ENGINEERING TRIPOS PART IIA

Friday 27th April 2007 9 to 12

Module 3A3

FLUID MECHANICS II

Answer not more than five questions.

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of the question is indicated in the right margin.*

There are no attachments.

STATIONERY
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Compressible flow Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The inlet of a convergent-divergent nozzle is connected to a large plenum of air at a stagnation pressure p_0. The nozzle exhausts to a second large plenum of pressure p_e. The stagnation pressure at the exit plane of the nozzle is p_{0e}. The flow through the nozzle is adiabatic and frictionless.

(a) Sketch the pressure distribution along the nozzle as p_e/p_0 is gradually reduced. Explain how p_{0e}/p_0 varies. If there is a 6.1% drop in stagnation pressure from duct inlet to exit show that the area of the duct at which the shock is located is 15.6% larger than the throat area of the nozzle. [40%]

(b) The ratio of the exit area to throat area of the nozzle is 1.2. The duct has the same drop in stagnation pressure as specified in part (a). Calculate the Mach number at the exit of the nozzle M_e and the pressure ratio p_e/p_0. [25%]

(c) The cross-sectional area of the divergent section of the nozzle varies linearly with distance downstream of the throat. The pressure ratio p_e/p_0 calculated in part (b) is altered so that the shock moves downstream by 20% of the length of the divergent section of the duct. Calculate the percentage change in p_e/p_0 from that calculated in part (b). [35%]

2 (a) By considering an infinitesimal surface wave on shallow water in a frame of reference where the wave is stationary, or otherwise, show that $V+2c$ is invariant across a wave propagating to the left. V is the velocity of the water, $c = \sqrt{gh}$ and is the speed of propagation of surface waves and h is the depth of the water. V is considered positive towards the right. [30%]

(b) A channel of constant width and finite length is closed at the left hand end and has a closed sluice gate at the right hand end. The channel is filled with water of depth h_0. On the other side of the sluice gate is a large reservoir containing water of depth h_1, where $h_0/h_1>1$. The bottom of the channel and reservoir are at the same level. At time t_0 the sluice gate instantaneously opens. Draw a space-time (s-t) diagram of the water in the channel marking on the wave and particle trajectories. Calculate the lowest ratio of h_0/h_1 that maximises the velocity of water leaving the channel immediately after the sluice gate opens. [35%]

(c) Prove that for $h_0/h_1>1$ and less than the limit calculated in part (b) the fraction of mass of the original water in the channel that has escaped by the time the water height at the left end of the channel just drops is given by

$$2\frac{h_1}{h_0}\left(1-\sqrt{\frac{h_1}{h_0}}\right)$$

Comment on physical interpretation of the variation of this function with h_0/h_1. [35%]

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3 (a) Write down the equations governing the steady adiabatic one-dimensional flow with friction of a perfect gas in a constant area duct. By differentiating these equations, or otherwise, show that

$$\delta\left(\frac{F}{A}\right) = p\left(M^2 - 1\right)\frac{\delta V}{V}$$

where F is the impulse function, A is the area of the duct, p is the static pressure, M is the Mach number and V is the velocity. [30%]

(b) A turbine is designed to operate with an inlet to exit stagnation pressure ratio of 4 and an inlet Mach number of 0.5. In practice when the turbine is installed in a plant a 100 m pipe is used to connect the inlet of the turbine to a large 4 bar supply tank. The pipe has a diameter of 2 m and the walls of the pipe have a skin friction coefficient C_f of 0.0025. The turbine exhausts to an ambient pressure of 1 bar. Assume that the turbine inlet Mach number remains unchanged and that the pipe exit Mach number and the turbine inlet Mach number are equal. Calculate the Mach number at the inlet to the pipe and the stagnation pressure ratio of the installed turbine. Assume that the gas has the properties of air throughout. [35%]

(c) If the isentropic efficiency of the turbine in part (b) remains unchanged when it is installed, determine the ratio of the turbine installed power output to design power output. [35%]

4 An aircraft technician accidentally drops a spanner on the upper surface of the wing of a supersonic aircraft. The thin skin of the wing is dented inwards between two heavy spars, as sketched in figure 1(a).

