ENGINEERING TRIPOS
PT IIA 2007
3D5 ENVIRONMENTAL ENGINEERING I

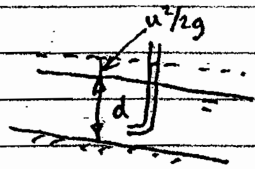
Q1 (a) Distribution %'s : 5, 17, 28, 23, 16, 11 for 3 hr intervals
Cumulative : 5, 22, 50, 73, 89, 100
Plot at ½ times : $t =$ 1.5, 4.5, 7.5, 10.5, 13.5, 16.5 hours
Connect by smooth curve (see graph)
Lag by 1 hour and subtract (i.e. measure difference between S-curves at new "½ time" points, $t = 0.5, 1.5, 2.5, \ldots$ hrs.

From graph, these are:
1, 4, 5, 5, 7, 8, 10, 10, 9, 7, 7, 6, 5, 5, 4, 3, 4, 0

Total volume of runoff from 1 hour rainfall at 15 mm/hr
$= 20\,\text{km}^2 \times 15\,\text{mm} = 20 \times 10^6\,\text{m}^2 \times 15 \times 10^{-3}\,\text{m}$
$= \underline{300 \times 10^3\,\text{m}^3}$

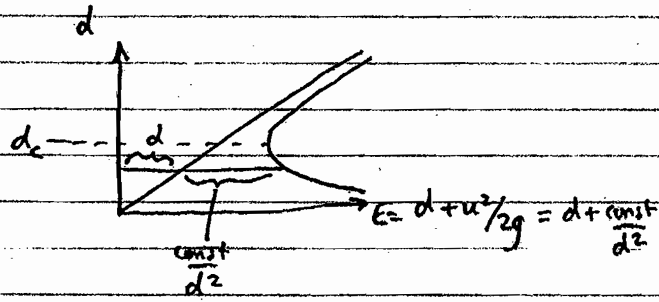
Now, over the hour of peak outflow, 10% of this runs off in that hour,
so $30 \times 10^3\,\text{m}^3$ runs off in the peak hour
so peak flow $= \dfrac{30 \times 10^3\,\text{m}^3}{3600\ \text{seconds}} = \underline{\underline{8.33\,\text{m}^3/\text{s}}}$ (8)

b) Specific energy (rel. to bed) is $E = d + \dfrac{u^2}{2g}$



$= d + \dfrac{1}{2g}\left(\dfrac{q}{d}\right)^2$ for constant q.

