ENGINEERING TRIPOS PART IIA

Thursday 10 May 2007 2:30-4:00

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: Data sheet (1 page).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: 3

2 Thin filament pyrometry uses a thin wire of diameter $D = 1.5 \times 10^{-3}$ m, and known emissivity $\varepsilon = 0.8$, embedded in a fluid whose temperature is to be measured. The temperature is determined by collecting the radiation emitted by the filament onto a detector, as shown in Fig. 1. The detector is sensitive to wavelengths from $\lambda_1 = 0.4\ \mu$m to $\lambda_2 = 0.8\ \mu$m only. The optical setup integrates the emission over an area corresponding to one half of the filament perimeter over segment $\Delta x = 1 \times 10^{-3}$ m of the length of the wire. The Table on the attached data sheet can be used to determine the fractional normalized radiation as a function of wavelength.

(a) Determine the total rate of energy emitted per unit area of wire at a temperature $T = 2000$ K. [20%]

(b) Determine the total rate of energy radiated by the segment Δx of the wire and intercepted by the detector. [20%]

(c) A second measurement is made using the same detector, and the signal is found to be 10 times higher than the original measurement at $T = 2000$ K. From this measurement, determine the temperature to within 100 K. [60%]

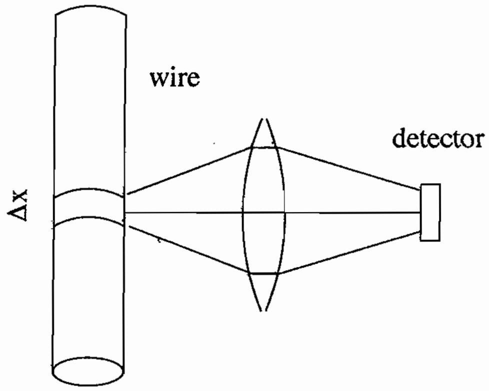

Fig. 1

What geometrical arrangement can be made to maximise this heat transfer rate for given $\dot{m}$, c, h, $T_{h,o}$, $T_{c,i}$ and a fixed radius for only one of the tubes? [25%]

(d) Calculate the length required to achieve $T_{c,o} = 324$ K if $T_{c,i} = 290$ K. Use the following values in your calculation: $\dot{m} = 0.02$ kg/s, $c = 4175$ J/(kg K), $h = 1950$ W/(m^{2}K), $r = 0.01$ m, $\theta = 30°$, $T_{h,i} = 360$ K, and $T_{h,o} = 310$ K. [20%]

Blackbody Radiation Functions[a]

λT (μm · K)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{\max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

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[a] The radiation constants used to generate these blackbody functions are:
$C_1 = 3.7420 \times 10^8$ W · μm^4/m^2
$C_2 = 1.4388 \times 10^4$ μm · K
$\sigma = 5.670 \times 10^{-8}$ W/m^2 · K^4.

$$F_{(0\rightarrow\lambda)} = \frac{\int_0^\lambda E_{\lambda,b}(\lambda,T)\,d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)}$$

$$F_{(\lambda_1\rightarrow\lambda_2)} = F_{(0\rightarrow\lambda_2)} - F_{(0\rightarrow\lambda_1)}$$