

ENGINEERING TRIPOS PART IIA

Thursday 3 May 2007 9.00 to 10.30

Module 3A5

THERMODYNAMICS AND POWER GENERATION

Answer not more than three questions.

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of the question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS

Single-sided script paper.

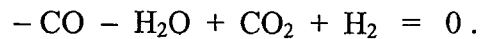
SPECIAL REQUIREMENTS

Engineering Data Book.

CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Figure 1 on the next page shows a schematic diagram of a chemical reformer to convert methane to hydrogen within a power plant. The reformer is supplied with methane and (excess) steam in the ratio of 1 kmol of CH_4 to 4 kmol of H_2O . At reformer outlet, it may be assumed that there is no CH_4 present and that a gaseous mixture of CO_2 , CO , H_2O and H_2 leaves in chemical equilibrium subject to the 'shift' reaction,



All streams enter and leave at a temperature of 1000 K and 1 bar pressure. It may be assumed that the reaction proceeds isothermally with the heat required being supplied externally by a hot air stream also at 1000 K, and that all species behave as ideal gases.

(a) Verify that each kmol of CH_4 supplied results in 0.5063 kmol of CO_2 at reformer outlet and find the corresponding numbers of kmol of CO , H_2O and H_2 . [40 %]

(b) With reference to the data given on the next page, show that \bar{Q} (the heat transferred from the air stream to the reformer per kmol of CH_4 supplied) is given by,

$$\bar{Q} = [-\Delta\bar{H}_{1000}^0]_{ref} + 0.5063[-\Delta\bar{H}_{1000}^0]_{shift}. \quad [20 \%$$

(c) With reference to the data given on the next page, derive an analogous expression for $\Delta\bar{S}_{irrev}$, the entropy created by irreversibility in the reformer per kmol of CH_4 supplied. In doing this, assume that the molar entropies of all gas species are to be evaluated at the mixture pressure of 1 bar rather than at their individual partial pressures (*i.e.*, neglect the entropy of mixing). [20 %]

(d) With reference to the data given on the next page and assuming an environment temperature of 25 °C, calculate the maximum rational efficiency of a power plant supplied by methane at 25 °C and 1 bar which contains a reformer operating at the conditions considered above. [20 %]

(Cont ...

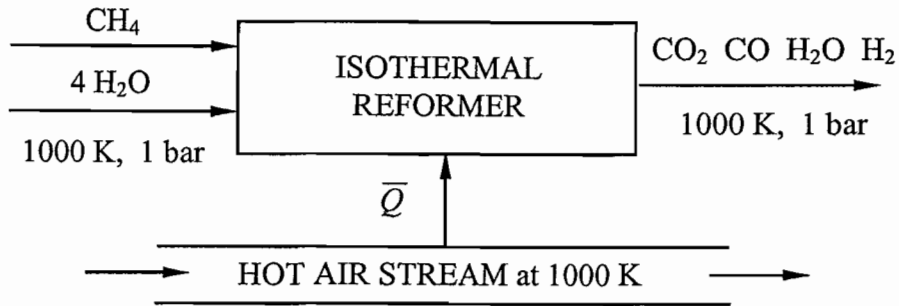


Figure 1

DATA FOR QUESTION 1

For the methane reforming reaction at 1000 K : $\text{CH}_4 + \text{H}_2\text{O} \rightarrow \text{CO} + 3\text{H}_2$

$$[\Delta\bar{H}_{1000}^0]_{ref} = 225.70 \text{ MJ}/(\text{kmol CH}_4), \quad [\Delta\bar{S}_{1000}^0]_{ref} = 0.2529 \text{ MJ}/(\text{kmol CH}_4) \text{ K}.$$

For the shift reaction at 1000 K : $\text{CO} + \text{H}_2\text{O} \rightarrow \text{CO}_2 + \text{H}_2$

$$[\Delta\bar{H}_{1000}^0]_{shift} = -34.74 \text{ MJ}/(\text{kmol CO}), \quad [\Delta\bar{S}_{1000}^0]_{shift} = -0.0317 \text{ MJ}/(\text{kmol CO}) \text{ K}.$$

For the oxidation of methane at 298.15 K : $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$

$$\Delta\bar{G}_{298}^0 = -800.70 \text{ MJ}/(\text{kmol CH}_4).$$

(TURN OVER)

2 Note that parts (a) and (b) of this question are unrelated.

(a) In characteristic form, the specific Helmholtz function f of a pure substance is expressed in terms of temperature and specific volume [*i.e.*, $f = u - Ts = f(T, v)$].

(i) Prove that the specific entropy is given by

$$s = -\left(\frac{\partial f}{\partial T}\right)_v.$$

(ii) The specific Helmholtz function of a pure substance is given by

$$f = f_0(T) - RT \left[\ln(v) - \frac{B(T)}{v} \right],$$

where R is the specific gas constant, and f_0 and B are functions of temperature only. Derive an expression for the specific internal energy u as a function of T and v . What condition must be fulfilled in order that c_v (the specific heat capacity at constant volume) is not a function of v ? [50 %]

(b) In a steam power plant, liquid water enters the boiler at state 1 (h_1, s_1), is heated, and leaves as superheated steam at state 2 (h_2, s_2). There is a pressure drop in the boiler tubes and hence $p_2 < p_1$. The water-steam mass flowrate is \dot{m} and changes in kinetic and potential energies can be neglected.