(a) If the aircraft is permitted to fly in this condition, and assuming that the flow is purely in the plane of the page, sketch the wave patterns due to the dent. You may assume that, local to the dent, the wing surface is otherwise flat. [25%]

(b) Calculate the Mach number downstream of the dent when the aircraft flies at such a speed that the Mach number immediately ahead of the dent is 2.20. [25%]

(c) The pressure forces resulting from this flowfield generate a net drag force on the wing. At the flow conditions in part (b) the static pressure immediately upstream of dent is 0.2 bar, calculate the net drag per unit span on the wing. [25%]

(d) If instead of the damage sketched in figure 1(a), the spanner had been dropped closer to the front spar than the rear, as sketched in figure 1(b), would you expect a greater, a lower or an equal net drag to that calculated in part (c)? Explain your answer. [25%]

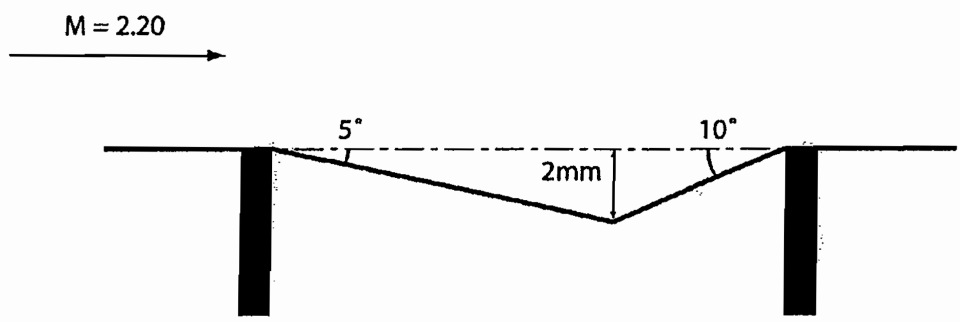

Fig. 1(a)

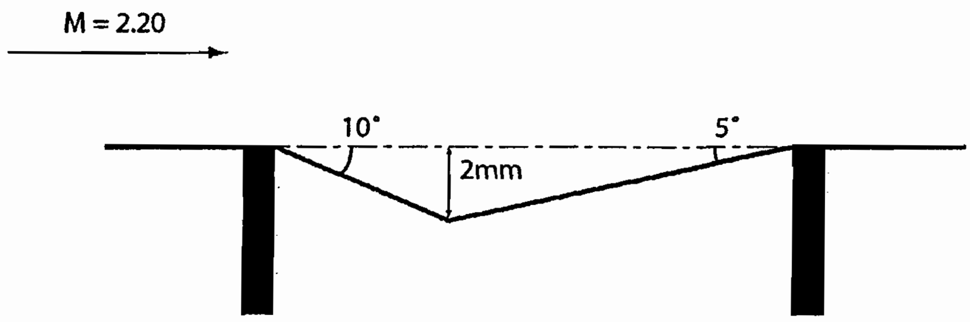

Fig. 1(b)

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5 (a) Describe the circumstances under which a family of characteristics in a supersonic flow will be straight and the consequences that straight characteristics have for the variation of flow properties along them. [10%]

(b) Figure 2 shows a bend in a duct (exaggerated for clarity), designed to turn a flow of air by 6° and to deliver a uniform flow downstream at section CT. The duct is 10 cm wide at AP and the Mach number of the flow there is 1.33. Only the segment QRS is curved; the other segments are straight. Find the Mach Number and the duct width at CT. [25%]

(c) Explain carefully how the curve QRS should be chosen in order to ensure uniform flow at CT. [15%]

(d) Find the co-ordinates of the point R, in a cylindrical-polar co-ordinate system based on B, where the angle of the wall is 3° relative to the original flow direction. [25%]

(e) Sketch the flow pattern when the inlet Mach Number is 1.5. [25%]

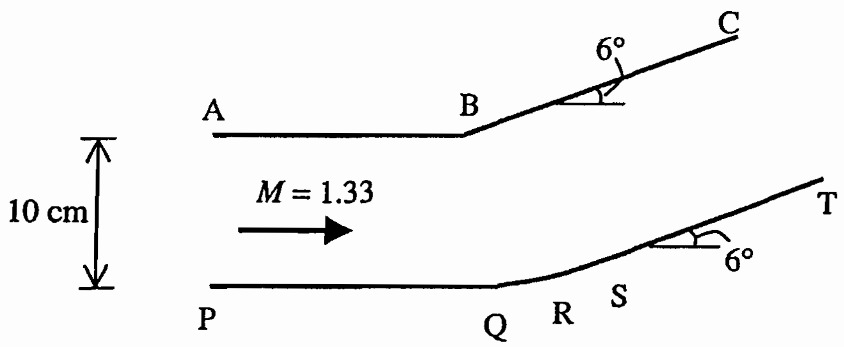

Fig. 2

6 The one-dimensional diffusion equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

where the viscosity ν is constant, is to be modelled using the following difference equation

$$u_i^{n+1} = u_i^n + \alpha\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$

where time $t = n\Delta t$, distance $u = i\Delta x$ and $\alpha = \nu\Delta t/(\Delta x)^2$.

