ENGINEERING TRIPOS PART IIA

Tuesday 8 May 2007 2.30 to 4

Module 3D1

SOIL MECHANICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Special datasheets (19 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Rock is to be quarried, crushed and screened so that it can be used as granular fill in a large earth dam to be built in central Asia. The resulting fill consists of angular fragments ranging in size from cobbles to sand. An engineer conducts various trials on the compaction and compressibility of the fill.

(a) Tests reveal that the density of the solid material is about 2650 kgm^{-3}. Considering that the material contains particles with dimensions up to 100 mm, the engineer constructed a test rig consisting of a 900 mm diameter steel tube welded on to a steel base, into which a soil sample could be dumped and then compacted: see Fig. 1. It was discovered that loose dumping of 182 kg of dry fill from a shovel gave a finished average depth of 200 mm. An episode of vibration ultimately led to its average depth reducing to 162 mm. The tube was then emptied and a fresh layer of fill was dumped inside at its natural water content of 4%, prior to being compacted with hammers to replicate the energy input per unit volume of soil being compacted by machine in the field. A mass of 216 kg of damp fill was compacted to the mean target depth of 200 mm. Find the maximum and minimum voids ratios, and the projected voids ratio of the compacted soil. Estimate the projected relative density of the compacted fill, and briefly discuss the significance of this in relation to possible earthquake effects in service. [40%]

(b) A full-size piston was fitted and jacked downwards so that the compacted fill could be compressed inside the tube. The data for the 200 mm thick sample are given in terms of applied stress σ_v and compression Δh, as follows:

σ_v kPa	0	100	200	400	800	1600	3200
Δh mm	0	9	12	16	23	36	45

(i) Plot specific volume v versus log σ_v. Mark values corresponding to the maximum and minimum voids ratios obtained in part (a). Suggest an explanation for the relatively large compression observed on first applying load to the piston. Briefly discuss the possible reasons for the shape of the remaining compression curve. Explain why compression led to voids ratios much smaller than could be obtained by vibration. [30%]

(ii) Plot the same data in terms of vertical strain ε_v versus vertical stress σ_v. Treating a column of fill beneath the crest of the dam as though it were simply in one-dimensional compression with negligible friction on its sides, estimate the total self-weight compression during construction of a 200 m high dam made of the compacted fill. What problems may be caused by this compression? [30%]

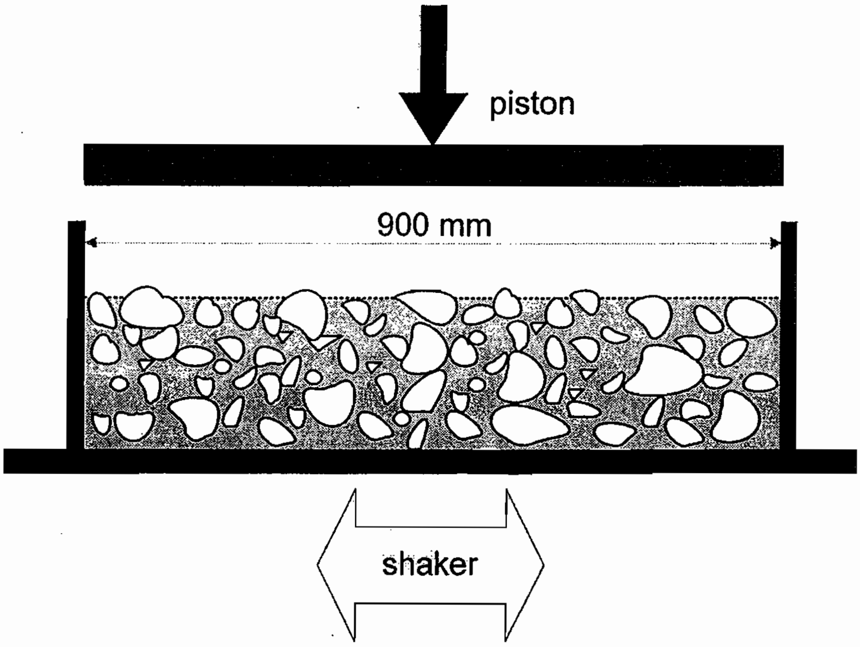

Fig.1

2 A rough rigid strip footing of width B is placed on top of clay as shown in Fig. 2. The undrained shear strength of the clay is s_u. A surcharge stress of σ_s is applied adjacent to the footing. A vertical load V and a horizontal load H (both per unit length) are applied at the centre of the footing.

(a) For the case of $H = 0$, an upper bound mechanism shown in Fig. 3 is proposed.

(i) The downward velocity of the footing is v. Complete the velocity diagram shown in Fig 3. [15%]

(ii) Show that the ultimate vertical force V evaluated from this mechanism is $[(10\sqrt{3}/3)s_u + \sigma_s]B$. [25%]

(b) A lower bound mechanism shown in Fig. 4 is proposed for combined V-H loading.

(i) Draw the Mohr's circle of stress for Zone A. [15%]

(ii) Draw the Mohr's circle of stress for Zone B. On the Mohr's circle of stress, plot the stress point ($\sigma_n(=V/B)$, $\tau_n(=H/B)$) acting on the plane parallel to the base of the footing. The angle between the major principal stress and the vertical direction is ψ as shown in the figure. [15%]

(iii) Using the stress fan concept, show that the combination of V and H when the footing fails is expressed by the following equation.

$$\frac{V}{Bs_u} = 1 + \pi + \frac{\sigma_s}{s_u} - \sin^{-1}\left(\frac{H}{Bs_u}\right) + \sqrt{1 - \left(\frac{H}{Bs_u}\right)^2}$$ [20%]

(iv) Evaluate H when the footing slides and find the corresponding maximum vertical load V using the above equation. [10%]

