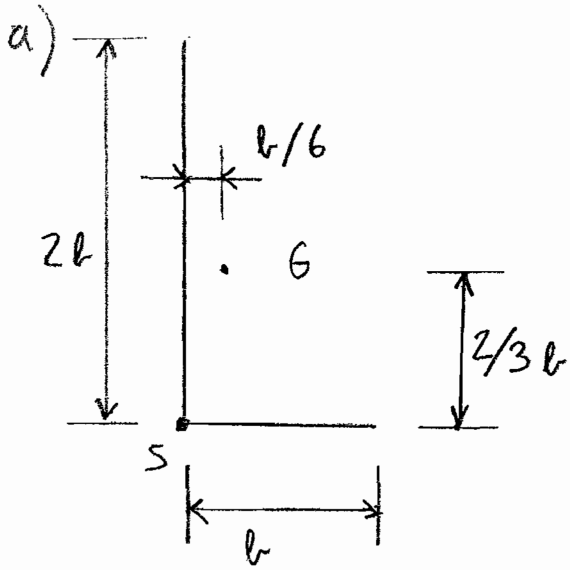
3D4 - Question 1

PART IIA 2007
3D4 Structural analysis and stability
Dr F Cirak

a)



$$I_x = \frac{8}{12} t b^3 + 2bt \frac{1}{9} b^2 + b \cdot t \cdot \frac{4}{9} b^2 = \underline{\frac{4}{3} t b^3}$$

$$I_y = \frac{b^3 t}{12} + b \cdot t \cdot \frac{b^2}{9} + 2 b \cdot t \cdot \frac{b^2}{36} = \underline{\frac{1}{4} t b^3}$$

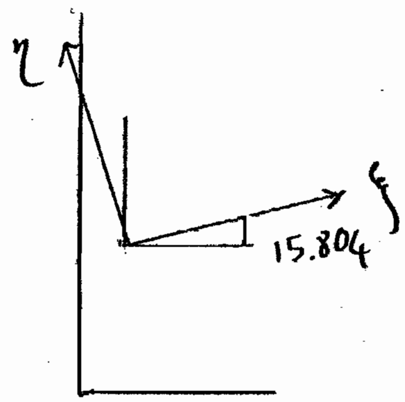
$$I_{xy} = 2bt\left(-\frac{b}{6}\right)\frac{b}{3} + bt\left(-\frac{2}{3}b\right)\left(\frac{b}{3}\right) = \underline{-\frac{1}{3} b^3 t}$$

Principal 2nd moments of area

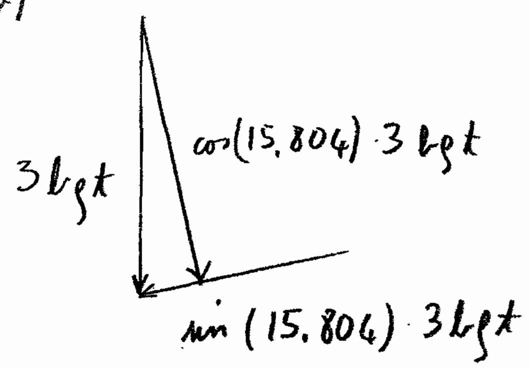
$$I_{\xi/\eta} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

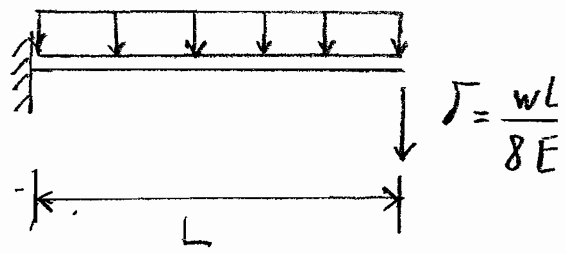
$$\rightarrow \quad I_\xi = \underline{1.428\, b^3 t} \qquad I_\eta = \underline{0.1557\, b^3 t}$$

$$\tan 2\alpha = \frac{I_{xy}}{\left(\frac{I_x - I_y}{2}\right)} \quad \rightarrow \quad \underline{\alpha = -15.804}$$



b)





