ENGINEERING TRIPOS PART IIA

Friday 11 May 2007 9.00 to 10.30

Module 3C6

VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Datasheet S32: 3C5 Dynamics and 3C6 Vibration (5 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 A stretched string of length L and mass per unit length M/L is under tension P and fixed at the two ends $x=0$ and $x=L$. The loaded string undergoes small transverse vibration.

(a) Assuming that the nth mode shape of vibration of the string has the form

$$u_n(x) = \sin n\pi x / L \ , \tag{1}$$

use Rayleigh's principle to obtain an expression for the corresponding natural frequency Ω_n. [20%]

(b) A small mass m is now fixed to the string at the position $x = X$. It may be assumed that $m \ll M$. Use the same approximate mode shape (Equation 1) in Rayleigh's principle to show that the nth natural frequency becomes:

$$\omega_n \approx \Omega_n \left[1 - \frac{m}{M} \sin^2 \left(\frac{n\pi X}{L} \right) \right].$$ [30%]

(c) Calculate the minimum value of m in order to reduce the second natural frequency by one semitone (i.e. one twelfth of an octave). Where should the mass be placed to have most effect? For this mass in this position, sketch a graph showing the frequency shift in semitones of the first six natural frequencies. [30%]

(d) Without detailed calculations, explain what would be the corresponding result to the answer of (b) if the mass m were attached to a pinned-pinned bending beam of total mass M instead of being attached to the stretched string. [20%]

2 (a) A beam of length $2L$, mass per unit length m and bending rigidity EI is pinned at both ends, which lie at $x=-L$ and $x=L$. At the two ends, rotation of the beam is restrained by identical torsional springs of stiffness K. The beam undergoes small transverse vibration with displacement $w(x,t)$.

(i) Explain why the vibration modes of this system must take one of the two forms

$$u(x)=A\cos\alpha x+B\cosh\alpha x \qquad \text{or} \qquad u(x)=A\sin\alpha x+B\sinh\alpha x$$

where

$$w(x,t)=u(x)e^{i\omega t}.$$

Specify an expression for α. [10%]

(ii) Explain why one boundary condition at $x=L$ can be written in the form

$$\frac{\partial^2 u}{\partial x^2}=-\lambda\frac{\partial u}{\partial x}.$$

Specify an expression for λ. [10%]

(b) For the symmetrical modes of the beam, show that the natural frequencies are determined by the roots of the equation

$$\tan\alpha L+\tanh\alpha L+2\alpha/\lambda=0.$$ [30%]

(c) Sketch a graphical solution to this equation. Explain what happens as the value of λ is varied. What is the physical interpretation of the limiting cases $\lambda=0$ and $\lambda\to\infty$? [30%]

(d) What is the significance of this calculation for the natural frequencies that you would expect to measure from a uniform beam held at its ends in two identical clamping grips? [20%]

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3 Part of a satellite is an unconstrained rod of length L, mass M, cross sectional area A, and Young's modulus E as shown in Fig. 1(a). A four-element discrete model of the rod, used to calculate axial vibration transmission, is shown in Fig. 1(b). Each element has mass m and is joined to adjacent elements with springs of stiffness k.

(a) Determine suitable values of m and k in terms of M, E, A and L, and write down mass and stiffness matrices for the discrete model. [20%]

(b) Without calculating eigenvalues or eigenvectors, sketch plausible mode shapes for each of the four natural frequencies of the discrete model. [20%]

(c) By inspection, or using Rayleigh's method, estimate the four natural frequencies of the discrete model. Which of the frequencies is exact? [30%]

(d) The base of the rod is excited by an axial sinusoidal force $F\sin\omega t$. Sketch a graph of the magnitude of the displacement of coordinate y_3 (on a dB scale) as a function of angular frequency ω. [20%]

(e) The base of the rod is now forced to vibrate with a vertical *displacement* of ±1 mm at a frequency corresponding to the lowest non-zero natural frequency. What is the amplitude and phase of the displacement at the top of the rod? [10%]

