ENGINEERING TRIPOS PART IIA

Thursday 26 April 2007 9 to 12

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (5 pages).
(i) Incompressible Flow (2 pages)
(ii) Boundary Layers
(iii) Applications to External Flows (2 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 A line sink and a line source of strength $-m$ and m respectively are located in the z-plane $(z = x + iy)$ on the x-axis at $(-a, 0)$ and $(a, 0)$ respectively as shown in Fig. 1.

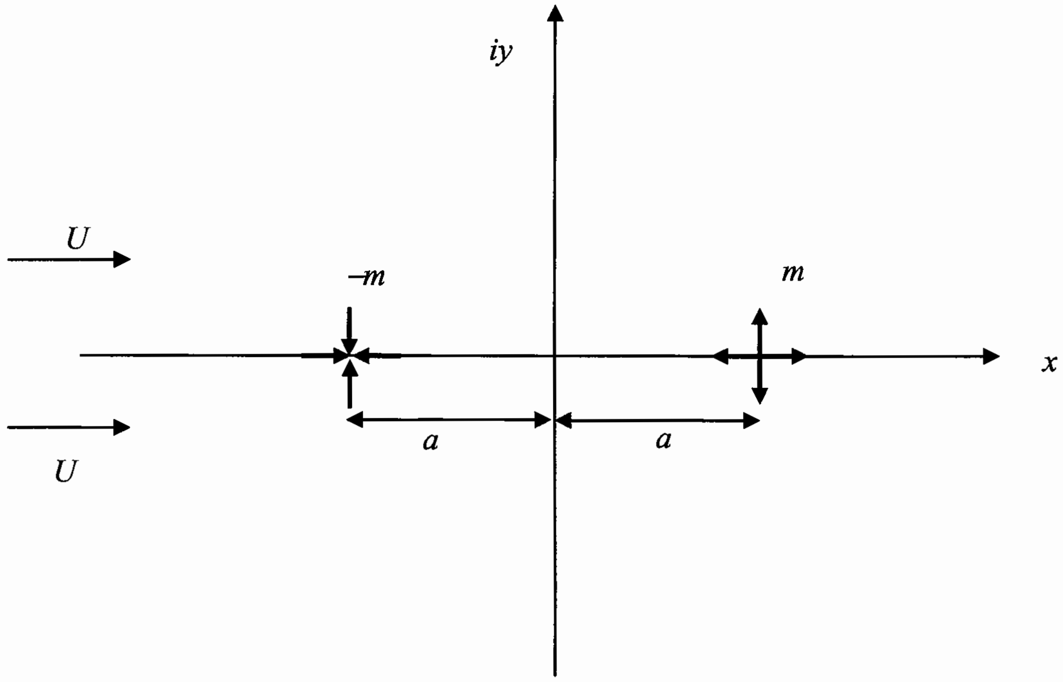

Figure 1

A uniform flow of velocity U flows from left to right (as shown).

(a) Write down the complex potential for this flow. [20%]

(b) Find the locations of the stagnation points for this flow in terms of the non-dimensional parameter $\frac{m}{\pi a U}$. Sketch the flow pattern for:

(i) $\frac{m}{\pi a U} < 1$, [10%]

(ii) $\frac{m}{\pi a U} = 1$, [10%]

(iii) $\dfrac{m}{\pi a U} > 1$. [10%]

(c) Using the results from (b), or otherwise, find the values of the parameter for which no fluid from the source could ever be ingested by the sink. [20%]

(d) A small amount of dye is released far upstream at a distance h from the x-axis i.e. at the point $(-\infty, ih)$. Determine the values of h for which none of this dye will be ingested into the sink for $\dfrac{m}{\pi a U} < 1$. [30%]

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2 (a) A pair of vortices are located on the z-plane $(z = x + iy)$ at the points $(0, ib)$ and $(0, -ib)$ and have circulations of $+\Gamma$ and $-\Gamma$ respectively as shown in Fig. 2.

Write down the complex potential for this flow and determine the velocity of the vortex pair (speed and direction). [20%]

NOTE: The velocity induced by a vortex on itself is zero.