(a) Determine the order of accuracy of this numerical scheme for a general value of α. [40%]

(b) By considering the change in amplitude over one time step of a small perturbation given by $u_i^n = (-1)^i \varepsilon$, determine the range of α for which the numerical scheme is stable. [30%]

(c) There is one value of α for which this scheme is more accurate than the order derived in part (a). Find this value of α, and the increased order of accuracy. [30%]

7 (a) A two-dimensional turbine cascade operating with air has a subsonic inlet flow and an exit Mach number of 1.2. The inlet flow angle is 29.5° from the axial direction. The flow is turned by 96.5° between the inlet and exit. The stagnation pressure loss coefficient is defined as

$$Y_p = \frac{P_{01} - P_{02}}{P_{02} - P_2}$$

and is equal to 0.08.

(i) Calculate the inlet Mach number of the cascade. [25%]

(ii) The inlet stagnation pressure is 1 bar. The inlet stagnation temperature is 300 K. The blade pitch is 50 mm. The span is 200 mm. Calculate the force acting on each blade in the tangential direction. [25%]

(b) Figure 3(a) shows a typical cell taken from a two-dimensional finite-volume grid. Explain how Gauss' theorem can be used to evaluate an approximation to the cell area-average value of $\frac{\partial u}{\partial y}$ using the values of u, the velocity in the x direction, at the nodes 1, 2 and 3. [25%]

(c) If integrals over the edges of the cell shown in Figure 3(b) are evaluated using the trapezium rule, show that the approximation to the cell-average value of $\frac{\partial u}{\partial y}$ formed in this way is

$$\frac{1}{d}\left(u_3 - \frac{u_1 + u_2}{2}\right).$$

[25%]

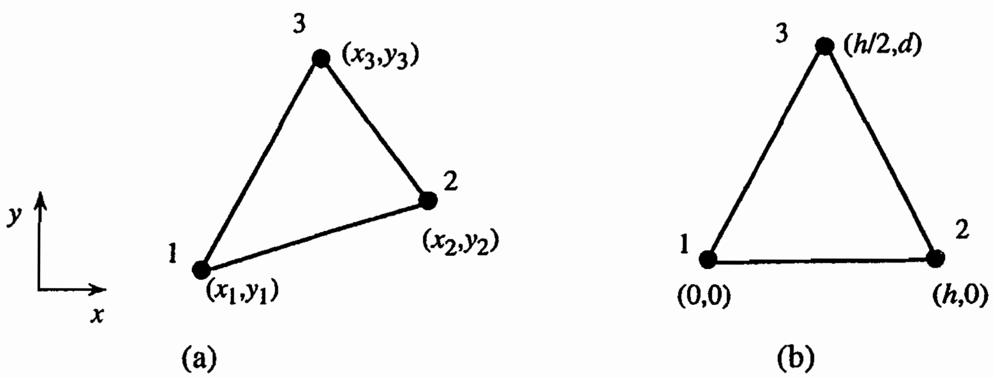

Fig. 3

8 A single-shaft multistage high pressure steam turbine consists of 8 stages where the velocity triangles and mean radius of each stage are constant through the turbine.

(a) The stator blades have an exit flow angle of 76°. The rotor relative inlet flow angle is 45°. The rotor relative exit flow angle is -72°. All angles are measured from the axial direction and are positive in the direction of rotation. Determine the absolute flow angle at rotor exit. Sketch the velocity triangles at the stator exit and at the rotor exit. [25%]

(b) Calculate the flow coefficient $\phi = V_x / U$ and the stage loading coefficient $\psi = \Delta h_0 / U^2$. [20%]

(c) At the inlet to the turbine, the stagnation enthalpy of the steam is 3450 kJ/kg. At the exit of the turbine the stagnation enthalpy is 3095 kJ/kg. The power output is 100 MW. The rotational speed of the turbine is 3000 rpm. Determine

- (i) The mass flow rate through the turbine.
- (ii) The change in stagnation enthalpy across each stage of the turbine.
- (iii) The span at the inlet to the first row of stator blades given that the density of steam at inlet is 43.6 kg/m^3.
- (iv) The reaction of each stage. [45%]

(d) Explain why the flow area must increase through the turbine. [10%]

END OF PAPER