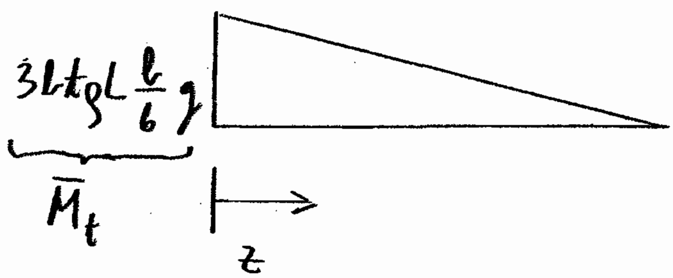
$$\delta_\xi = \frac{1}{E \cdot 0.1557 b^3 t} \; \frac{\sin(15.804)\, 3b\rho g t L^4}{8} = \underline{0.656 \frac{g\rho L^4}{E b^2}}$$

$$\delta_\eta = \frac{1}{E \cdot 1.42768\, b^3 t} \; \frac{\cos(15.804)\, 3b\rho g t L^4}{8} = \underline{0.2527 \frac{g\rho L^4}{E b^2}}$$

$$\delta = \sqrt{\delta_\xi^2 + \delta_\eta^2} = \underline{0.703 \frac{g\rho L^4}{E b^2}}$$

Twist at the tip

$$G J_t \frac{d\varphi}{dz} = M_t$$



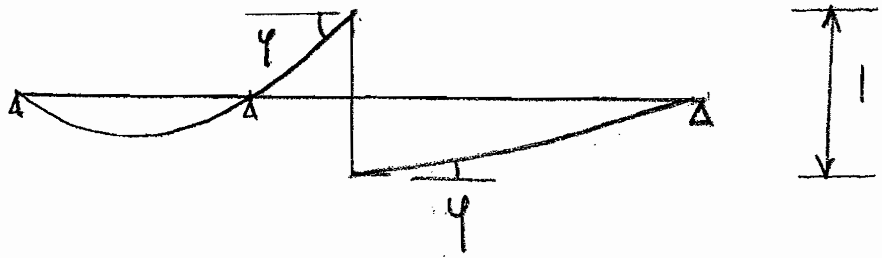
$$J_t = \frac{1}{3} 2bt^3 + \frac{1}{3} bt^3 = bt^3$$

$$G J_t \varphi = \bar{M}_t \left(z - \frac{z^2}{2L}\right) + \cancel{\phi} = 0$$

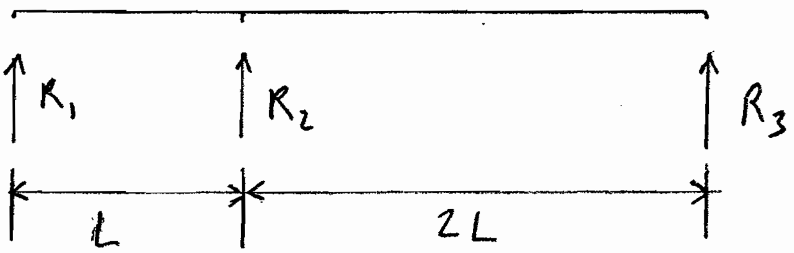
$$G J_t \varphi(L) = \bar{M}_t \frac{L}{2} \longrightarrow \underline{\varphi(L) = \frac{b\rho g L^2}{4 t^2 G}}$$