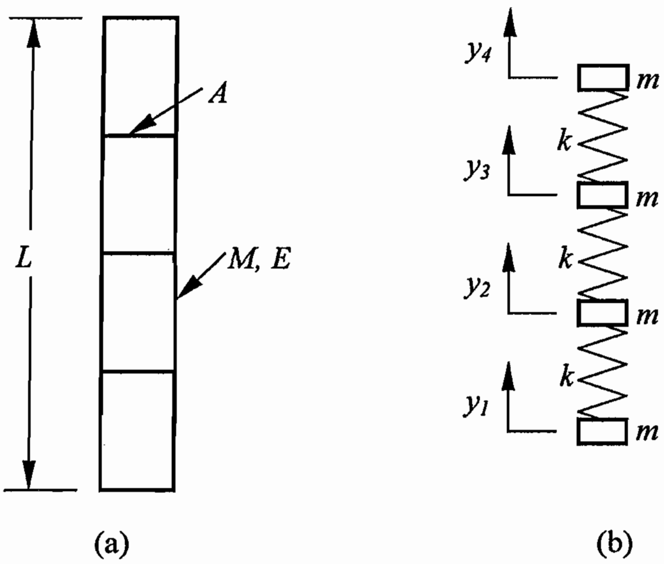

Fig. 1

4 A uniform straight rigid rod of length L and mass M is supported at its ends by two equal springs of stiffness k, whose other ends are attached to horizontal ground. A mass m is attached to the rod at position x as shown in Fig. 2. The system undergoes small vibration confined to the vertical plane.

(a) For the case where $m = 0$, write down the mode shapes of the system. Calculate the corresponding natural frequencies. [30%]

(b) For the case where $m = \varepsilon M$ ($\varepsilon << 1$), use Rayleigh's principle to show that the lower natural frequency becomes:

$$\omega^2 = \left(\frac{2}{1+\varepsilon}\right)\frac{k}{M}$$

and find the corresponding value of the higher natural frequency. [50%]

(c) What position of the additional mass m makes the difference between the two natural frequencies (i) largest; and (ii) smallest? Justify your answers. [20%]

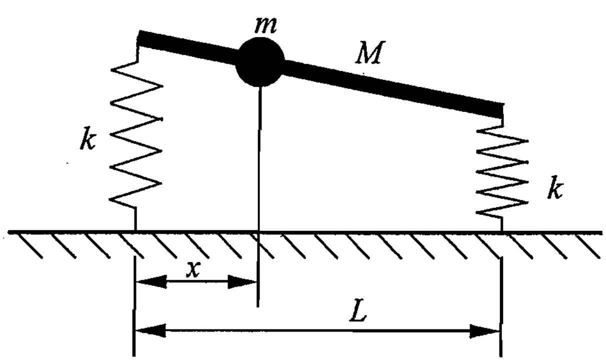

Fig. 2

END OF PAPER

ENGINEERING TRIPOS PART IIA

Module 3C6 Examination, 2007

Answers

1 (a) $\Omega_n^2 = \frac{Pn^2\pi^2}{ML}$ (c) $m = 0.056\,M$ at L/4 or 3L/4 (d) Same as (b)

2 (a)(i) $\alpha^4 = \frac{m\omega^2}{EI}$ (a)(ii) $\lambda = \frac{K}{EI}$

3 (a) $k = \frac{3EA}{L}$, $m = \frac{M}{4}$, $\mathbf{M} = m\mathbf{I}$ (4x4); $\mathbf{K} = k\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

(c) $\omega_1^2 = 0$ (exact); $\omega_2^2 = 0.6\frac{k}{m}$; $\omega_3^2 = 2\frac{k}{m}$ (exact); $\omega_4^2 = 3\frac{k}{m}$

(e) ±1mm at 180° out of phase with excitation

4 (a) Pure 'bounce' $\omega_1^2 = 2\frac{k}{m}$; Pure 'pitch' $\omega_2^2 = 6\frac{k}{m}$

(b) $\omega_2^2 \approx 6\frac{k}{m} - \frac{2k}{m}\left(\frac{1}{2} - \frac{x}{L}\right)^2 \varepsilon$

(c) largest difference when $x = L/2$; smallest difference when $x = 0$ or $x = L$.