$= d + \dfrac{\text{const}}{d^2}$

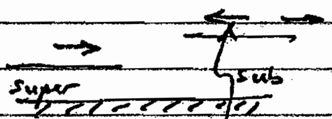


$\dfrac{dE}{dd} = 0$ when $d = d_c = \dfrac{u^2}{g}$

$\therefore\ u = \sqrt{g d_c}$

Also celerity of small amplitude waves $c = \sqrt{gd}$

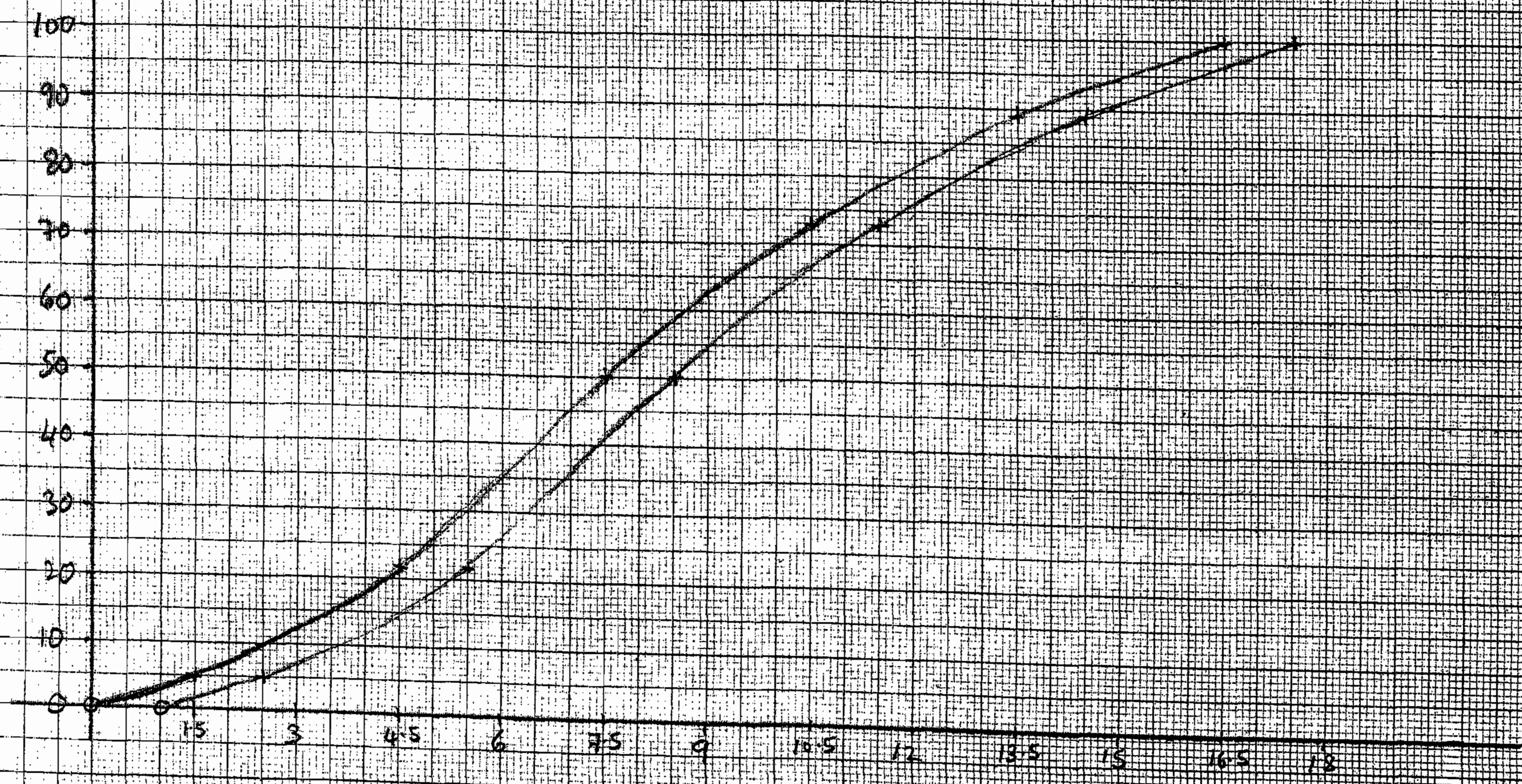
$\therefore\ \dfrac{c}{u} = \sqrt{d/d_c}$ so for $d > d_c$ (sub), $c > u$ $\therefore$ Waves both ways
$d < d_c$ (super) $c < u$ $\therefore$ Waves downstream only



(8)

these cannot propagate upstream into supercritical region $\therefore \rightarrow$ finite amplitude disturbance (hydraulic jump)

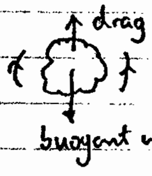
3D5
2007
Q1



t	S	S LAG	Diff.
0.5	1		1
1.5	5	1	4
2.5	10	5	5
3.5	15	10	5
4.5	22	15	7
5.5	30	22	8
6.5	40	30	10
7.5	50	40	10
8.5	59	50	9
9.5	66	59	7
10.5	73	66	7
11.5	79	73	6
12.5	84	79	5
13.5	89	84	5
14.5	93	89	4
15.5	96	93	3
16.5	100	96	4
17.5	100	100	0

$\Sigma = 100$

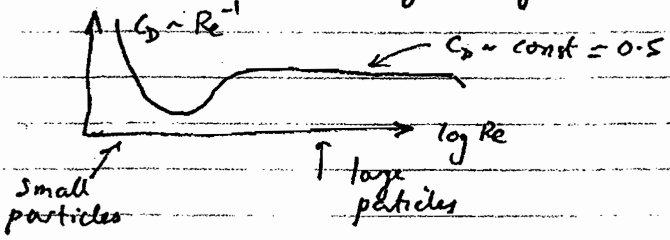
3D5 2007, Q1(c)



At terminal velocity, no acceleration, so no net force

$$\therefore \quad \underbrace{\frac{\pi D^3}{6}(\rho_s-\rho)g}_{\text{submerged weight}} = \underbrace{C_D \tfrac{1}{2}\rho w^2 \frac{\pi D^2}{4}}_{\text{drag}} \qquad (1)$$

but C_D



$Re = \frac{wD}{\nu}$

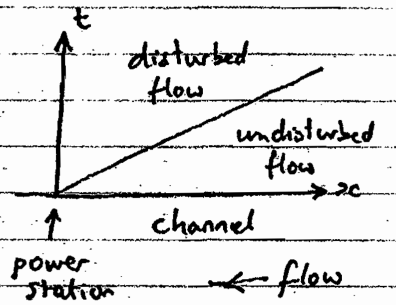
Small particles: D small, w small, → Re small → $C_D \sim 1/Re \propto \frac{1}{wD}$