Question 2

a)



b)



$$-EJ\frac{d^2y}{dz^2} = -R_1 z - R_2\{z-L\}$$

$$-EJ\frac{dy}{dz} = -\frac{R_1 z^2}{2} - R_2\frac{\{z-L\}^2}{2} + A$$

$$-EJy = -\frac{R_1 z^3}{6} - R_2\frac{\{z-L\}^3}{6} + Az + B - EJ\{z - 1.4L$$

Moments around R_3-boundary

$$3LR_1 + R_2 2L = 0 \longrightarrow \underline{R_1 = -\frac{2}{3}R_2}$$

$$z=0;\ y=0 \longrightarrow \underline{B=0}$$

$$z=L;\ y=0 \longrightarrow \underline{A = \frac{R_1 L^2}{6}}$$

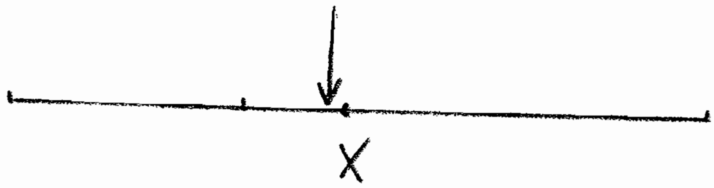
$$z=3L;\ y=0 \longrightarrow \underline{R_1 = -\frac{1}{2}\frac{EJ}{L^3}}$$

$$A = -\frac{1}{12}\frac{EJ}{L}$$

$$y = -\frac{1}{12}\frac{z^3}{L^3} + \frac{1}{8}\frac{\{z-L\}^3}{L^3} + \frac{1}{12}\frac{z}{L} + \{z-1.4L\}^0$$

x	0	$\frac{L}{2}$	L	$1.4-\varepsilon$	$1.4+\varepsilon$	2L	3L
y	0	0.0313	0	-0.104	0.896	0.625	0

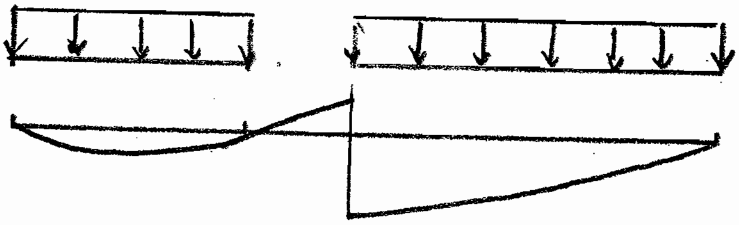
c) (i)



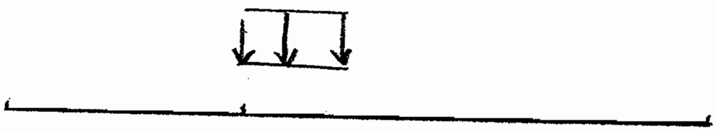
for min

for max

(ii)