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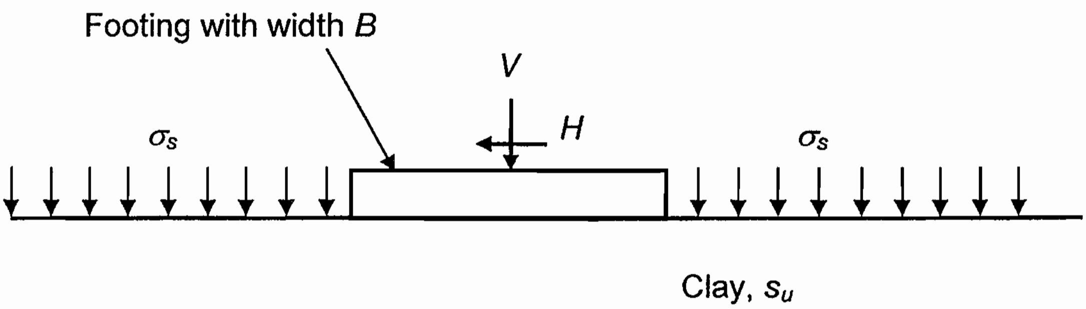

Fig. 2

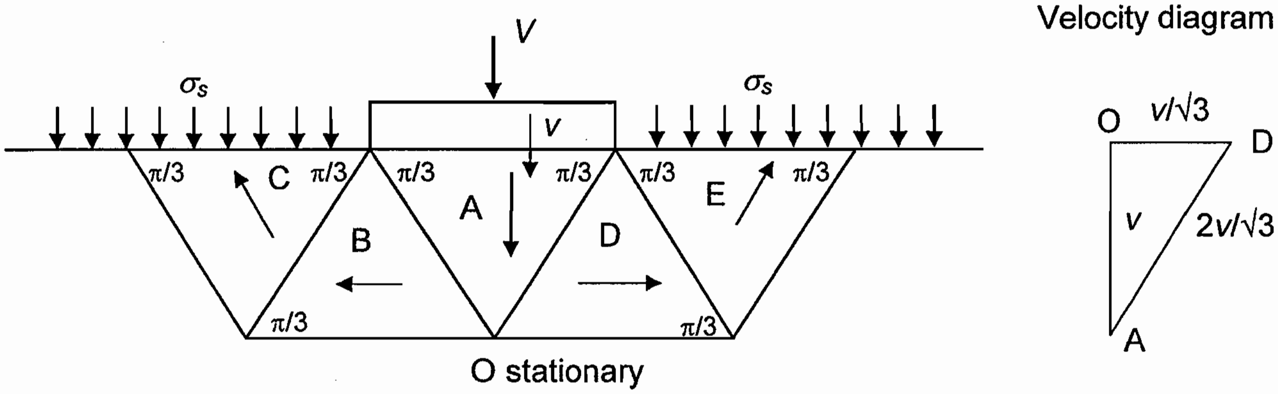

Fig. 3

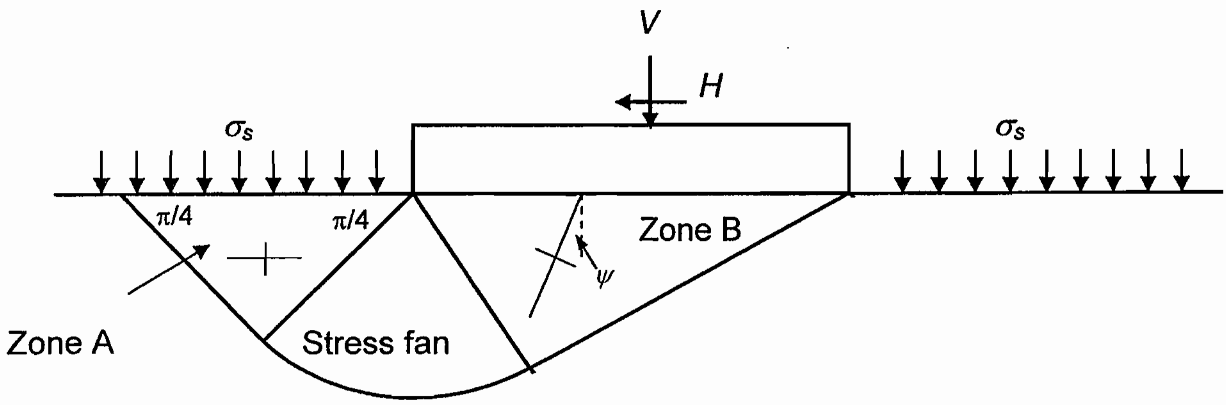

Fig. 4

(TURN OVER

3 (a) The bearing capacity q_f of a foundation placed on sand shown in Fig. 5 can be estimated from the following equation.

$$q_f = V/B = (1/2)\, N_\gamma \gamma' B + N_q\, \sigma'_{v0}$$

where V is the ultimate vertical load, B is the width of the foundation, γ' is the effective unit density of the sand, σ'_{v0} is the effective surcharge stress and N_γ and N_q are the bearing capacity coefficients that are a function of friction angle ϕ'. The first term on the right hand side corresponds to the contribution from the self-weight of the soil, whereas the second term corresponds to the contribution from the surcharge load.

