3E8 EXAM CRIBS 2007 - Dynamics

1 (a)

(i)

Variable	Type	Units
Baby Mice per Breeding Pair	Constant	mice/mice/week
births	Flow – **inflow** to YOUNG MICE	mice/week
deaths	Flow – **outflow** from OLD MICE	mice/week
MATURE MICE	Stock	mice
maturing mice	Flow – **outflow** from YOUNG MICE, **inflow** to MATURE MICE	mice/week
mice becoming old	Flow – **outflow** from MATURE MICE, **inflow** to OLD MICE	mice/week
OLD MICE	Stock	mice
Time for Old Mice before Death	Constant	weeks
Time to Get Old	Constant	weeks
Time to Mature	Constant	weeks
Total Mice	Variable	mice
YOUNG MICE	Stock	mice

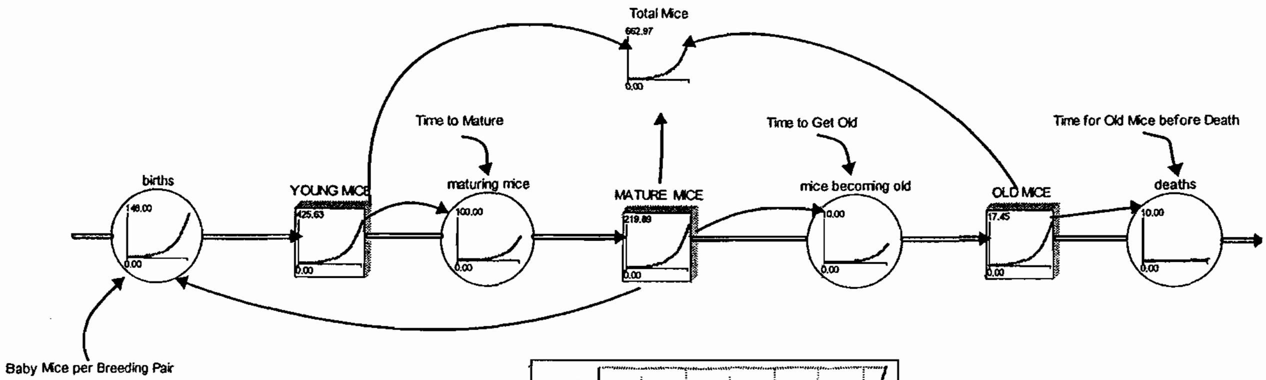
The only elements necessary in a dynamic model are Stocks and Flows; all other variables are for convenience: clearer representation, disaggregating flow variables or setting initial conditions.

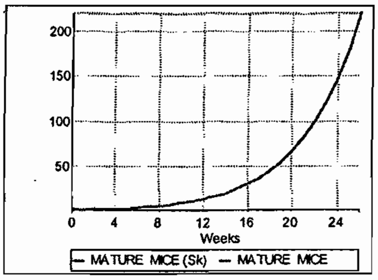
"Baby Mice per Breeding Pair" has unit of mice/mice/week, which makes it a dimensionless proportion per time period.

An old convention is to CAPITALISE stocks, use lower case for rates and Initial Capitals on other variables, which sometimes helps in equation lists but this is not necessary.

"Total Mice" is not a stock but a variable aggregating three stocks. It would be incorrect to model this as an additional stock as this would flout the general rule of mutual exclusivity: mice in the system can only be in one stock at any time.

(ii) Model Diagram





(iii) Model equations:

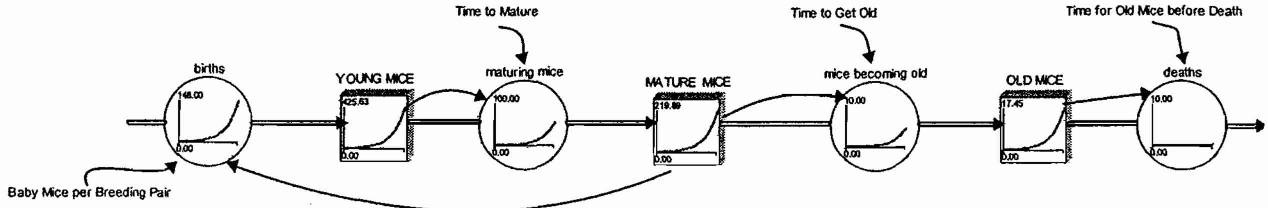
Variable	Equation
Baby Mice per Breeding Pair	70/52
births	(MATURE__MICE /2) * Baby_Mice_per_Breeding_Pair
deaths	OLD_MICE / Time_for_Old_Mice_before_Death
MATURE MICE	init(2) ; + maturing mice * dt - mice becoming old * dt
maturing mice	YOUNG_MICE / Time_to_Mature
mice becoming old	MATURE__MICE / Time_to_Get_Old
OLD MICE	init(0) ; + mice becoming old * dt - deaths * dt
Time for Old Mice before Death	52
Time to Get Old	52
Time to Mature	8
Total Mice	YOUNG_MICE + MATURE__MICE + OLD_MICE
YOUNG MICE	init(0) ; + births * dt - maturing mice * dt

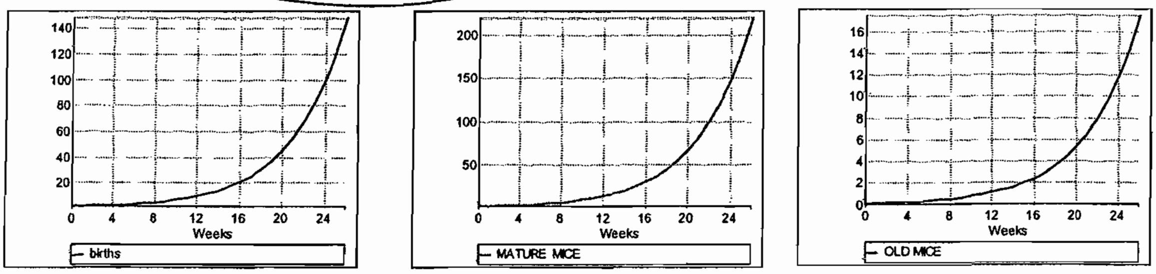
Notes:

1. **Baby Mice per Breeding Pair** Gestation of 20 days is superfluous (but see below for alternative view) since we can simply take the average 7 baby mice per litter x 10 litters per year = 70 mice/year and divide this by 52 to get mice per breeding female per week. A weekly time unit seems most appropriate: a lot of breeding can be done in a year or even a month!
2. **births**: to get the number of breeding pairs, MATURE MICE has to be divided by 2.
3. Outflow from the stocks is simply stock/(appropriate) average time.

4. (iv) Reference Modes for stocks:

Mature Mice





The behaviour of the model is very simple. The two initial mice are a breeding pair who in the first week start to produce baby mice. Although gestation is nearly three weeks, we can generally tolerate a smoothed flow of births since we are dealing with a natural process based on average numbers. An alternative would be to have another stock "mice gestating" out of which flow births exactly 20 days from conception.

Births flow into the Stock of YOUNG MICE. The young mature over an average 8 week period. Again, since this is a natural and averaged process, it is appropriate that "maturing mice" is an infinite first order delay: YOUNG MICE / 8 {weeks}.

"maturing mice" then adds to the stock of MATURE MICE, which is depleted only much more slowly by "mice becoming old" since this too is a first order delay but where the stock, MATURE MICE, is divided through by 52 not 8 weeks: MATURE MICE / 52 {weeks}.