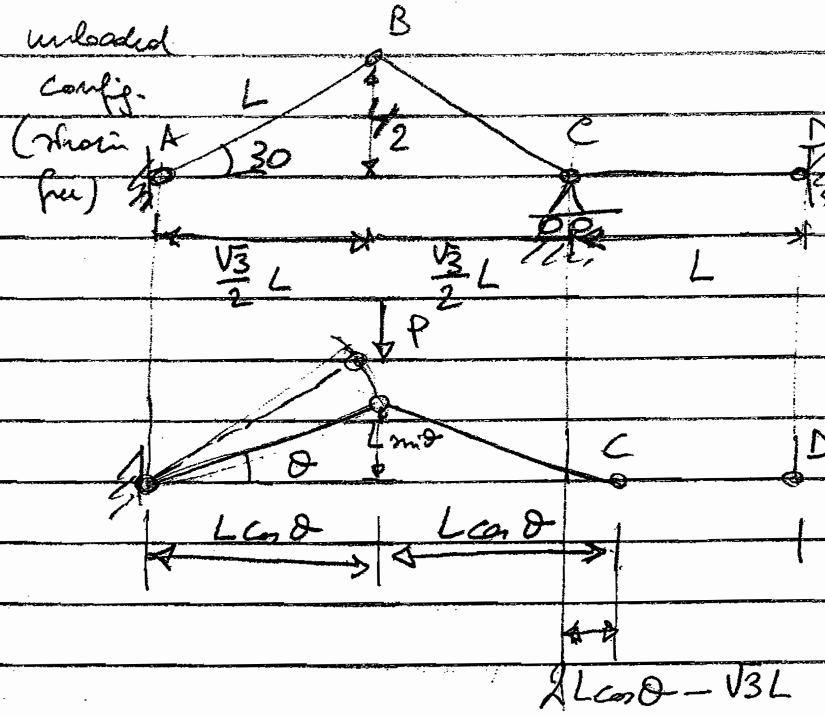


for max



for min

Q.1 (a)



- Bar CD has got shorter by $(2\cos\theta - \sqrt{3})L$. Hence the Strain Energy is $\frac{1}{2}k(2\cos\theta - \sqrt{3})^2L^2$
- Potential Energy is $P\cdot\left(\frac{L}{2} - L\sin\theta\right)$

$$T.P.E. = \Pi(\theta) = S.E - \text{Work done}$$
$$= \frac{1}{2}kL^2(2\cos\theta - \sqrt{3})^2 - PL\left(\frac{1}{2} - \sin\theta\right)$$

(b) Equilibrium required $\frac{d\Pi}{d\theta} = 0$

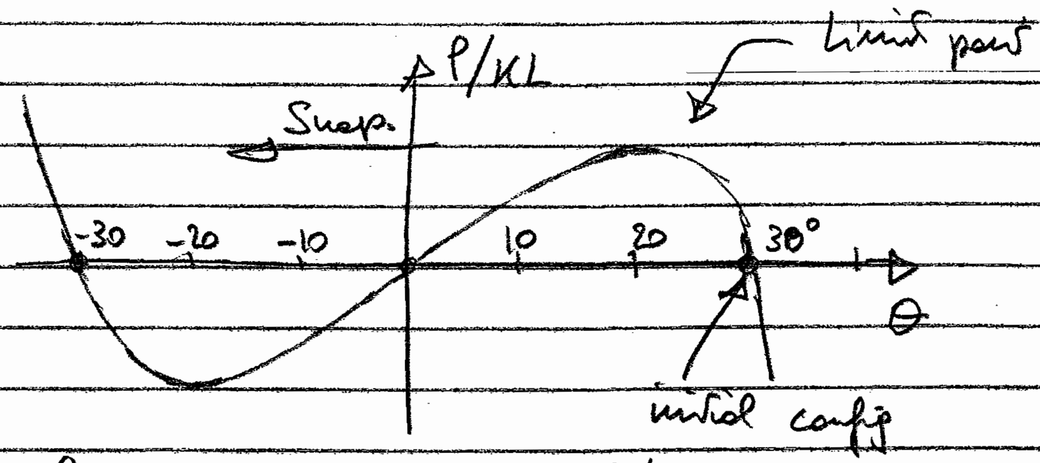
$$\frac{d\Pi}{d\theta} = \frac{1}{\cancel{2}}kL^{\cancel{2}}\cancel{2}(2\cos\theta - \sqrt{3})(-2\sin\theta) - P\cancel{L}(-\cos\theta) = 0$$

$$\therefore \quad -4kL\sin\theta\cos\theta + 2\sqrt{3}kL\sin\theta + P\cos\theta = 0$$

$$\therefore \quad P = \frac{4\sin\theta\cos\theta - 2\sqrt{3}\sin\theta}{\cos\theta}kL$$

$$= \underline{\underline{(4\sin\theta - 2\sqrt{3}\tan\theta)kL}}$$

θ	0	10	20	30°	40
P/KL	0	0.084	0.107	0	−0.336



To find limit point, consider $dP/d\theta = 0$

$$\frac{dP}{d\theta} = \left(4\cos\theta - 2\sqrt{3}\,\frac{1}{\cos^2\theta}\right) KL = 0$$

