ENGINEERING TRIPOS PART IIA

Wednesday 9 May 2007 9 to 10.30

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Special datasheets (10 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Outline the concept of a 'load path' in the design of structures to carry given applied loads, illustrating your answer by an example. In what circumstances, and with what limitations in the different structural materials, may an engineer designing or checking a structure for the ultimate limit state (ULS) choose a load path arbitrarily, without having to know or calculate the actual behaviour of the structure under load? Are any characteristics desirable in a load path, to make it better than other possible paths? [25%]

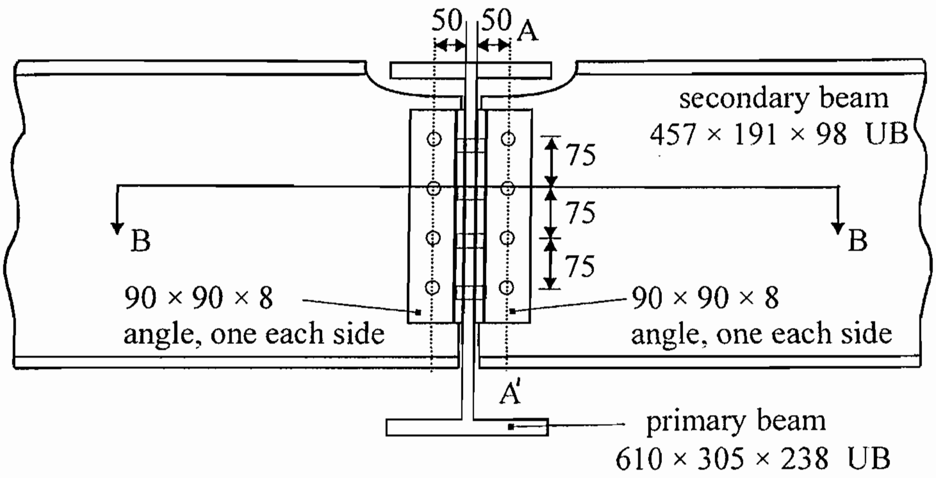
(b) Figure 1 shows a joint, made using four pieces of angle, between a primary beam and two secondary beams (in the same plane, one each side of the primary), all in S355 steel. There are sixteen 20 mm diameter bolts in 22 mm holes. The ULS shear capacity of each bolt per shearing plane is 375 MPa on its cross-section, and the ULS bearing capacity in each plate is an average stress of 550 MPa on the nominal bearing area.

(i) An engineer checks a load path through the joint in which each secondary beam applies to the primary beam only a single downward force. This force is assumed to act along the vertical line of bolts through the secondary web (e.g. A-A'). Determine the maximum (ULS) value of this force.

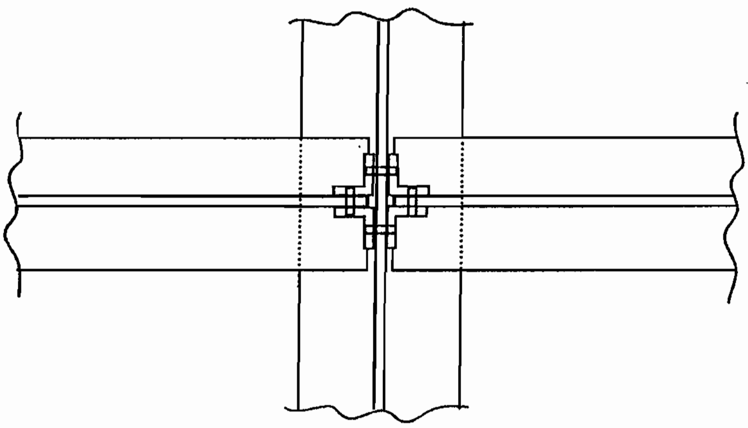
Supposing that this maximum force is applied simultaneously by both secondary beams, what are the design forces on the bolts through the primary web? If the ULS tensile capacity of a bolt is 500 MPa on its nominal area, and the sum of the ratios of applied shear and tensile forces to the respective bolt capacities must not exceed unity, is the connection to the primary web adequate? What other checks on the load path through this joint should be carried out? [50%]

(ii) In a particular case the required ULS vertical reaction is only 350 kN on each secondary beam. An engineer points out that it might then be advantageous to consider another load path taking a hogging moment through the joint to restrain the secondary beams against rotation. Considering one vertical line of bolts through a secondary web, estimate the maximum value of this hogging moment at A-A'. What are now the ULS design forces on the bolts in the primary web? [25%]

(cont.



side view
(all dimensions in mm)



section B-B

Fig. 1

(TURN OVER

2 (a) An engineer is interested in the maximum span over which a uniform beam, fixed at both ends against rotation and bending about its major axis, can carry its own weight plus a uniform live load of 20 kNm^{-1} applied across the entire span. The extra deflection at midspan due to the working load is not to exceed $1/300^{th}$ of the span, and the beam is to carry without collapse an ultimate load with factors γ_f of 1.4 on dead load and 1.6 on live load. Take $\gamma_m = 1.05$ for ULS, and neglect any effects due to shear force or buckling. Note that the central deflection for a uniformly-loaded fixed-ended beam is $1/5^{th}$ of that for a simply-supported beam.