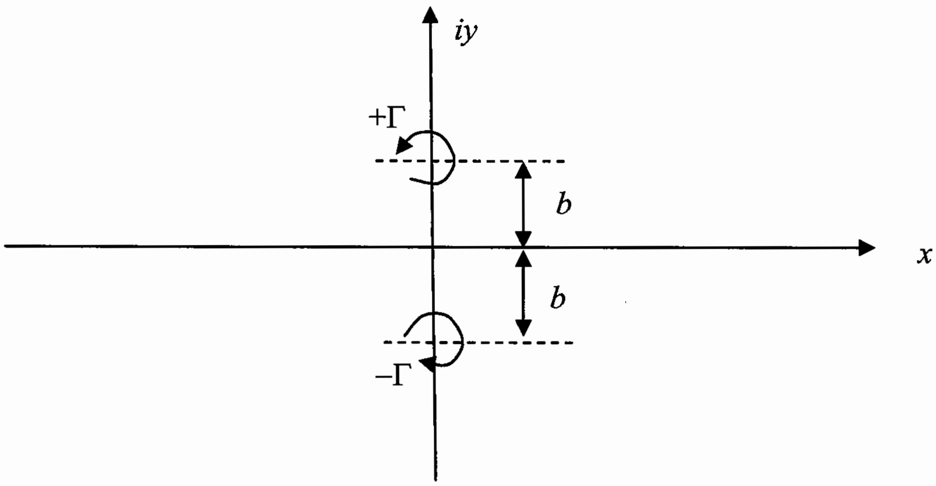

Figure 2

(b) A vortex pair is now placed in the vicinity of a wall along the x-axis as shown in Fig. 3.

The vortices are located at $(0, il)$ and $(0, i(l+2b))$. Write down the complex potential for this flow. Calculate the velocity of each vortex. [40%]

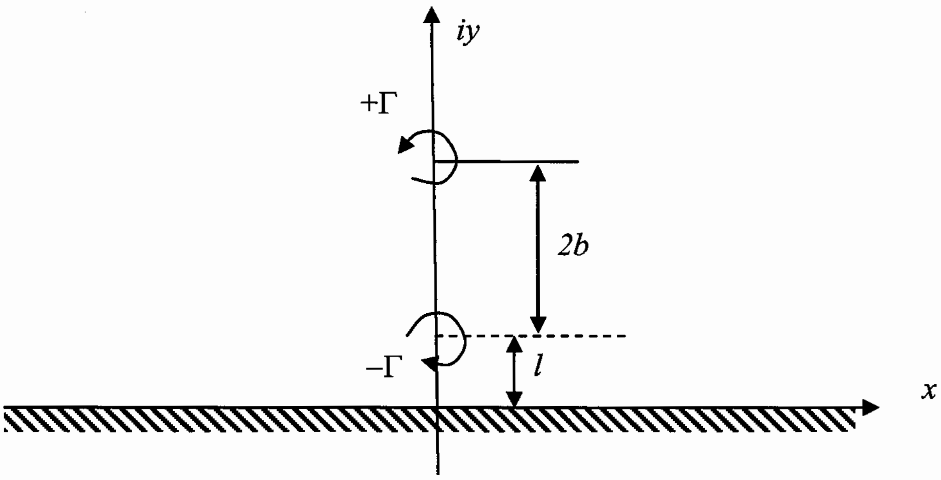

Figure 3

(c) Another wall is now added above the pair as shown in Fig. 4 where the axes have been moved to make the problem symmetrical.

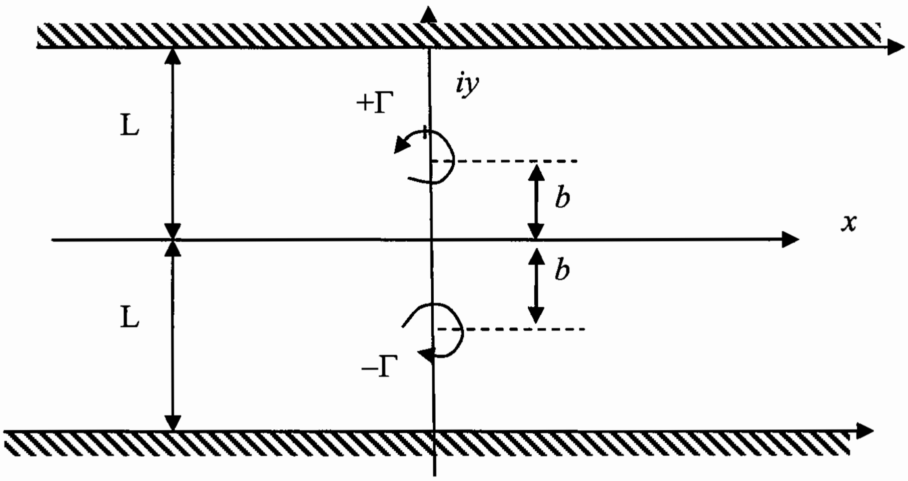

Figure 4

Write down an expression for the complex potential for this flow. [40%]

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3 A two-dimensional contraction for a proposed water tunnel is shown in Fig. 5.