(i) The bearing capacity coefficient N_q can be estimated from the lower bound mechanism shown in Fig. 6. By ignoring the self-weight of the sand, sketch the Mohr's circle of stress for Zone A. [15%]

(ii) On the same graph, sketch the Mohr's circle of stress for Zone B. [15%]

(iii) Using the stress fan concept, relate the two Mohr's circles of stress and show that N_q can be expressed by the following equation.

$$N_q = \frac{(1+\sin\phi')}{(1-\sin\phi')} \exp(\pi \tan\phi')$$ [20%]

(b) A rigid concrete block of 3 m width and 3 m depth is embedded in sand as shown in Fig. 7. The embedment depth is 2 m. The length of the block is much larger than the width and hence the analysis is performed assuming two dimensional plane strain conditions. The saturated unit weight of the sand is 19 kNm^{-3}, whereas the dry unit weight is 16 kNm^{-3}. The sand has a friction angle of 30 degrees. The unit weight of the concrete is 24 kNm^{-3}. In Eurocode7 (2004), the bearing capacity coefficient N_γ is related to N_q by $N_\gamma = 2(N_q - 1)\tan\phi'$. Assume the unit weight of water is 10 kNm^{-3}, determine the ultimate load V that can be applied on top of the concrete block for the following conditions (see Fig. 7):

(i) when the sand is dry; [15%]

(ii) when the water table is at the base of the block; [15%]

(iii) when the water level is at the top of the block. [20%]

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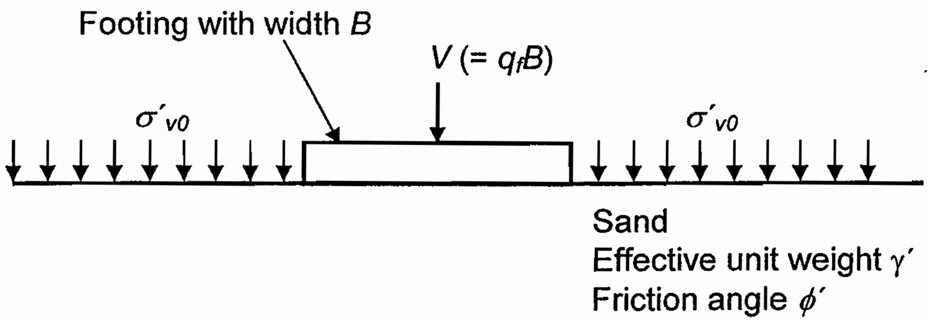

Fig. 5

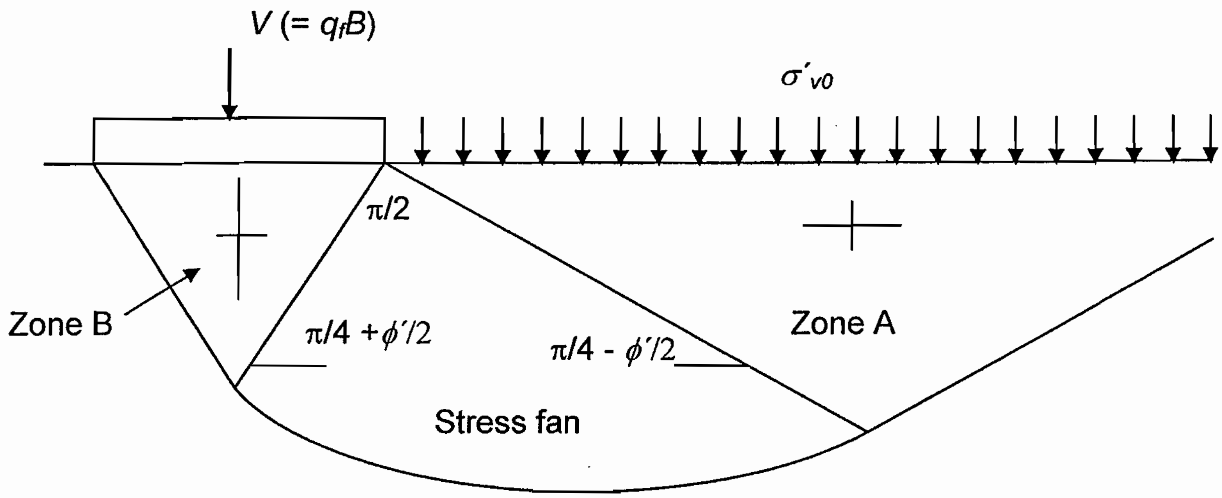

Fig. 6

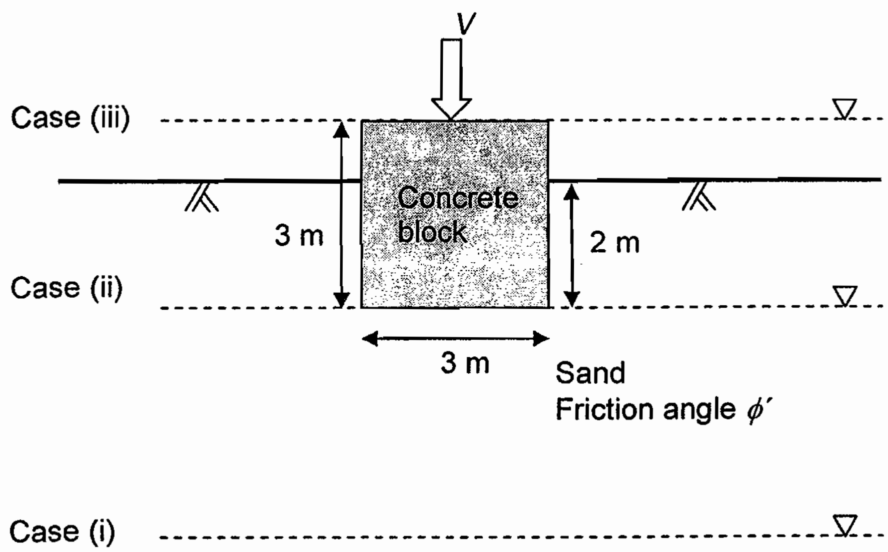

Fig. 7

(TURN OVER

4 (a) A normally consolidated clay lies just submerged under a horizontal free surface. Explain why both its permeability k and tangent stiffness E_o should be expected to vary strongly with depth z, whereas its bulk density would vary only weakly. Find an approximate expression for the clay's vertical coefficient of consolidation expressed as a function of z, $C_v(z)$, solely in terms of the specific gravity of soil solids G_s, the plastic compression index λ, the permeability $k(z)$, and z. [40%]

