

assuming linear distribution of forces

$$F_{TOP} = M \times \frac{112.5}{2(112.5)^2 + 2(37.5)^2} = M\,(0.004)$$

$$F_{net} = \sqrt{(V/4)^2 + (F_{TOP})^2} = \sqrt{\left(\frac{350}{4}\right)^2 + (M \times 0.004)^2}$$

as bearing controls (see pt b(i))

$F_{allow} = 20 \times 11.4 \times 550 = 125.4\ kN = F_{NET}$

$\therefore\ (125.4)^2 = (87.5)^2 + (M \times 0.004)^2 \Rightarrow M = 22.5\ kNm$

design forces in primary

$M = 22.5 + 350 \times 0.05 = 40\ kNm$

new tension in top bolt $= \frac{100.4}{2} \times \frac{40}{25.1} = 80\ kN$

$\left(\text{or } \frac{71.7}{2} \times \frac{40}{25.1} = 57.1\ kN\right)$

shear $= \frac{V}{8} = \frac{350}{8} = 43.8\ kN$

2a) (i)

762×267×134 UB steel

plastic design strength $\frac{w_{uls} L^2}{16} = M_p = \frac{\sigma_y Z_p}{\gamma_m}$

$$\therefore L = \sqrt{\frac{\sigma_y Z_p 16}{w_{uls} \gamma_m}} \quad (1)$$

deflection $\frac{w_{sls} L^4}{384 EI} \le \frac{L}{F}$

$$\therefore L \le \sqrt[3]{\frac{384 EI}{F \cdot w_{sls}}}$$

self weight = 134 kg × 9.8 m/s² = 1313 N/m

w_{uls} = 1.31 × 1.4 + 20 × 1.6 = 33.83 kN/m

w_{sls} (working) = 20 kN/m

σ_y = 220 MPa, $Z_p = 4644 \times 10^3 \text{ mm}^3$, $I = 150700 \times 10^4 \text{ mm}^4$

E_s = 210000 MPa, F = 300, $\gamma_m = 1.05$

(1) $L = \sqrt{\frac{220 \times 4644 \times 10^3 \times 16}{33.83 \times 1.05}} = 21452 \text{ mm} = 21.5 \text{ m}$

(2) $L \le \sqrt[3]{\frac{384 \times 210000 \times 150700 \times 10^4}{300 \times 20}} \le 27259 \text{ mm} = 27.3 \text{ m}$

∴ strength controls, L_{max} = 21.5 m

a)(ii)

500 mm deep × 200 mm wide oak section

elastic design strength $\frac{w_{uls} L^2}{12} = \frac{\sigma_t Z_e}{\gamma_m}$

$$\therefore L = \sqrt{\frac{\sigma_t Z_e 12}{w_{uls} \gamma_m}} \quad (1)'$$

deflection (as in part (i))

2(a) ii) continued

self wt = $.2 \times .5 \times .8 \times 1000 \times 9.8 = 784\ N/m$

$w_{ULS} = 1.4 \times 0.784 + 1.6 \times 20 = 33.1\ kN/m$

$\sigma_f = 60\ MPa$, $Z_e = \frac{bd^2}{6} = \frac{200 \times 500^2}{6} = 8.33 \times 10^6\ mm^3$

$E = 22000\ MPa$, $I = \frac{bd^3}{12} = 2.083 \times 10^9\ mm^4$, $\gamma_m = 1.05$

①' $L = \sqrt{\frac{60 \times 8.33 \times 10^6 \times 12}{33.1 \times 1.05}} = 13137 = 13.1\ m$

②' $L \leq \sqrt[3]{\frac{384 \times 22000 \times 2.083 \times 10^9}{300 \times 20}} \leq 14314 = 14.3\ m$

$\therefore$ strength controls $L_{max} = 13.1\ m$

b) i)

check shear force at ULS $V = w_{ULS} \cdot L_{max}/2$
$= 33.8 \times 21.5/2 = 363.4\ kN$

shear stress on web (ave) $= \frac{363.4 \times 1000}{719 \times 12} = 42\ MPa$

$D = 750 - 2 \times 15.5 = 719$
$t_w = 12$

allowable stress = $0.6\sigma_y = 0.6 \times \frac{210}{1.05} = 120\ MPa > 42\ MPa$

so not close to yield

check buckling in thin webs

$q_{cr} = (0.75 + (\overset{0}{d/a})^2)(1000\, t/d)^2$

$= 0.75\,(1000 \times 12/719)^2 = 209\ MPa > 42\ MPa$

so not significant

buckling could occur locally in flange also
possibity of web buckling at support

2(b)(ii)

lateral torsional buckling (neglecting J, assume uniform moment)