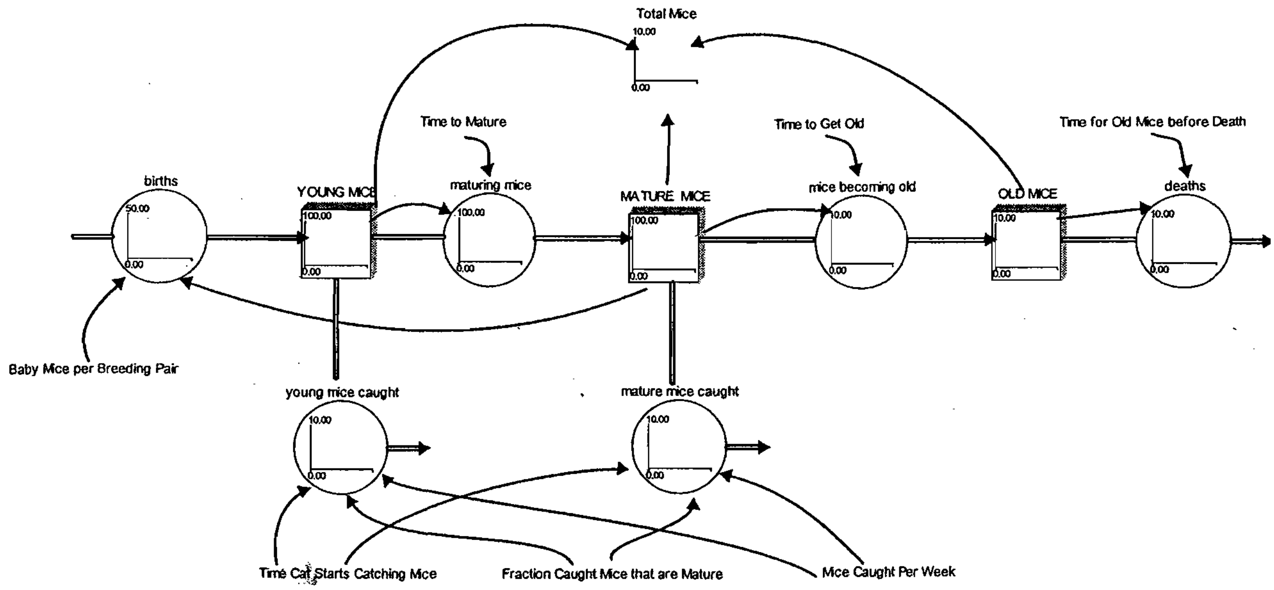
As mature mice become old, there is a further average delay of 52 weeks before OLD MICE die: OLD MICE/ 52 {weeks}. It takes much less time for mice to be born and mature than it does to grow old and die.

We can easily see the reinforcement mechanism that will drive this simple system: maturing mice add to the stock of MATURE MICE that, in the next time step, drives up the number of births; after only 8 weeks, these newly born mice are mature, pairing up and producing their own young. Each pair of mice contributes 1.35 mice (70/52) to the total mice population each week; since old mice play no part in reproduction and merely contribute to the total population of mice, deaths from MATURE MICE could be seen as a second order delay with average delay time of 104 days (the sum of the delay constants "Time for Old Mice before Death" and "Time to Get

Old"). Again, since maturing (8 weeks) happens much more quickly than deaths (104 weeks), we can expect exponential reinforcement.

The reference charts here are drawn to 26 weeks, which is sufficient to show the problem of mice: 350 or so mice in a house would be a real health hazard as well as a destructive nuisance. In fact, going beyond 26 weeks, the numbers of nice rapidly escalate to many thousands. Order of magnitude estimation to six weeks is all that is required here.

1 (b) Adding a cat simply creates more routes for mice to exit the population other than a natural death. Since the number of OLD MICE will always be relatively small, most mice caught will be either young (mostly) or mature. This is observed and reported by the people in the house. . The model might be modified so:



To understand whether the cat will win, the additional outflows need to be subtracted from the stocks of YOUNG MICE and MATURE MICE but only after the 7 weeks it takes for the "kitten" to start catching. The best way to do this is to simulate (computer or manually) the first two stocks of the model (ignoring insignificant deaths & OLD MICE). Constructing a table, it looks like this:

(Weeks)	births	YOUNG MICE	young mice caught	maturing mice	MATURE MICE	mature mice caught
0	1.35	0	0	0	2	0
1	1.35	1.35	0	0.17	2	0
2	1.46	2.52	0	0.32	2.17	0
3	1.67	3.67	0	0.46	2.48	0
4	1.98	4.88	0	0.61	2.94	0
5	2.39	6.25	0	0.78	3.55	0
6	2.92	7.86	0	0.98	4.33	0
7	3.58	9.8	0	1.22	5.32	0
8	4.4	12.15	3.75	1.52	6.54	1.25
9	4.58	11.30	3.75	1.41	6.81	1.25

Although not accurate in a third order model, a time step of 1 can be assumed for ease of calculation. Work from left to right. In Week 0, 2 mice generate 1.35 births [(MATURE MICE/2) * 70/52]. In Week 1, and we can record the 1.35 mice born as YOUNG MICE. The same number are born again (1.35) as we still have two MATURE MICE but 0.17 are flowing out from YOUNG MICE to MATURE MICE: 1.35/8 {weeks}, which are added to MATURE MICE next time period.