(b) The functional relationship $k(z)$ for a normally consolidated clay site can often be written sufficiently accurately as an inverse relationship normalised by the permeability k_1 of the clay at 1 m depth:

$$k = k_1 \, \frac{1}{z}$$

Modify the expression derived in (a) to establish an expression for C_v applicable to high plasticity normally consolidated clays with mechanical properties similar to London Clay. [20%]

(c) Filling is to take place to a depth of 2 m on a site already comprising 2 m of old fill which has been placed over 4 m of normally consolidated clay, below which there is a deep stratum of dense sand. The long-term water table is expected to remain where it has been for many years, at the base of the old fill. Both old and new fill is sandy gravel which can be taken to have a unit weight of 20 kNm^{-3}. The clay can be taken to have properties similar to London Clay. A continuous core was taken from the clay which revealed an additional 20 mm layer of sand at its centre. An oedometer sample was trimmed from the clay core 1 m below its interface with the old fill. It was brought into equilibrium under a vertical stress of 50 kPa after which its thickness was estimated as 20 mm. The imposition of a further 40 kPa of vertical stress caused a compression of 0.5 mm after 45 minutes and an ultimate compression tending towards 0.85 mm. Making clear your assumptions, estimate the settlement due to the new fill after 3 months, 12 months and ultimately. [40%]

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Soil Mechanics Data Book

Data Book 2005/2006

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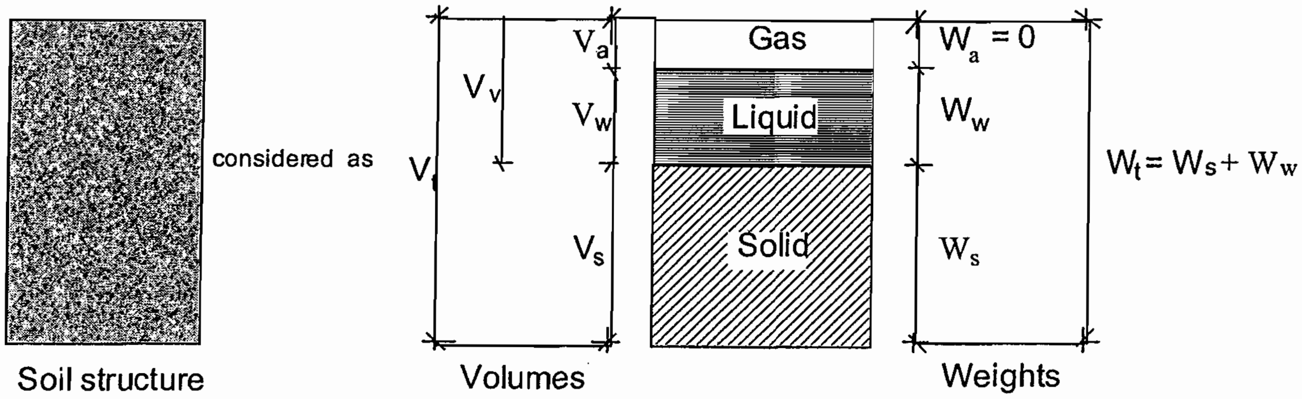
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

Classification of particle sizes:–

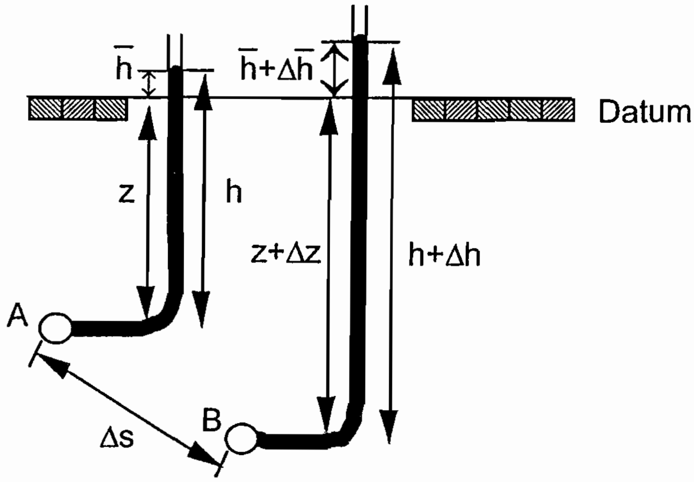
Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D equivalent diameter of soil particle

D_{10}, D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U uniformity coefficient D_{60} / D_{10}

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} >$ 1μm	:	$k \cong 0.01\,(D_{10}\text{ in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