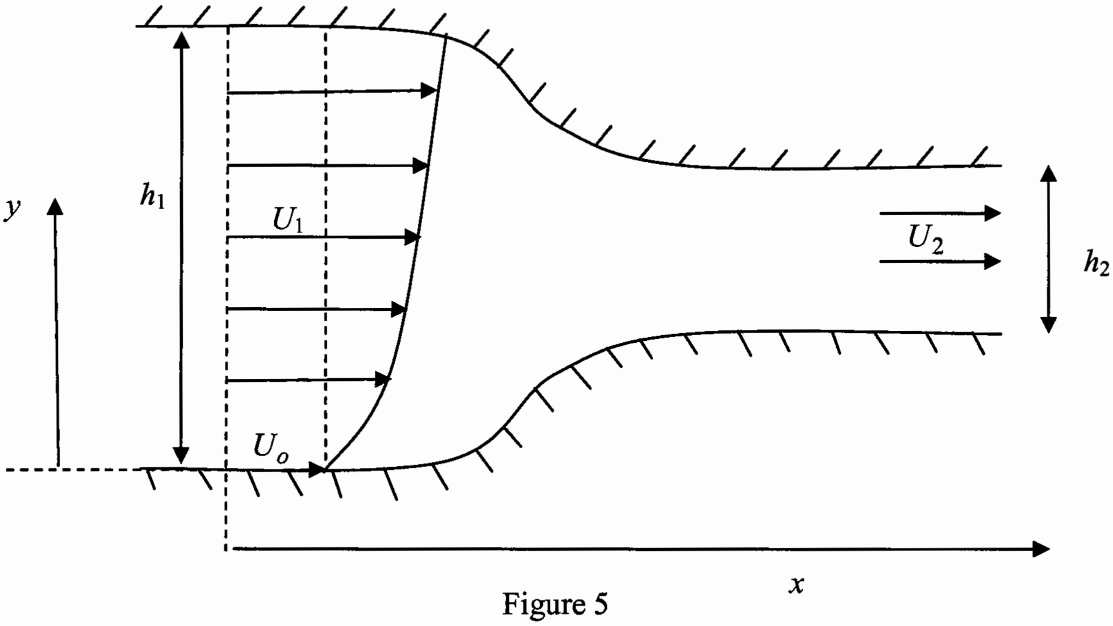

Figure 5

Flow coming from upstream has passed round a corner and hence is non-uniform as shown, and is given by $U_1 = U_o + U_e\left(\frac{y}{h_1}\right)^2$ where U_o and U_e are constant. Ignore boundary layers in this analysis.

(a) What is the profile of vorticity prior to the contraction? [20%]

(b) Find an expression for the velocity U_2 downstream in terms of y, h_2, h_1 and U_1. [50%]

(c) The non-uniformity of the flow upstream is given by the maximum variation of the velocity from uniform i.e. $\frac{U_e}{U_o}$. Assume that this is 0.10 in the real water tunnel.

The designer wants the maximum non-uniformity of the flow at the outlet of the contraction to be 0.01. Find the contraction ratio $\frac{h_1}{h_2}$ needed to achieve this condition. [30%]

4 (a) What is a boundary layer and why is it a useful concept in fluid mechanics and heat transfer? [30%]

(b) Compare and contrast a laminar and a turbulent boundary layer on a smooth flat plate each with the same free-stream velocity.

Your answers should include comments on:

(i) the mechanisms of momentum transfer, [10%]

(ii) the mechanisms of heat transfer (if the plates were at a constant temperature above ambient and the Prandtl number of the fluid was unity), [10%]

(iii) the relative growth rates of the boundary layer, [10%]

(iv) the relative surface shear stresses, [10%]

(v) the relative heat fluxes from the surfaces (if the plates are at a constant temperature above ambient and the Prandtl number of the fluid is unity). [10%]

(c) How would your answers to (b) (ii) and (b) (v) above change for the cases of very large and very small Prandtl numbers? [20%]

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5 Consider a flat plate at zero incidence to a uniform incompressible flow with velocity U_0, density ρ and viscosity μ. The plate is porous and fluid is removed by suction with uniform velocity V_0. A boundary layer develops, as shown in Fig. 6, and a fully-developed regime is reached where all flow properties, velocity (u, v) and pressure p are independent of x.