The calculations are repeated, adding/subtracting flows to/from stocks and calculating next periods flows from stocks – basic integration and how all System Dynamics software works – until the cat gets going after 7 weeks i.e. in Week 8. At which point, we can divide the 5 mice the cat with catch in the observed proportion: 25% mature, 75% young. Thus, to find the stock of YOUNG MICE in Week 9, from YOUNG MICE we subtract maturing young and caught young: [YOUNG MICE/8 + (0.75 x 5)] and add births, 1.35 * 6.54/2; or this can be expressed simply: 12.15 – (12.15/8) – 3.75 + (1.35 * 6.54/2) = 11.30. This is an absolute decline in YOUNG MICE.

Repeating the calculation steps again for MATURE MICE where the cat catches relatively fewer mice, we find that the stock still increases from Week 8 to 9 BUT at a lower rate than previously.

We can see that in the next time period, YOUNG MICE will fall further (since the outflows now consistently exceed the inflows). Furthermore, the "maturing mice" will also lessen since there are fewer YOUNG MICE, which means the growth in MATURE MICE will decline once again and eventually the outflow will exceed the inflow (actually around Week 13). It can be seen that the cat will eventually triumph and the mouse population, even excluding natural deaths from old age, will be eliminated.

Note: It is not necessary to draw the amended model.

1(c) Eventually there are limits to growth. While our first, no cat, model showed the mouse population experiencing a dramatic increase, some other factors will limit the rise or even cause it to fall again by the operation of balancing forces. The availability of food is one example. As the mouse population grows so, with a fixed supply, food per head would reach a level where mice cannot be sustained and the death rate would increase. Further with less food per head, the fecundity of the mice could drop (i.e. fewer mice per litter, fewer litters) thus driving down population inflow as well as driving up the outflow.

Another balancing factor common in populations that are too large is the incidence of disease. As the population density rises, so does the chance of infections spreading leading to increased mortality. Depending on the characteristics of the epidemic, the population could stabilise or collapse.

Yet another example is that large populations of animals often attract an influx of predators (the cats from next door!). These predators might have their own population dynamics, driven in part by the availability of mice.

In these examples, if the outflows from the stocks are increased by some means so that they exceed the inflows, the population total will decline.

2 (a)

(i) The doubling time for a first-order positive-feedback system is 0.7 / time constant

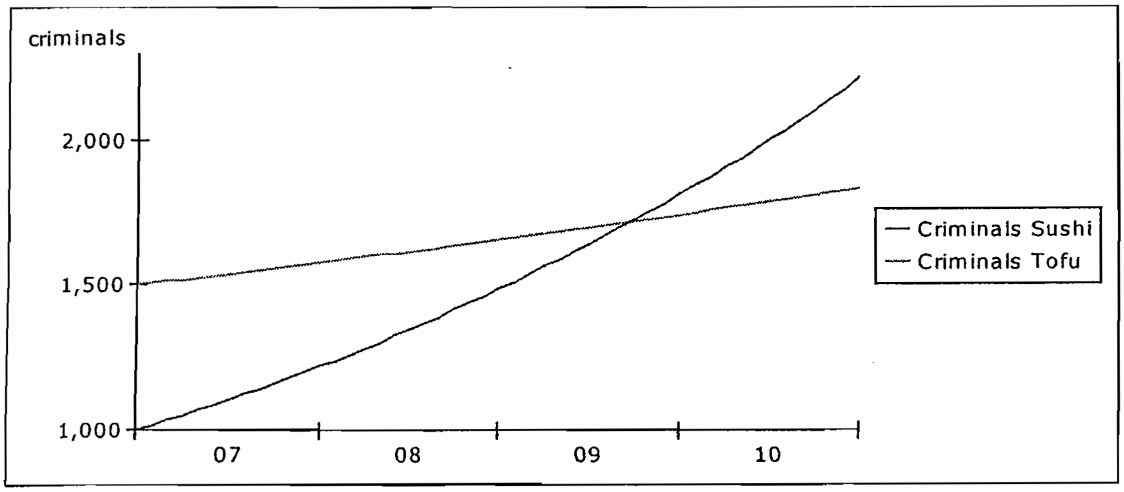
Since in both islands criminals are not caught and nor do they cease being crime; and since over three years we can ignore death/demographics, we can use the rule of 70 to estimate the doubling time for each population.