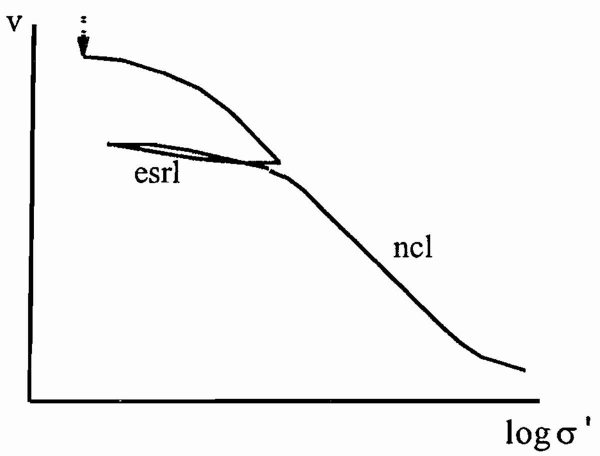
Saturated capillary zone

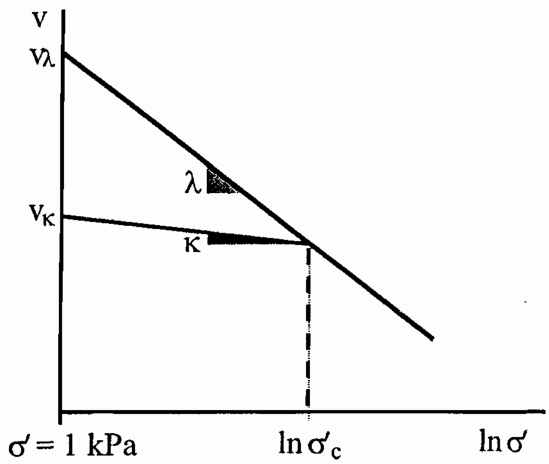
$$h_c = \frac{4T}{\gamma_w d}$$: capillary rise in tube diameter d, for surface tension T

$$h_c \approx \frac{3\times10^{-5}}{D_{10}}\ \text{m}$$: for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

ensional Compression

• Fitting data





Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

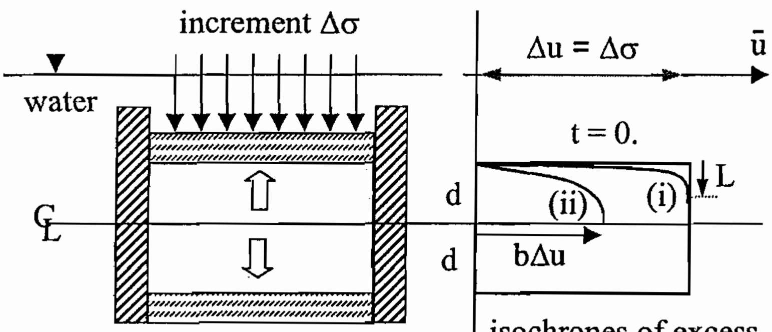
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

ıal Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\frac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\frac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

l strain components

- **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

- **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma_1', \sigma_2', \sigma_3'$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

ıtus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	$= \sigma'_a + u$
total radial stress	σ_r	$= \sigma'_r + u$
total mean normal stress	p	$= (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	$= (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	$= \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	$= q/p'$
axial strain	ε_a	
radial strain	ε_r	
volumetric strain	ε_v	$= \varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	$= \frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	$= \sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	$= p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

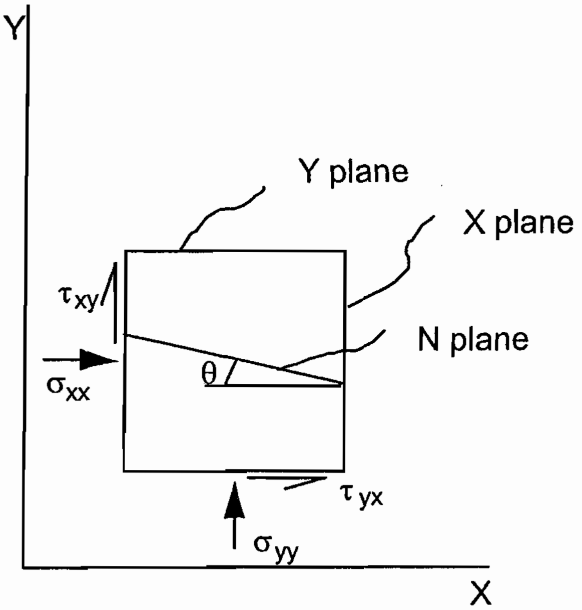
isotropic compression in which p' increases at zero q

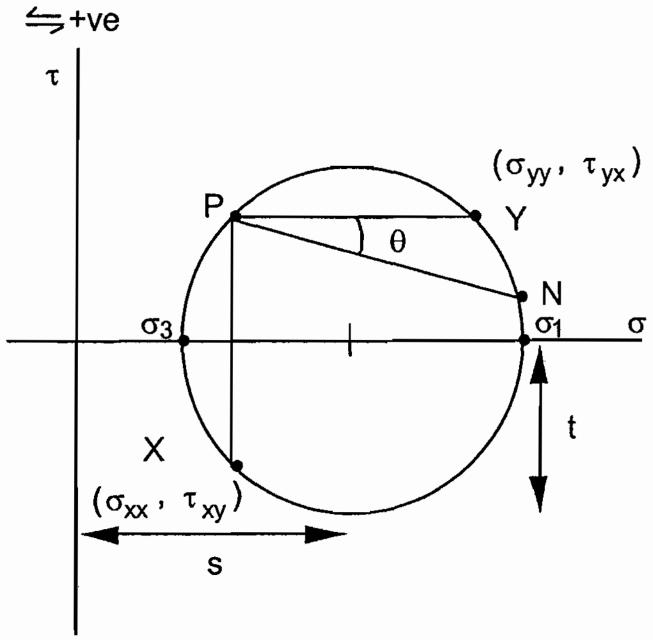
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

- **Mohr's circle of stress (1–3 plane)**

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P: the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

ffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

- **Interchangeable parameters for stress combinations at yield, and plastic strain increments**

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan \phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin \phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

- **General equations of plastic work**

Plastic work and dissipation $\quad \sigma^* \delta\varepsilon^* + \tau^* \delta\gamma^* = \mu^*_{crit} \sigma^* \delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*} \cdot \dfrac{d\gamma^*}{d\varepsilon^*} = -1$