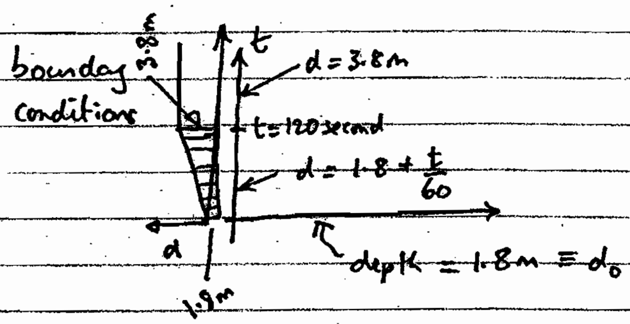
$\therefore \quad D^3 \propto \frac{1}{wD} w^2 D^2$ from (1) → $\underline{w \propto D^2}$ QED.

Large particles: D large, w large, → Re large → $C_D \sim$ const

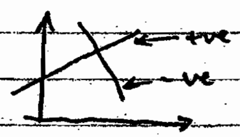
$\therefore \quad D^3 \propto w^2 D^2$ from (1) → $\underline{w \propto D^{1/2}}$ QED. (4)

Q2.





From data sheet: $u - 2c = k_1$ const on any −ve characteristic (1)
$u + 2c = k_2$ const .. .. +ve .. (2)

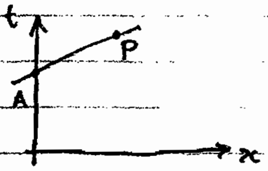


Any −ve characteristic can be traced back to undisturbed region, where $u = u_0$ and $c = c_0 = \sqrt{g d_0}$

$\therefore \quad u - 2c = k_1 = u_0 - 2c_0$ EVERYWHERE in R.H. quadrant.

3DS 2007

Q2 cont'd:



On +ve characteristic AP, for any P on AP

$u_A + 2c_A = u_p + 2c_p = k_2$ from (2)

but $u_p - 2c_p = k_1$ from (1)

Adding → $u_p = (k_1 + k_2)/2$ = const on AP

Subtracting → $c_p = (k_2 - k_1)/4$ = const on AP.

$\therefore$ $\frac{1}{\text{slope of +ve charac.}} = \left(\frac{dx}{dt}\right)_{\text{on AP at P}} = u_p + c_p$ = const + const = const

$\therefore$ +ve characteristics are straight.

So $\left(\frac{dx}{dt}\right)_{\text{on AP at P}} = u_p + c_p = u_p - 2c_p + 3c_p$

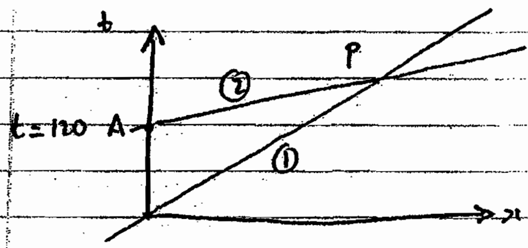
$= u_0 - 2c_0 + 3c_p$ from (1)

$= u_0 - 2\sqrt{gd_0} + 3c_p$

But c_p = const on AP $= c_A = \sqrt{gd_A}$

$\therefore$ $\frac{1}{\text{+ve slope}} = u_0 - 2\sqrt{gd_0} + 3\sqrt{gd_A}$

$\therefore$ slope $= \frac{dt}{dx} = \left[u_0 - 2\sqrt{gd_0} + 3\sqrt{gd_A}\right]^{-1}$ with $d_A = d_0 + \frac{t_0}{60}$ (when $t < 120$) 10



Along (2), c_p is const $\therefore$ d_p is const.

$\therefore$ $d = 3.8$ m everywhere on (2).

Undisturbed boundary (1)

$\frac{dx}{dt} = u_0 + \sqrt{gd_0}$

$u_0 = -0.6$ m/s (NOTE SIGN)

$\sqrt{gd_0} = \sqrt{9.81 \times 1.8} = \sqrt{17.65} = 4.2$ m/s.