Doubling time for Tofu is 70/100g where g = 5/100/yr or 5%/yr: 70/5 = 14 years;
Doubling time for Sushi is therefore 20/100 or 20%/yr = 70/20 = 3.5 years.

The prison population of each island is given by the crime level figures:

Tofu: 0.015 x 100,000 = 1,500
Sushi: 0.010 x 100,000 = 1000.

Even rudimentary, linear plotting shows that Sushi's criminal population is likely to match Tofu's before 2010 and then rapidly exceed it with a doubling time ¼ of that in Tofu.



This can also be modelled using a first order linear rate equation:

$$dS/dt = rS$$

where S is the stock of drug users in Tofu or Sushi and r is the fractional influence rate. We solve for S,

$$S = \exp(rt + C)$$

For Sushi, r = 0.2 and at t = 0, S = 1000; for Tofu, r = 0.05 and at t = 0, S=1500;

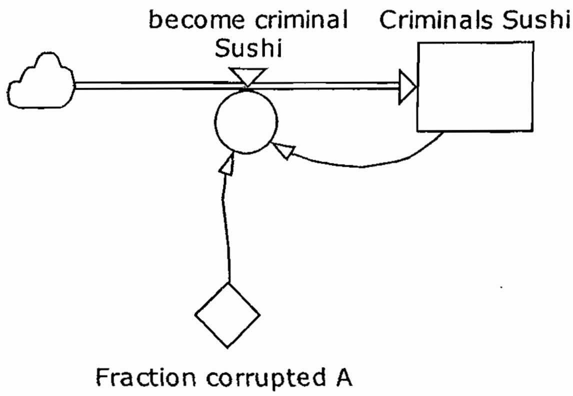
so after solving for C, we find

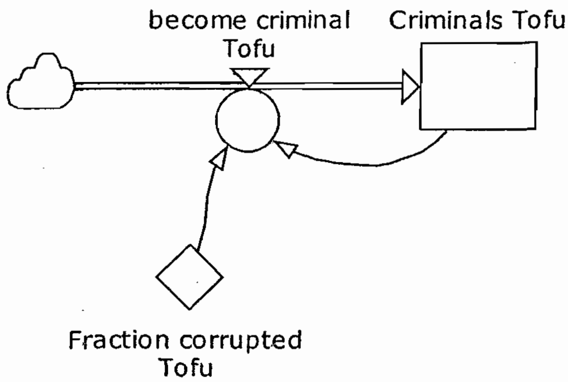
$$S_S = 1000 * \exp(rt) = 1000 * \exp(0.2t)$$
$$S_T = 1500 * \exp(rt) = 1500 * \exp(0.05t).$$

t=3 is year 2010.

Therefore, Sushi: $S_S(t{=}3) = 1000 * \exp(0.6) = 1{,}822$; and Tofu: $S_T(t{=}3) = 1500 * \exp(0.15) = 1{,}742$, so Sushi is clearly the most deserving of the islands for United Nations money.

(ii) Model diagram:





Fraction corrupted Tofu	criminals/criminals/yr	0.05
Fraction corrupted A	criminals/criminals/yr	0.2
Criminals Tofu	criminals	1500
become criminal Tofu.in		'become criminal Tofu'
Criminals Sushi	criminals	1000
become criminal Sushi.in		'become criminal Sushi'
become criminal Tofu	criminals/yr	'Criminals Tofu'*'Fraction corrupted Tofu'
become criminal Sushi	criminals/yr	'Criminals Sushi'*'Fraction corrupted A'