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_{yy}\frac{\pi^2}{L^2}EC_w} \quad \text{where } C_w = \frac{D^2 I_{yy}}{4}$$

$$= \frac{\pi}{L}\sqrt{EI_{yy}\frac{\pi^2}{L^2}E.D^2\frac{I_{yy}}{4}}$$

$$= \frac{\pi^2}{L^2}EI_{yy}\frac{D}{2}$$

$D = 750 - 15.5 = 734.5$ mm, $I_{yy} = 4788 \times 10^4$ mm^4,
$E = 210\,000$ MPa

$$M_{cr} = \frac{\pi^2}{L^2} \times 210000 \times 4788 \times 10^4 \times \frac{734.5}{2}$$

$$= \frac{3.644 \times 10^{16}\ \text{N.mm}^3}{L^2} = \frac{36440\ \text{kN.m}^3}{L^2}$$

$$M_p = \sigma_y Z_p = \frac{220}{1.05} \times 4644 \times 10^3 = 973\,\text{kNm}$$

$$\lambda_{LT} = \left(\frac{M_p}{M_{cr}}\right)^{1/2}$$

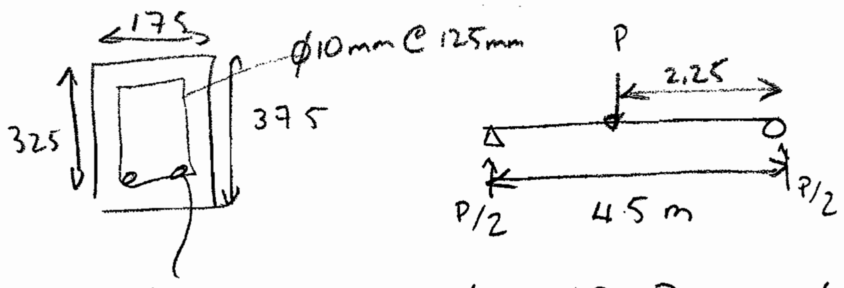
for $\chi_{LT} = 0.9$, $\lambda_{LT} \approx 0.57$

$$\therefore M_{cr} = \frac{M_p}{\lambda_{LT}^2} = \frac{973}{(0.57)^2} = 2995\ \text{kNm}$$

$$L^2 = \frac{36440}{M_{cr}} = \frac{36440}{2995} = 12.17, \quad L = 3.49\ \text{m}$$

need a spacing of ~ 3.5m (quite close)

3.



2 Ø 20mm

$A_s = \frac{\pi \times 20^2}{4} \times 2$

$= 628.3\ mm^2$

$f_y = 460\ MPa$ $\quad f_{yv} = 250\ MPa$

$f_{cyl} = 30\ MPa$ $\quad f_{cu} = 37\ MPa$

$\gamma_c = 1.5$ $\quad \gamma_s = 1.15$ $\quad \varepsilon_{cu} = 0.0035$

$\varepsilon_y = 0.002$

a) under-reinforced

$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} b d} = \frac{1.5 \times 628.3 \times 460}{1.15 \times 0.6 \times 37 \times 175 \times 325} = 0.299$

$\therefore x = 97.2\ mm$
$< d/2$
under-reinf ✓

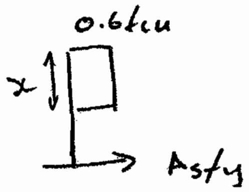
$M_u = A_s f_y\, d\,(1 - 0.5x/d)/\gamma_s$