$\therefore$ $\frac{dx}{dt} = -0.6 + 4.2 = 3.6$ m/s

$\therefore$ $\left(\frac{dt}{dx}\right)_{1} = \frac{1}{3.6} = \underline{0.278 \text{ s/m}}$

On AP (2)

$d_A = 3.8$m

$\sqrt{gd_A} = 6.1$ m/s

$\frac{dx}{dt} = -0.6 - 2(4.2) + 3(6.1)$

$= 9.3$ m/s

$\therefore$ $\left(\frac{dt}{dx}\right)_{2} = \underline{0.107 \text{ s/m}}$

4

Q2 (cont'd)

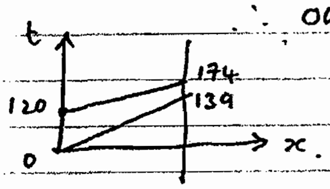
Time required to travel $\cancel{1}\ 0.5$ km is thus $\frac{107}{2}$ seconds $= 53.5$ 8

$\therefore$ Maximum reaches u/s end at time $t = 120 + \cancel{107}\ 53.5 = \cancel{227}\ 174$ seconds. (18)

[Check undisturbed boundary hasn't crossed ① by then.
Undisturbed bdy reached u/s end after $t = \left(\frac{dt}{dx}\right)_{①} \times \cancel{1000}\ 500\,m$

$= 0.278 \times 500 = 139$ seconds.

$\therefore$ OK]



(2)

Q3. $u_* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$ $\quad \tau_0 = \rho g R S \quad \rightarrow \quad u_* = \sqrt{gRS}$

Wide channel $R \approx d \rightarrow u_* = \sqrt{gdS} = \sqrt{9.81(1.2)\,0.00015} = 0.042$ m/s. 2

$\frac{u_* k_s}{\nu} = \frac{(0.042)(0.015)}{10^{-6}} = 630 > 70$ $\therefore$ hydraulically rough. 2

a) use $u = u_*\, 2.5 \log\left(\frac{30.2\,y}{k_s}\right) = \overset{0.042}{u_*}(2.5)\log_e\left(\frac{30.2\,(1.2)}{0.015}\right)$

$= \underline{\underline{0.82\ m/s}}$ 2

(6)

b) $D = 0.09$ mm $= \cancel{0.0000} < 0.5$ mm $\therefore$ small

$\therefore$ Fall velocity $w \simeq (56 \times 10^4)(0.09)^2 \times 10^{-6} \times 1.65 = 0.0075$ m/s.

$\frac{w}{K u_*} = \frac{0.0075}{(0.4)(0.042)} = 0.45$ 2

$\frac{c}{c_a} = \left[\left(\frac{d-y}{y}\right)\left(\frac{a}{d-a}\right)\right]^{w/Ku_*}$ not needed.

Use $\int_b^d C u\, dy = 11.6\, u_*\, C_b\, b \left[I_1 \log_e A d + I_2 \right]$

$A = \frac{30.2}{k_s}$ (rough bed) $= \frac{30.2}{0.015} = 2013\ m^{-1}$

Q3 cont'd.

$\therefore \quad d = 1.2\,m \rightarrow \log Ad = \log(2013 \times 1.2) = 7.79$

$$\frac{b}{d} = \frac{0.015}{1.2} = 0.0125$$

I_1		$w/\kappa u_*$	0.2	0.625	0.45	0.375	0.6
b/d	0.02	0.75	5.003	→	2.83	←	1.527
	0.0125				4.23		
	0.01	0.25	8.892	→	4.69	←	2.174

2

I_2		$w/\kappa u_*$	0.2	0.625	0.45	0.375	0.6
b/d	0.02	0.75	5.960	→	3.914	←	2.687
	0.0125	0.25			6.12		
	0.01		11.20	→	6.859	←	4.254

2

$\rightarrow \quad I_1 \log Ad + I_2 = 4.23(7.79) + 6.12$

$= \underline{\underline{39.1}}$ 2

$\therefore \quad \int_b^d Cu\,dy = 11.6\, u_* C_b\, b\,(39.1)$

$= 11.6\,(0.042)(8.6)(0.015)(39.1)$

$= (0.0628)(39.1) = \underline{\underline{2.46\ kg\,m^{-1}\,s^{-1}}}$ 2 (10)

Q3 (c)