- **General yield surface**

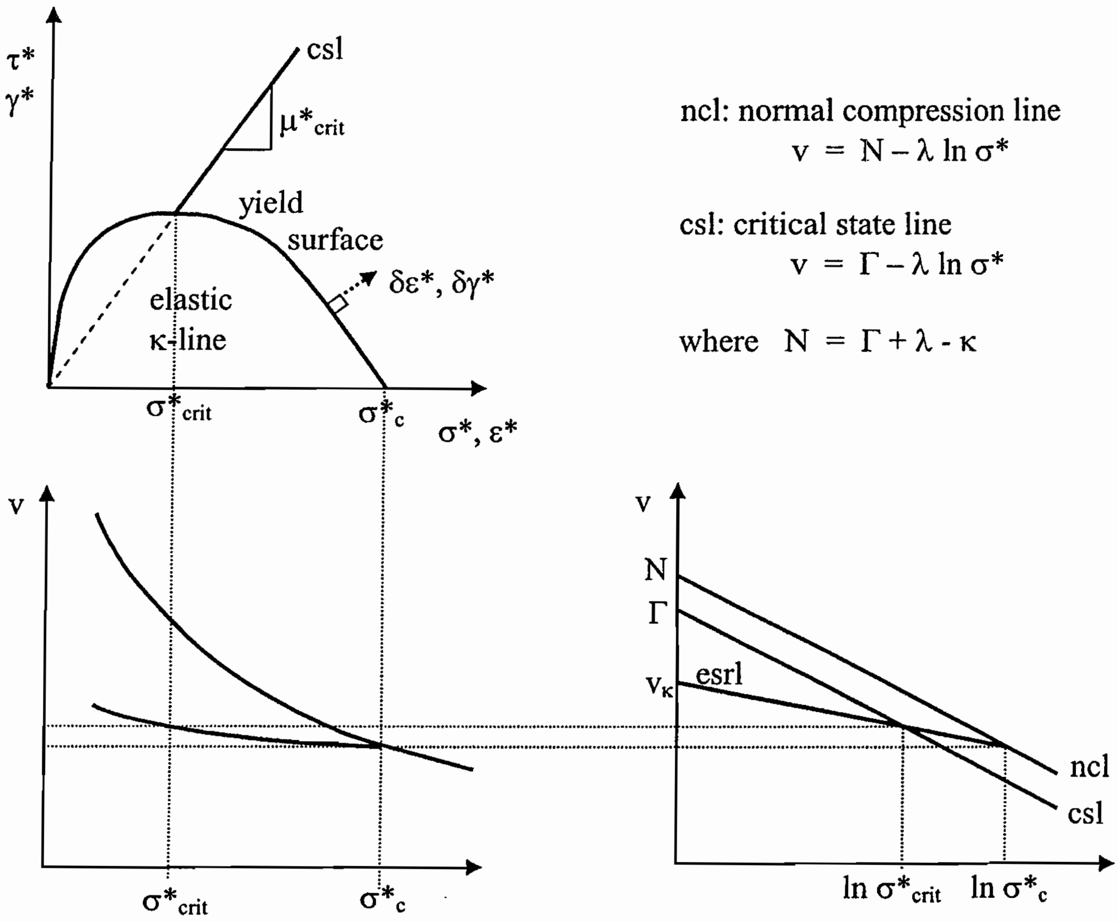
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit} . \ln \left[\frac{\sigma_c^*}{\sigma^*}\right]$$

- **Parameter values which fit soil data**

	London Clay	**Weald Clay**	**Kaolin**	**Dog's Bay Sand**	**Ham River Sand**
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

- **The yield surface in (σ*, τ*, v) space**



ncl: normal compression line

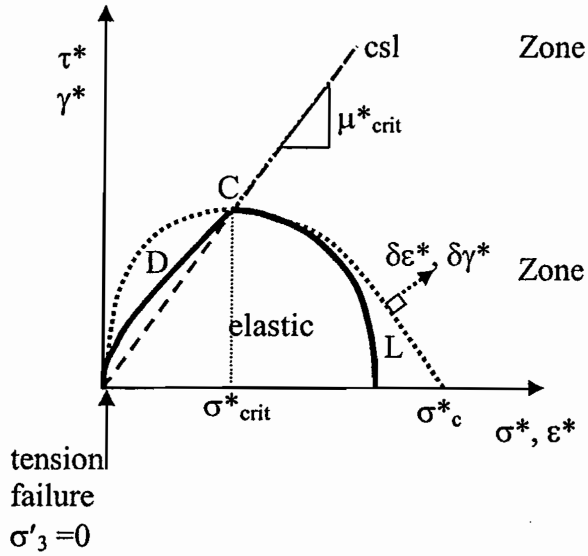
$$v = N - \lambda \ln \sigma^*$$

csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$

- **Regions of limiting soil behaviour**



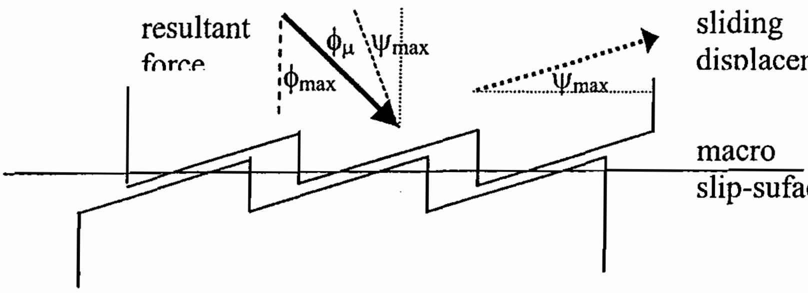
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