(i) Assuming heat transfer only with the environment at temperature T_0 , write down an expression for $[\dot{W}_X]_{\max}$, the maximum shaft power obtainable from an external steady-flow circuit connecting the boiler outlet at state 2 to its inlet at state 1.

(ii) If \dot{Q} is the rate of heat supply in the boiler, show that

$$[\dot{W}_X]_{\max} = \int_1^2 \left(1 - \frac{T_0}{T}\right) d\dot{Q} - \left(-\dot{m} \int_1^2 \frac{T_0}{T} v dp \right),$$

where the integrations apply to the water-steam in the boiler tubes at local temperature T and specific volume v . Explain the physical meaning of the terms on the right hand side of the equation. [50 %]

3 (a) State briefly how the performance of a simple-cycle gas turbine changes with the addition of,

- (i) an intercooler;
- (ii) a recuperator;
- (iii) an intercooler and a recuperator.

[25 %]

(b) An industrial gas turbine has a single stage of intercooling and a recuperator. Air enters the low-pressure (LP) compressor at a pressure and temperature of 1 bar and 25 °C. After intercooling, the air enters the high-pressure (HP) compressor at a temperature of 40 °C. Both compressors are adiabatic and both have a pressure ratio of 3 and a polytropic efficiency of 0.9.

After the HP compressor, the air passes through the cold side of the recuperator. It then enters the combustion chamber where it is mixed with fuel supplied at 25 °C. The combustion process takes place adiabatically and the products of combustion leave the chamber at 1575 K and expand adiabatically through the turbine with a polytropic efficiency of 0.9. The gas then passes through the hot side of the recuperator before exhausting to atmosphere. The recuperator effectiveness is 0.75.

Pressure losses in the intercooler, combustor and recuperator may be neglected. Assume that air behaves as a perfect gas with $c_p = 1.005$ kJ/kg K and $\gamma = 1.40$, and that the products of combustion can also be treated as a perfect gas with $c_p = 1.100$ kJ/kg K and $\gamma = 1.35$. For the combustion of the fuel with air, $\Delta H_{298}^0 = -50.0$ MJ/kg.

- (i) Calculate the temperature of the air entering the combustion chamber. [25 %]
- (ii) Calculate the air-fuel ratio by mass. [25 %]
- (iii) Calculate the overall efficiency of the plant. [25 %]

(TURN OVER)

4 A combined-cycle power plant consists of a gas turbine (GT) topping a single-pressure steam cycle. At exit from the GT turbine, the gas pressure and temperature are 1 bar and 650 °C, and the mass flowrate is 150 kg/s. The gas then enters the heat recovery steam generator (HRSG) where it is cooled at constant pressure as it transfers heat to the steam. The pinch point temperature difference in the HRSG is 15 °C and it may be assumed that the gas behaves as a perfect gas with $c_p = 1.25$ kJ/kg K. The dead state pressure and temperature are defined to be 1 bar and 25 °C.

In the steam cycle, saturated liquid water at a pressure of 0.08 bar (state 1) enters the feed pump and is compressed isentropically to a pressure of 80 bar (state 2). The water then enters the HRSG where it is heated at constant pressure to the saturated liquid temperature (state 2S). After further heating at constant pressure, it leaves the HRSG as superheated steam at a temperature of 500 °C (state 3). The steam is then expanded adiabatically but irreversibly in the steam turbine to a pressure of 0.08 bar (state 4). Finally, the steam is condensed at constant pressure to state 1.

The steam property data for this question are given in Table 1 on the following page so it is unnecessary to refer to the steam tables in the thermofluids data book.

- (a) Calculate the steady-flow exergy in kW (*i.e.*, the maximum available shaft power) of the gas at exit from the GT turbine. [15 %]
- (b) Calculate the mass flowrate of steam circulating in the bottoming cycle. [10 %]
- (c) (i) Calculate the power input to the feed pump. [5 %]
- (ii) Calculate the steady-flow exergy (*i.e.*, the maximum available shaft power) of the gas at exit from the HRSG. [20 %]
- (iii) Calculate the ‘lost power’ for the HRSG. [15 %]
- (iv) Calculate the power output and the ‘lost power’ for the steam turbine. [10 %]
- (v) Calculate the steady-flow exergy (*i.e.*, the maximum available shaft power) of the heat transferred from the steam in the condenser. [15 %]
- (d) Demonstrate that the power, ‘lost power’ and exergy terms calculated in (c) account fully for the available exergy at exit from the GT turbine as calculated in (a). [10 %]

(Cont ...

	Pressure bar p	Temperature °C T	Specific enthalpy kJ/kg h	Specific entropy kJ/kg K s
State 1	0.08	41.51	173.8	0.592
State 2	80.0	41.75	181.9	0.592
State 2S	80.0	295.01	1317.2	3.208
State 3	80.0	500.00	3399.5	6.727
State 4	0.08	41.51	2215.8	7.082

Table 1 Properties of water and steam for Question 4.

END OF PAPER