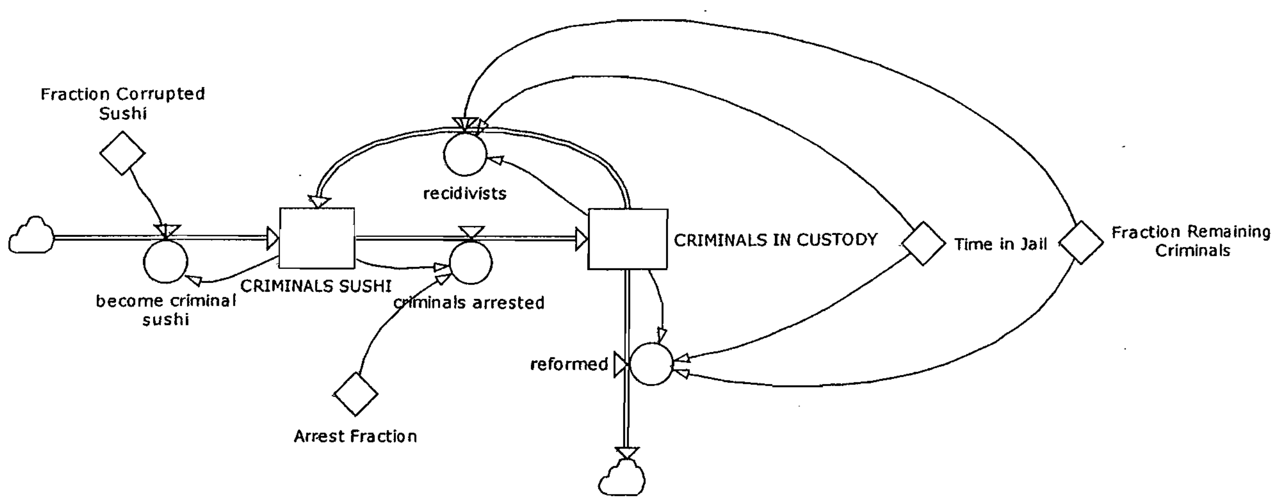
(iii) See plot as above. The plot shows the number of Sushi criminals growing faster than that in Tofu.

2 (b)

(i) With the building of a prison, we have to represent criminals in custody as a new stock. The flow into this stock will come from the stock CRIMINALS IN SUSHI (hopefully!). After serving their sentence (Time in Jail), prisoners (units = criminals) are released with a fraction (25%) returning to crime and 1-Fraction Remaining Criminals becoming reformed criminals. We can assume reformed criminals return to the general population and return to criminality with the same probability of all non-criminals – so there is no need to record them in a third stock.

Thus CRIMINALS IN CUSTODY will have two outflows, one returning to the stock of CRIMINALS SUSHI.

Model diagram:

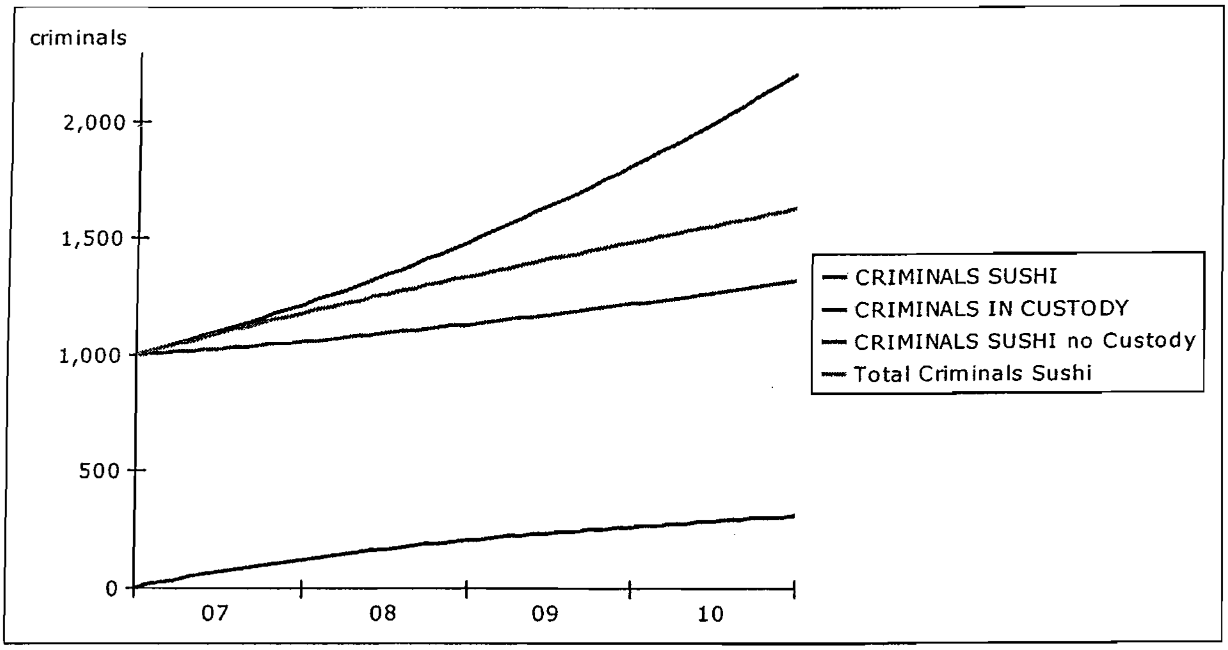


(ii) Model equations:

Arrest Fraction	criminals/criminals/yr	0.15
become criminal sushi	criminals/yr	'CRIMINALS SUSHI'*'Fraction Corrupted Sushi'
criminals arrested	criminals/yr	'CRIMINALS SUSHI'*'Arrest Fraction'
CRIMINALS IN CUSTODY	criminals	0
CRIMINALS SUSHI	criminals	1000
Fraction Corrupted Sushi	criminals/criminals/yr	0.2
Fraction Remaining Criminals	criminals/criminals	0.25
recidivists	criminals/yr	('CRIMINALS IN CUSTODY'/'Time in Jail')*'Fraction Remaining Criminals'
reformed	criminals/yr	('CRIMINALS IN CUSTODY'/'Time in Jail')*(1-'Fraction Remaining Criminals')
Time in Jail	yr	2

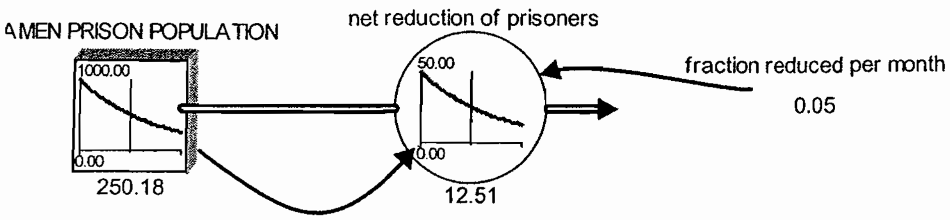
Notes: The only tricky bit is splitting the outflows of criminals leaving custody but this is done simply by using '0.25 * CRIMINALS SUSHI/Time in Jail' for recidivists and '(1-0.25) * CRIMINALS SUSHI/Time in Jail' for reformed.

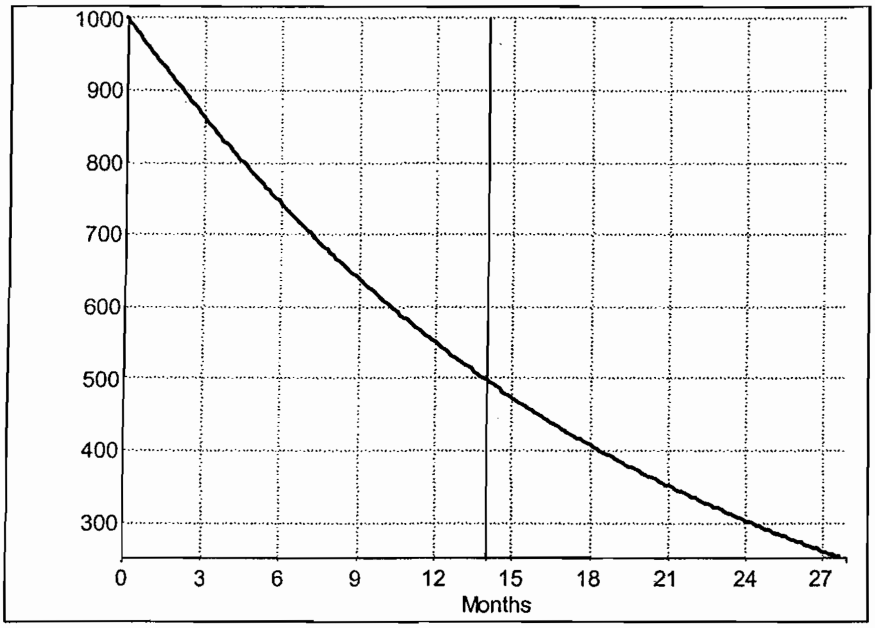
Model behaviour:



It can be assumed that adding another stock to contain criminals, which removes some of them from influencing the general public, will reduce criminality. Additionally, custodial sentencing is thought to reform criminals. Thus, inflow to criminals is reduced and outflow is increased and the overall number of criminals will decline (Total Criminals Sushi) (c)_This problem again utilises the rule of 70 but instead of doubling type we are looking at halving time. 250 is two periods of halving: 1000 : 500 : 250. Thus, we need to know the halving time and double it to know the number of months before the prison population falls to 250.

Thus: 2 x 70/(100 x 0.05) = 140/5 = 28 months. So we can expect the prison population to reach 250 by January 2009.





3.

a. Confidence interval is a range of values constructed from sample data, so that the population parameter is likely to occur within that range at a specified probability. This probability is the level of confidence.

b. the point estimate of the population mean is 2.84

The 95% interval of confidence for this mean: $2.84 \pm 1.96 \cdot {}^{0.8}/_{10}$

c. To solve this problem we follow these steps: a) state the null hypothesis and the alternate hypothesis; b) select the level of significance; c) select the test statistic and formulate the decision rule (select the critical region); d) interpret the results; e) take a decision. a)Ho: μ=2; Ha: μ>2; b)5%; c) test statistic is the sample mean. The rule: if it falls further $2 + 1.645 \cdot {}^{0.8}/_{10}$ we reject the null. (Notice it is a one-way test). d) the statistic is 2.84, which falls further the limit fixed in step c), that is, in the critical region. So we reject the null. The sample mean is so far from the hypothesis (so 'statistically unlikely') that the null has to be wrong. e) There was an improvement.

d. $P(b_2 - se(b_2) \cdot t_{0.975} \leq \beta_2 \leq b_2 + se(b_2) \cdot t_{0.975}) = 0.95$

$0.0949276-0.00827*2.201 \leq \beta_2 \leq 0.0949276+0.00827*2.201$

$0.08 \leq \beta_2 \leq 0.11$

The confidence interval indicate that the price that the market pays is statistically within the interval 9 cents and 11 cents per AA degree, more would be unfair. The price suggested by the Elena, though, is too low.

4.

a) A time series y_t is non-stationary if its distribution changes along time. It is weakly stationary if $E[y_t]$, $Var[y_t]$ and $Cov[y_t, y_{t-n}]$ are constant along time. The potential problem is that of estimating spurious correlations.

b) It is correctly designed because the variable seem clearly to present a trend and a constant, and these are included in the Dickey-Fuller analysis. The variable is I(1) as the null hypothesis of presence of unit root is not rejected with the variable in levels and it is rejected for the variable in differences.

c) It's an error correction model, so despite the variables being I(1), it is possible to identify a long run relationship between them. The long run relationship, depicted in square brackets, indicates that the higher the price the lower the demand and the higher the disposable personal income the higher the demand. All these variables are significant.

Since the variables are defined in logs, the coefficients convey elasticity concepts, price and income. That is, proportional changes are the results. The time term indicates whether we can identify a constant (unexplained) proportional change through time, but it is not significant. Short term elasticities are estimated significantly through the variables in differences.

d) In the long run relationship some variables could be added. The log of the prices of the cars could be used as to capture cross-elasticities. The sign expected for this variable is negative: the lower the prices for cars (a complement good) the higher the demand. Also a dummy could be included in order to capture the effects of the agreement with Japan. The sign in this case should be positive: further the agreement more cars would be available and therefore more usage of the motorways. Subsidies to the train system could be included as they could be oriented to improve quality and reduce individual ticket at the same time. Should this happen, as trains are a substitute good, the sign of the variable subsidies should be negative: given all the rest of the variables, more subsidies determine a higher demand for trains and therefore a lower one for the motorways.