: friction and dilation

- **Friction and dilatancy: the saw-blade model of direct shear**

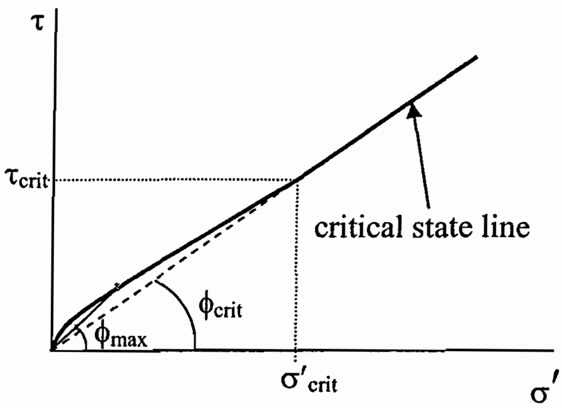


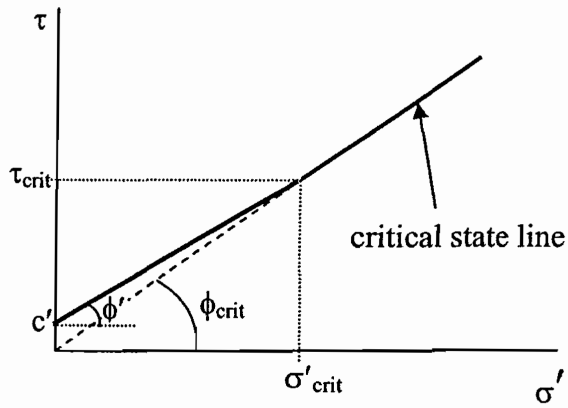
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

- **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

ınd dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

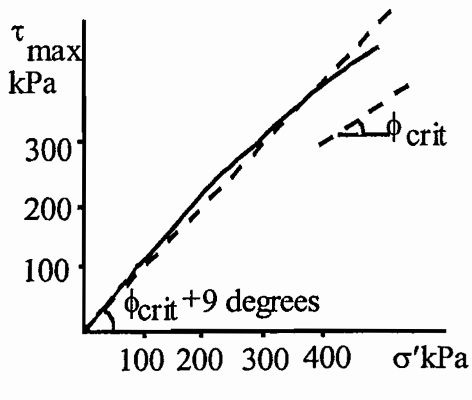
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

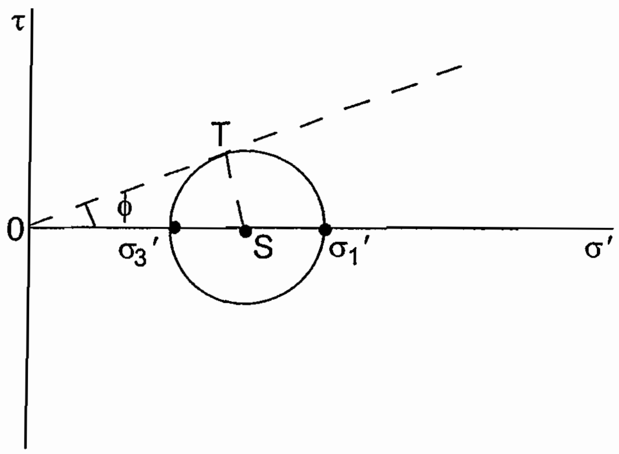
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1, \sigma' = 400$ kPa

nt) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

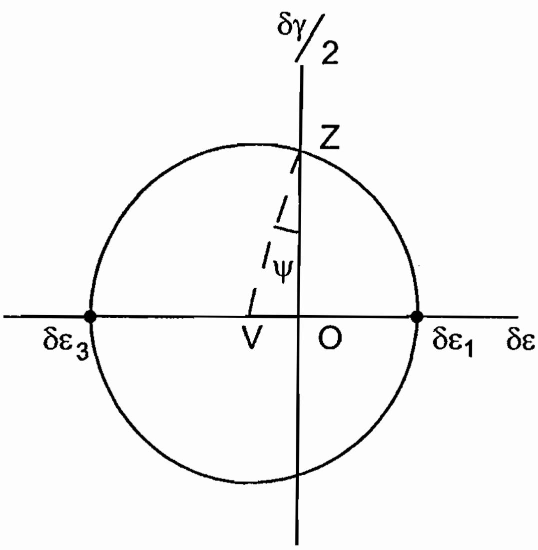
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

- **Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane**



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Cohesive material $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

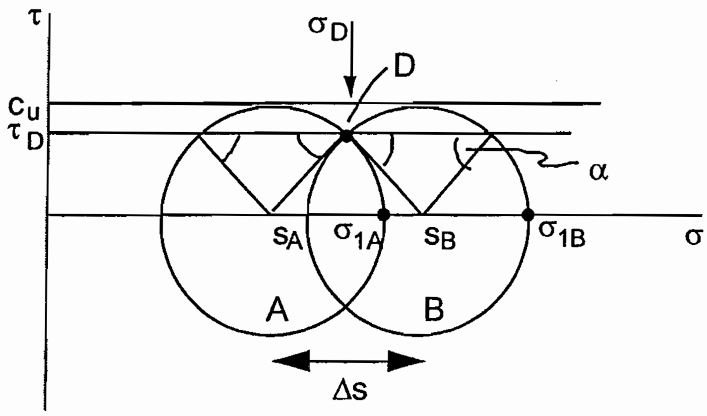
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = Ac_u x$$