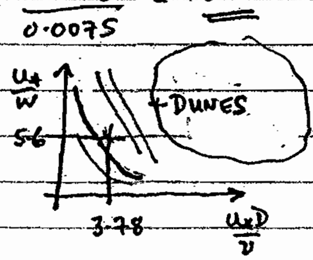
Shields $\dfrac{\tau_0}{(\rho_s - \rho) g D} = \dfrac{(1000)(\cancel{9.81})(1.2)(0.00015)}{(1650)(\cancel{9.81})(0.09 \times 10^{-3})} = \underline{\underline{1.21}}$ Shields. 1

Froude $\dfrac{\bar{u}}{\sqrt{gd}}$ $\quad \bar{u} = 2.5\, u_* \log\left(\dfrac{12.1\,R}{k_s}\right) = (2.5)(0.042)\log\left(\dfrac{12.1 \cdot 1.2}{0.015}\right) = 0.72\ m/s$

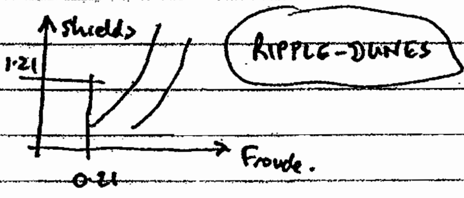
$\dfrac{\bar{u}}{\sqrt{gd}} = \dfrac{0.72}{\sqrt{9.81(1.2)}} = \underline{\underline{0.21}}$ Froude 1

$\dfrac{u_*}{w} = \dfrac{0.042}{0.0075} = \underline{\underline{5.6}}$ $\qquad \dfrac{u_* D}{\nu} = \dfrac{(0.042)(0.09 \times 10^{-3})}{10^{-6}} = 3.78$ 1

Albertson Simons Richardson



Garde + Albertson



1 (4)

University of Cambridge
Department of Engineering

3D5: Environmental Engineering 1

Exam Solution 2007

By the use of variable speed motors the discharge of a single pump can be varied to suit the operating requirements of the systems.

Use $Q_2 = Q_1 (N_2/N_1)$
And $H_2 = H_1 (N_2/N_1)^2$

Plot the system and pump curves to find the duty point

Discharge and Head at the duty point are:

$Q = 17$ l/s and $H = 25.6$ m and $\eta = 80\%$

So input power $P = \rho g HQ / \eta = 1000 \times 9.806 \times 25.6 \times 0.017 / 0.8 = 5.33$ kW (10)

b) At the new duty point on the revised system curve the head is 28.6 m (at the same discharge of 17 l/s)

Need to find a curve that connects all corresponding points through the new duty point at different speeds.

Combining $Q_1/Q_2 = N_1 / N_2$ and $H_1/H_2 = N_1^2 / N_2^2$

So: $N_2 / N_1 = (H_2 / H_1)^{0.5}$
Thus: $Q_2/Q_1 = (H_2 / H_1)^{0.5}$

So $Q_2 = Q_1 (H_2 / H_1)^{0.5} = 17 (H_2/28.6)^{0.5} = 3.18 \sqrt{H_2}$

Use this to plot a curve:

Q l/s	0	5	9.5	14	18.5	23
$H = (Q/3.18)^2$	0	2.47	8.92	19.38	33.84	52.30

Intersection of this curve with the original pump characteristic gives the equivalent operating point of the pump at 750 rpm to the desired new duty point on the incrustated pipeline.

So using $Q_1/Q_2 = N_1 / N_2$ Or $N_2 = N_1 \times Q_2/Q_1 = 750 \times 17/16.4 = 777$ rpm

(Similarly $N_2 = N_1 (H_2/H_1)0.5 = 750 (28.6/26.7)^{0.5} = 776$ rpm

So the pump needs to be run at a speed of 777 rpm to maintain a discharge of 17 l/s on the encrusted pipeline 7