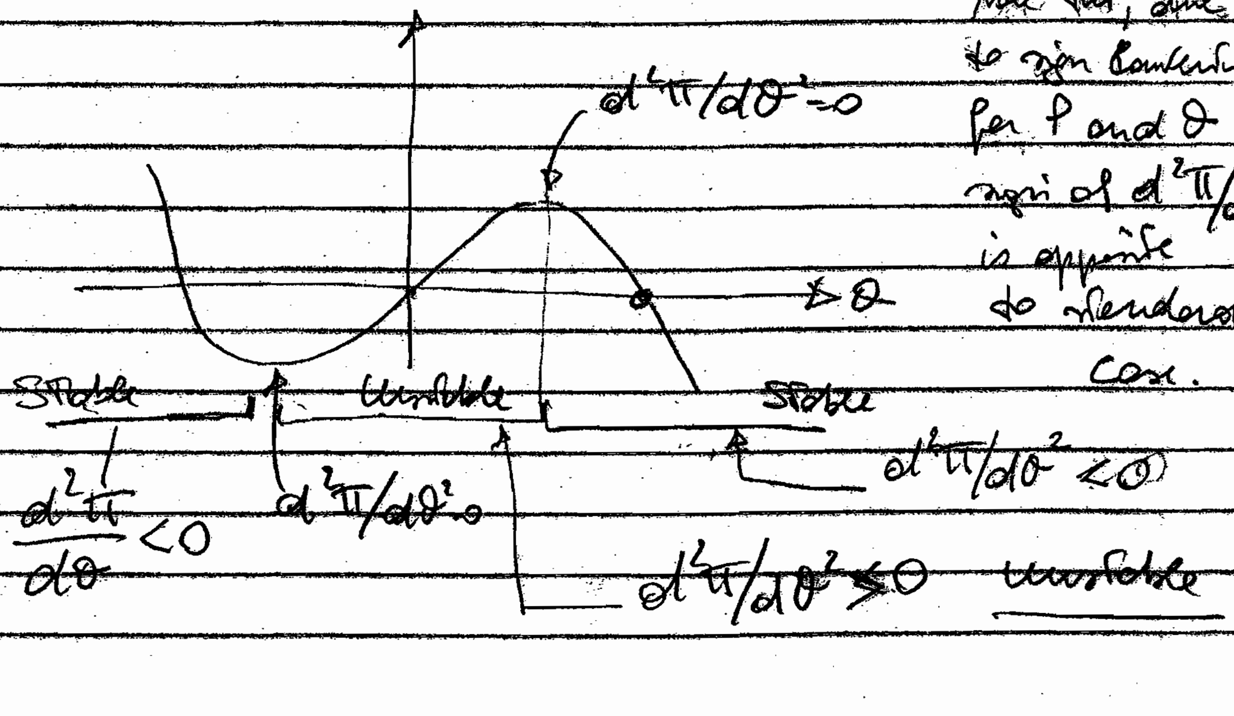
$$\therefore\ 4\cos\theta = 2\sqrt{3}/\cos^2\theta \qquad \therefore\ \cos\theta = \left(\sqrt{3}/2\right)^{1/3}$$

$$\therefore\ \theta = 17.6^\circ$$

and so $P_{crit} = \left(4\sin 17.6^\circ - 2\sqrt{3}\tan 17.6^\circ\right) KL$

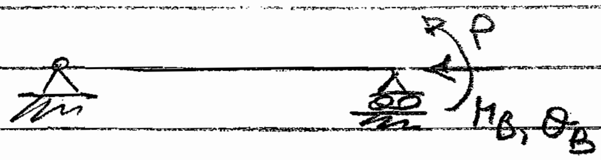
$= \underline{\underline{0.1106\ KL}}$

(c)



Note that, due to sign convention for P and θ sign of $d^2\Pi/d\theta^2$ is opposite to standard case.