- **Stress conditions across a discontinuity**

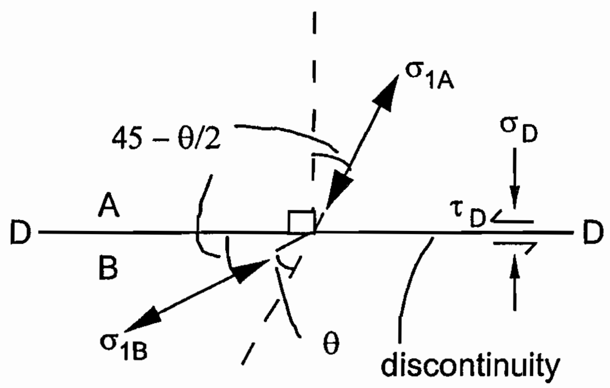


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$\theta = 30°$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

tional material $(\tau/\sigma')_{max} = \tan\phi'$

- **Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

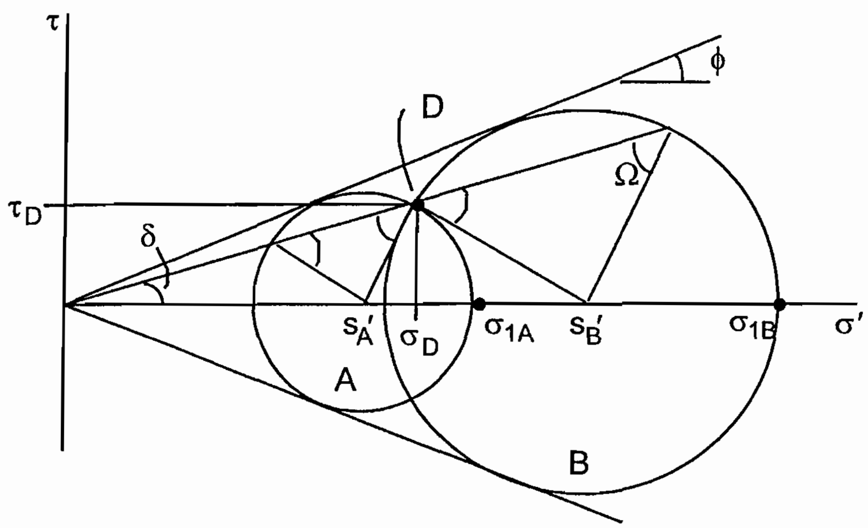
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

- **Stress conditions across a discontinuity**



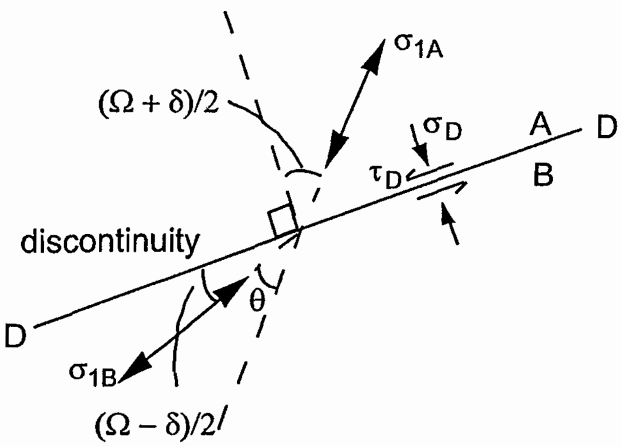
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta\tan\phi)$

l earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \frac{\delta A}{2\pi r}$

small shear strain $\gamma = \frac{2\rho}{r}$

Radial equilibrium: $r\frac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\frac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$

ation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2\, B / L$$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1}(h/B)$ (or h/D for a circular foundation)

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}}$ or $\frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$

If $V/V_{ult} < 0.5$: $H = H_{ult} = Bs_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$ (Taiebet & Carter 2000)

'Coulomb) soil, with friction angle ϕ.

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

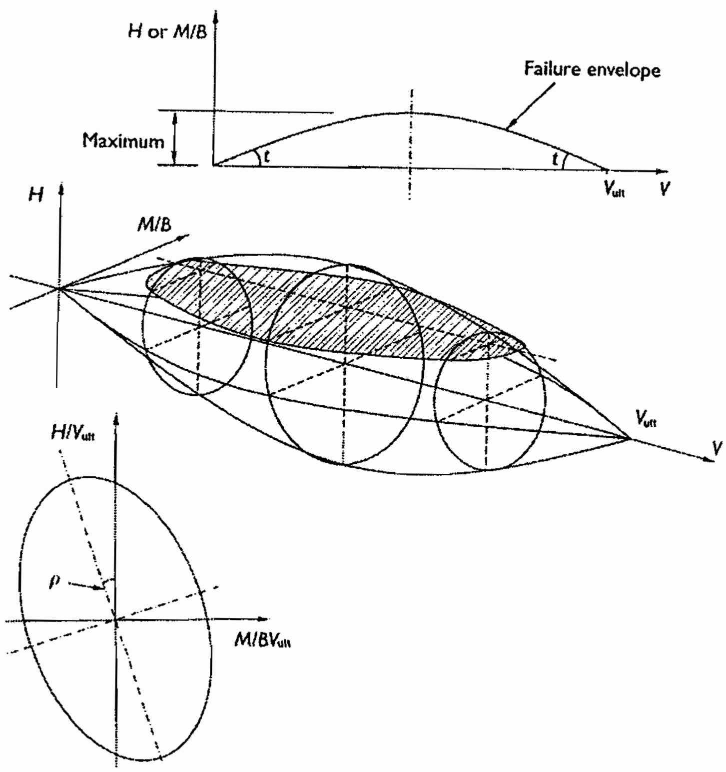
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1) \tan \phi$$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054 e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663 e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, $H/V = \tan\phi$, during sliding.

3D1 SOIL MECHANICS
ANSWERS

1. (a) $e_{max} = 0.849$, $e_{min} = 0.500$, $e_{comp.} = 0.620$, $I_D = 66\%$
   (b)(i) –
   (ii) 27 m

2. (a) (i), (ii) –
   (b) (i), (ii), (iii) –
   (iv) $H = Bs_u$, $V = (1+\pi/2+2\sigma_s/s_u)Bs_u$

3. (a) (i), (ii), (iii) –
   (b) (i) 3.00 MN
   (ii) 2.36 MN
   (iii) 1.68 MN

4. (a) $(G_s - 1)kz/\lambda$
   (b) $10.9k_1$ m^2/s
   (c) Settlement at 3 months = 50 mm
   Settlement at 12 months = 98 mm
   Ultimate Settlement = 156 mm