$= 628.3 \times 460 \times 325\,(1 - 0.5 \times 0.299)/1.15$

$= 69.5\ kNm$

$M_{a\,max} = \frac{P \times 2.25}{2}$ $\quad \therefore P_{max} = \frac{69.5\,kN.m \times 2}{2.25\,m} = 61.7\ kN$

b) assume bottom steel still yields and is in tension



$N = 0.6 f_{cu} \cdot b \cdot x/\gamma_c - A_s f_y/\gamma_s$

$x = \frac{N + A_s f_y/\gamma_s}{0.6 f_{cu} b/\gamma_c} = \frac{200000 + 628.3 \times 460/1.15}{0.6 \times 37 \times 175/1.5} = 174\ mm$

check steel has yielded

$\varepsilon_s = \frac{0.0035}{174} \times (325 - 174) = 0.003 > 0.002$ ✓

take moments about centreline

$M_u = \frac{A_s f_y}{\gamma_s}(d - h/2) + \frac{0.6 f_{cu}}{\gamma_c} b x\,(h/2 - x/2)$

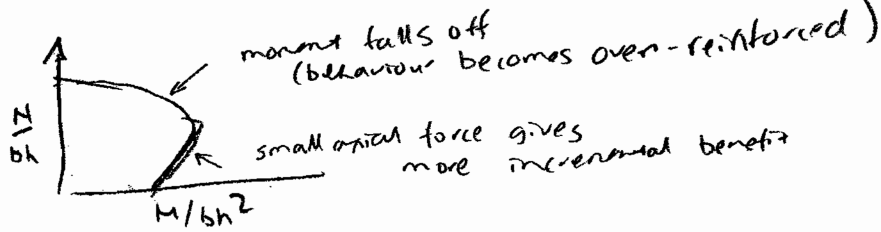
$= \frac{628.3 \times 460}{1.15}(325 - 375/2) + \frac{0.6 \times 37 \times 175 \times 174}{1.5}\left(\frac{375}{2} - \frac{174}{2}\right)$

$= 34.6 \times 10^6 + 45.3 \times 10^6 = 79.8 \times 10^6\ kNm$

3 b) cont'd

$P_{max} = 71.0\text{ kN} > 61.7\text{ kN}$

The allowable value of P has increased. The axial forces acts to relieve the tension in the steel and allows for a higher moment to be applied before the steel yields. Hence an increasing axial load will act to increase the allowable moment capacity up to the point where the section becomes balanced. Thereafter, although there is a region where the axial force remains beneficial in terms of the moment capacity, the benefit decreases until a point where the behaviour is dominated by the compressive capacity of the beam



c) 10 mm Ø @ 125 spacings $\quad f_{yv} = 250\text{ MPa} \quad A_{sw} = \frac{\pi 10^2}{4} \times 2$ (2 legs) $= 157\text{ mm}^2$

check stirrup capacity

$V_{Rd,s} = A_{sw} f_{yv} (0.9d)(\cot\theta) / (s\gamma_s)$

$= 157 \times 250 \times 0.9 \times 325 \times (\cot 21.8) / 125 \times 1.15$

$= 200\text{ kN}$

check concrete capacity $\quad f_{ck} = 30\text{ MPa}$

$f_{c,max} = 0.6\,(1 - 30/250) \times 30/1.5 = 10.6\text{ MPa}$

$V_{Rd,max} = f_{c,max}\,(b_w\, 0.9d) / (\cot\theta + \tan\theta)$

$= 10.6 \times 175 \times 0.9 \times 325 / (\cot 21.8 + \tan 21.8)$

$= 187\text{ kN}$

5 min

$V_{allow} = 187\text{ kN}$

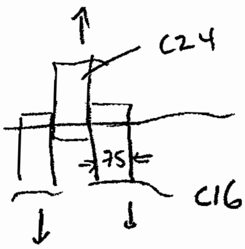
SDS 2007
Q3 3/3

3. Limits are placed on the allowable strut angle to avoid local failure modes and inefficient use of either the steel or the concrete. A designer seeks to choose the minimum angle that would satisfy the applied shear force. By choosing the minimum, the designer reduces the amount of steel required.

4a) book work

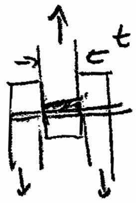
- anisotropic materials have different material properties in different directions
- examples include timber + composites.
- need to be concerned about load direction vs fibre direction
- stress concentrations a concern, shear, bearing
- joining etc., splitting, shear stiffness

b)