Q.2 (a)



Set $M_A = 0$ in $\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{EI}{L}\begin{bmatrix} s & sc \\ sc & s \end{bmatrix}\begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$

first eqn gives: $\cancel{s}\theta_A + \cancel{s}c\ \theta_B = 0 \quad \therefore \quad \theta_A = -c\,\theta_B$

Substitute into 2nd eqn:

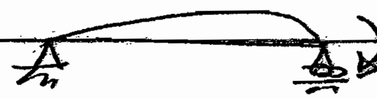
$$M_B = \frac{EI}{L}\left(sc(-c\,\theta_B) + s\,\theta_B\right)$$
$$= \frac{EI}{L}\left(s - sc^2\right)\theta_B$$
$$= \frac{EI}{L}\, s\left(1 - c^2\right)\theta_B$$

For $P = 0$, $s = 4$, $c = 0.50 \quad \therefore \quad M_B = \frac{EI}{L}\, 4(1 - 0.25)\theta_B$

$$= \frac{3EI}{L}\theta_B$$

This result agrees with Structures DB.

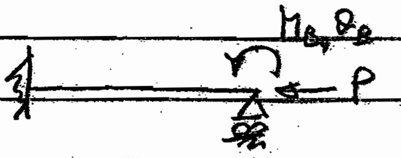
diff. coeff:



$M \quad \theta = ML/3EI$

(b. i)

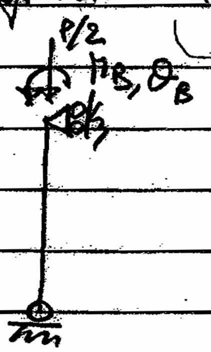
Member AB:



$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{EI}{L}\begin{bmatrix} s & sc \\ sc & s \end{bmatrix}\begin{bmatrix} 0 \\ \theta_B \end{bmatrix}$

from second eqns: $\underbrace{M_B = \frac{EI}{L}\, s\, \theta_B}$

Member BD:

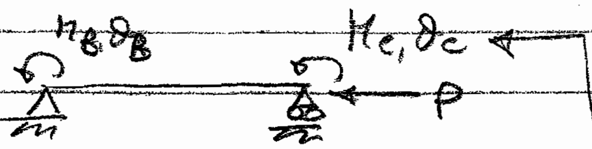


from part (a)

$\boxed{M_B = \frac{EI}{L}\, s(1-c^2)\,\theta_B}$

all referring to $P/2$, hence use notation $s_{1/2}$, $c_{1/2}$

Member BC:



Note that we could set $M_c = 0$ as a boundary condition, but then we would lose the variable θ and so we would end up with 1×1 matrix

$$\begin{bmatrix} M_B \\ M_c \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s & sc \\ sc & s \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix}$$

Adding up all contributions:

$$\begin{bmatrix} M_B \\ M_c \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s + s_{1/2}(1 - c_{1/2}^2) + s & sc \\ sc & s \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix}$$

hence stiffness matrix is:

$$K = \frac{EI}{L} \begin{bmatrix} 2s + s_{1/2}(1 - c_{1/2}^2) & sc \\ sc & s \end{bmatrix}$$

(b.ii) To find the critical value we need to set $\det(\underset{\sim}{K}) = 0$

Neglecting EI/L, we compute values of:

$$2s^2 + s_{1/2}\, s\,(1 - c_{1/2}^2) - s^2 c^2$$

P/P_E	s	c	$s_{1/2}$	$c_{1/2}$	det
2	0.14	24.68	2.47	1.00	−11.90
2.2	−0.52	−7.51	2.28	1.11	−14.43
2.4	−1.3	−3.37	2.09	1.25	−14.28
2.6	−2.25	−2.23	1.89	1.42	−10.73
2.8	−3.44	−1.71	1.68	1.66	−0.79
3.0	−5.03	−1.42	1.46	1.97	20.74

$\frac{P}{P_E} \simeq 2.8$

$$\therefore P_{crit} \cong 2.8 \frac{\pi^2 EI}{L^2}$$