(a) Show that $v = -V_0$ across the boundary layer. Hence integrate the x- momentum equation to show that $u = U_0\left[1 - \exp\left(-\frac{V_0\rho}{\mu}y\right)\right]$. [30%]

(b) From the result of part (a), calculate the displacement thickness δ^* and the shear stress at the wall τ_w. [20%]

(c) Take a control volume enclosing the boundary layer (dotted line in Fig. 6) and apply the steady flow momentum equation to find an expression for τ_w. Show that this is identical to the expression derived in part (b). [30%]

(d) Discuss whether the result of part (c) is valid when the boundary layer is turbulent. Sketch the velocity profiles for laminar and turbulent boundary layers. [20%]

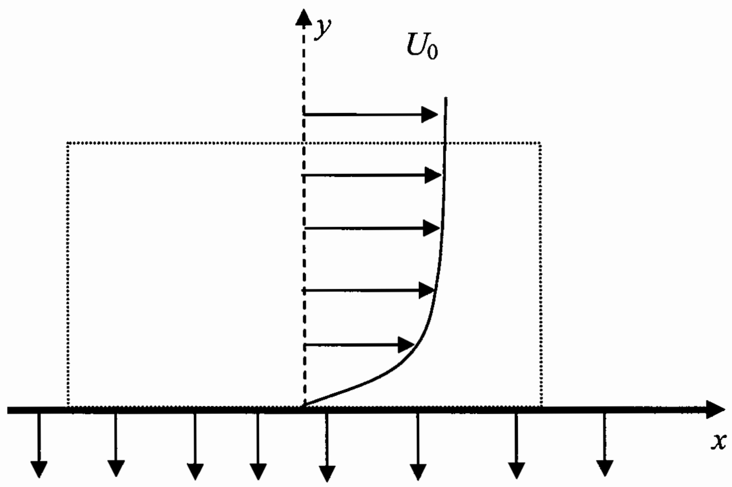

Figure 6

6 (a) Describe, with the aid of diagrams, how the pressure distribution on a typical cambered aerofoil varies over the incidence range from zero lift up to the limit of attached flow. [30%]

(b) A thin aerofoil has the camber-line

$$y_c = h_1 \frac{x}{c}\left(1 - \frac{x}{c}\right),$$

where x is the streamwise co-ordinate, c the chord length and h_1 a parameter controlling the magnitude of the camber. The aerofoil is to generate a lift coefficient of $\frac{\pi}{10}$ at zero incidence. Find the value of h_1 that is required. [20%]

(c) An alternative design, with fixed centre of pressure, is proposed. This has a camber-line

$$y_c = h_2 \frac{x}{c}\left[7 - 15\frac{x}{c} + 8\left(\frac{x}{c}\right)^2\right].$$

Find the value of h_2 that gives this aerofoil a lift coefficient of $\frac{\pi}{10}$ at zero incidence. [25%]

(d) (i) Explain why the second design has a less desirable pressure distribution at zero incidence.

(ii) Describe, without calculations, how the camber-line of the second aerofoil would be modified to give an improved pressure distribution without losing its property of fixed centre of pressure. You may assume that the designer is free to vary the incidence at which the required lift coefficient is attained. [25%]

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7 (a) An unswept, rectangular wing of uniform cross-section is twisted so that it has an elliptical lift distribution at its design lift coefficient of 0.5. If the corresponding induced drag coefficient is 0.01, what is the aspect ratio of the wing? [15%]

(b) The wing cross-section is uncambered. The circulation distribution at the design condition is written as

$$T = UsG_1 \sin\theta ,$$

where U is the free-stream velocity, s is the wing semi-span, and θ is linked to the spanwise coordinate y via

$$y = -s\cos\theta .$$

(i) Find the value of G_1.

(ii) Find the flow downwash angle at the wing.

(iii) The slope of the lift curve for this aerofoil section is 2π, and the twist angle is zero at the wingtips. Find, in radians, the wing incidence and the twist distribution; the latter as a function of θ. [35%]

(c) Formulate the lifting-line equation for the change in lift distribution due to a departure from the design incidence. Without attempting to solve this equation, briefly discuss its implications. [30%]

(d) It is proposed to replace the twisted, rectangular wing with an untwisted, elliptical wing that has the same aspect ratio and cross-section shape. What are the advantages and/or disadvantages of this idea? [20%]

8 The Engineering Department's Markham wind tunnel is of the closed return (closed loop) type with a working section measuring 1.52 m x 1.22 m. At the maximum operating speed of 60 ms^{-1} (measured in the working section) the fan supplies a mechanical power of 75 kW. Breathers are installed at the working section.

(a) Sketch the basic lay-out of the wind tunnel and identify the main components. [20%]

(b) Estimate the power factor of the wind tunnel. Where does the energy supplied by the fan go? [20%]

(c) The flow quality of the Markham wind tunnel is surveyed by traversing a hot wire probe through the working section. What is the basic principle of measuring flow velocity with hot-wires? [30%]

(d) The survey of (c) is performed by traversing in a large number of horizontal sweeps starting at the ceiling of the working section and ending at the floor approximately 6 hours later. When analysing the results it is found that a vertical velocity gradient appears to exist in the flow. Can you suggest a cause for this apparent velocity variation and propose strategies to rectify the problem? [30%]

END OF PAPER