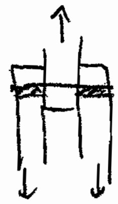
$k_{mod} = 0.9$
$\gamma_m = 1.3$

i) mode 1 failure in C24



$R = f_{h,1,d} \cdot t_1 \cdot d$

mode 2 failure in C16



$R = 2 f_{h,2,d} \cdot t_2 \cdot d$

ii)

$$f_{h,1,d} \cdot t_1 \cdot \cancel{d} = 2 f_{h,2,d} \cdot t_2 \cdot \cancel{d}$$

$$t_1 = \frac{2 f_{h,2,d} \cdot t_2}{f_{h,1,d}}$$

$\rho_{k\,C24} = 350\ kg/m^3$
$\rho_{k\,C16} = 310\ kg/m^3$

$$f_{h,2,d} = 0.082(1-0.01d)\rho_{k2}$$

for 10mm bolt $= 0.0738\,\rho_{k2}$

$$\therefore\ t_1 = \frac{2\rho_{k2} \cdot t_2}{\rho_{k1}} = 2 \cdot \frac{310}{350} \cdot 75 = 133\ mm$$

use mode 1 → mode 2 should give same thing

$$R_{max} = 0.0738 \times 350 \times \frac{0.9}{1.3} \times 133 \times 10 = 23.8\ kN$$

4 (c)(i)

C16

indoor, permanent loading → $k_{mod} = 0.60$
(class 1)

$k_{ls} = 1$, $k_h = 1$, $k_{crit} = 1$, $\gamma_m = 1.3$

C16, $f_{v,k} = 1.8$ MPa, $f_{m,k} = 16$ MPa

· shear - $V = 10$ kN

$$f_{v,d} = \frac{k_{mod}\, k_{ls}\, f_{v,k}}{\gamma_m} = \frac{0.6 \times 1 \times 1.8}{1.3} = 0.83 \text{ MPa}$$

$$f_{v,d} = \frac{1.5V}{bh} \quad \text{(rectangular)}$$

$$h = \frac{1.5 \times 10000}{75 \times 0.83} = 241 \text{ mm}$$

· flexure · $M = 6$ kNm

$$f_{m,d} = k_{mod}\, k_h\, k_{crit}\, k_{ls}\, f_{m,k} / \gamma_m$$
$$= 0.6 \times 1 \times 1 \times 1 \times 16 / 1.3$$
$$= 7.38 \text{ MPa}$$

$$M = \sigma Z_e = f_{m,d} \frac{bh^2}{6}$$

$$h^2 = \frac{6M}{f_{m,d}\, b} = \frac{6 \times 6 \times 1000 \times 1000}{7.38 \times 75}$$

$$h = 255 \text{ mm}$$

∴ flexure controls need height of 255 mm

$h/b = 255/75 = 3.4$ so k_{crit} likely to be less than 1 but not significantly so.

(ii) by notching beam you reduce overall height of structure

- however, have also reduced section locally which may be problematic in ~~particular~~ in terms of shear capacity
- need to think about bearing at support
- stress in corner?

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Paper 3D3 Structural Materials and Design

Answers

1. (b) (i) secondary beam V_{max} (bearing) = 501 kN, V_{max} (shear) = 942 kN
so bearing controls and maximum force is 501 kN
primary beam - $V_{applied}$ = 62.6 kN, $T_{applied}$ (top bolt) = 35.9 kN
connection is adequate
(ii) M_{max} = 22.5 kNm, $V_{applied}$ = 43.8 kN, $T_{applied}$ (top bolt) = 57.1 kN

2. (a) (i) L_{max} (strength) = 21.5m, L_{max} (deflection) = 27.3m
$\therefore$ strength controls – L_{max} = 21.5 m
(ii) L_{max} (strength) = 13.1m, L_{max} (deflection) = 14.3m
$\therefore$ strength controls – L_{max} = 13.1 m
(b) (ii) max spacing of restraints ≈ 3.5 m

3. (a) P = 61.7 kN
(b) P = 71.0 kN
(c) $V_{Rd,maxc}$ = 187 kN, $V_{Rd,s}$ = 200 kN $\therefore$ concrete capacity controls – V_{allow} = 187 kN

4. (b) (i) failure in C24 – $R = f_{h,1,d} \cdot t_1 \cdot d$, failure in C16 - $R = 2 f_{h,2,d} \cdot t_2 \cdot d$
(ii) t_1 = 133 mm, R_{max} = 23.7 kN
(c) (i) h_{shear} = 241 mm, $h_{flexure}$ = 255 mm, k_{crit} likely to be < 1 but not significantly so

J.L. June, 2007