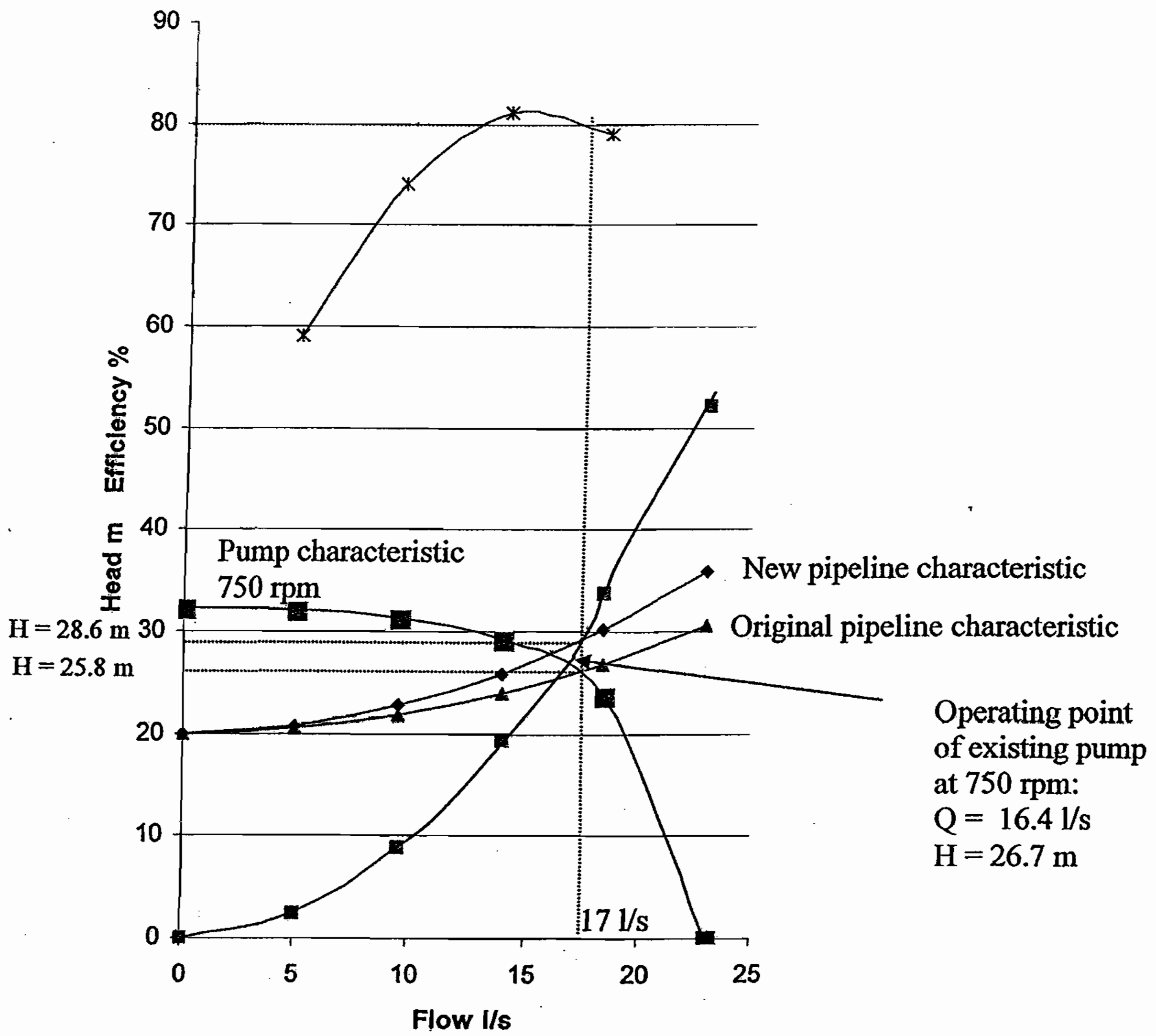
The new Power in is: 3

Power $_{in} = \rho gQH/\eta = 1000 \times 9.806 \times 17 \times$ ~~0.0286~~ $/ 0.8$ 5.96 kW

0.017 28.6

This means an extra power requirement of $5.96 - 5.33 = 0.63$ kW

So extra cost each day of running the pump $= 0.63 \times 24 \times 6\text{p} = 91$ p (10)



90
80
70
60
50
40
30
20
10
0
Head m Efficiency %
Pump characteristic
750 rpm
H = 28.6 m
H = 25.8 m
New pipeline characteristic
Original pipeline characteristic
Operating point
of existing pump
at 750 rpm:
Q = 16.4 l/s
H = 26.7 m
17 l/s
0
5
10
15
20
25
Flow l/s

ENGINEERING TRIPOS PART IIA 2007
3D5: ENVIRONMENTAL ENGINEERING 1

Answers

1. a) 1,4,5,5,7,8,10,10,9,7,7,6,5,5,4,3,4,0 and 8.33 m^3/s

2 174 seconds

3 a) 0.82 m/s
b) 2.46 kg/s per metre width
c) Dunes (Albertson, Simons, Richardson)
Ripple-dunes (Garde and Albertson)

4 a) 17 litres/sec and 5.33 kW
b) 777 rpm and 91p per day