Estimate the maximum span for

(i) a 762 × 267 × 134 UB in ductile mild steel, where the Young's modulus and characteristic yield stress of mild steel can be taken as 210 GPa and 220 MPa respectively and [25%]

(ii) a 500 mm deep by 200 mm wide rectangular section in oak stressed along the grain. For oak, the Young's modulus can be taken as 22 GPa, the characteristic ultimate stress as 60 MPa and the density as 0.8 Mgm^{-3}. [25%]

(b) For the 762 × 267 × 134 UB steel beam in part (a):

(i) Discuss the possibility that the strength of the beam could be reduced by various forms of buckling, thus reducing the allowable span. Is shear force likely to be significant in this regard? [20%]

(ii) Estimate roughly (neglecting J) the maximum spacing of devices restraining the flanges against lateral movement which would be necessary to ensure that 90% of the full plastic moment of this steel beam can be attained. [30%]

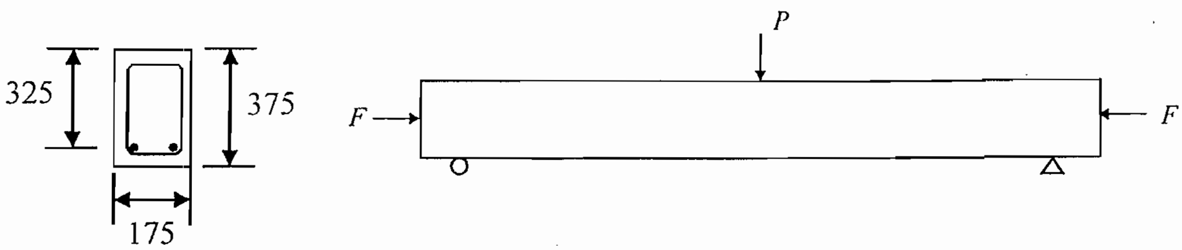
3 An existing reinforced concrete beam of height 375 mm, effective depth 325 mm and width 175 mm is shown in Fig. 2(a). The longitudinal reinforcing steel consists of two 20 mm diameter bars and the internal shear reinforcement consists of 10 mm diameter stirrups at 125 mm spacing throughout the length of the beam. The characteristic yield stresses of the longitudinal and the shear reinforcement are 460 MPa and 250 MPa respectively. The concrete has a characteristic compressive cube strength of 37 MPa which is analogous to a characteristic cylinder strength of 30 MPa. The partial material safety factors for concrete and steel are 1.5 and 1.15 respectively and $\varepsilon_{cu} = 0.0035$ and $\varepsilon_y = 0.002$. The beam is simply supported and has a span of 4.5 m. A point load P is applied at the mid-span of the beam. It can be assumed that the beam fails in flexure, the self-weight of the beam can be neglected and the load factor $\gamma_f = 1$.

(a) Assuming the beam is under-reinforced and that no external axial force is applied, find the maximum value of P that the beam can carry. [20%]

(b) An axial load, F, of 200 kN is now applied to the beam along its centreline (as shown in Fig. 2(b)). Find the maximum value of P that can be sustained in combination with this axial load. Discuss briefly how this answer compares with that found in (a) and reasons why there may, or may not, be a difference. Without carrying out any further calculations, explain how you would expect the allowable moment capacity of the beam to change if the axial load continues to increase. [40%]

(c) For the case with no axial load, find the shear capacity of the beam assuming a strut angle of 21.8°. (Note that on the 3D3 Concrete datasheet for shear $f_{cd} = f_{ck} / \gamma_m$). [25%]

In the design of shear reinforcement for concrete, discuss reasons why there are limits on the allowable strut angle. What factors will influence a designer's choice of angle within these limits? [15%]



all dimensions in mm

(a)

not to scale, internal reinforcement not shown

(b)

Fig. 2

4 (a) Briefly discuss the nature of anisotropic materials and factors of particular importance when designing with such materials. [20%]

(b) A double lap tension joint is shown in Fig. 3(a). The central C24 timber is connected on either side to 75 mm wide C16 timbers using a single bolt of diameter 10 mm in a pre-drilled hole. The load is applied in the direction of the grain. In the strength formulae for joints in the 3D3 Timber datasheet, assume k_{mod} =0.9 and γ_m= 1.3.

(i) Assuming the bolt is rigid (no hinges form in the bolt), derive expressions reflecting likely failure modes. [15%]

(ii) Using your answers from (i), find an optimum value of thickness for the C24 timber to maximise the strength of the joint. For this calculated thickness, find the load capacity of the joint. [15%]

(c) A simply-supported rectangular C16 timber beam with a width of 75 mm and a span of 3.5 m is in an indoor environment and subjected to permanent loading. Assume k_{ls} =1, k_h =1, k_{crit} =1 and γ_m= 1.3. At a cross-section of interest, a maximum shear force of 10 kN occurs simultaneously with a bending moment of 6 kNm.

(i) Find the minimum height of beam required to satisfy shear and flexural strength requirements. Based on your answer, do you think in practice k_{crit} is likely to be 1? Explain your reasons but do not carry out any further calculations. [30%]

(ii) There is a proposal to notch the beam at the support (as shown in Fig. 3(b)). Why might such a notch be desirable? What checks would need to be carried out to ensure the integrity of the system? [20%]

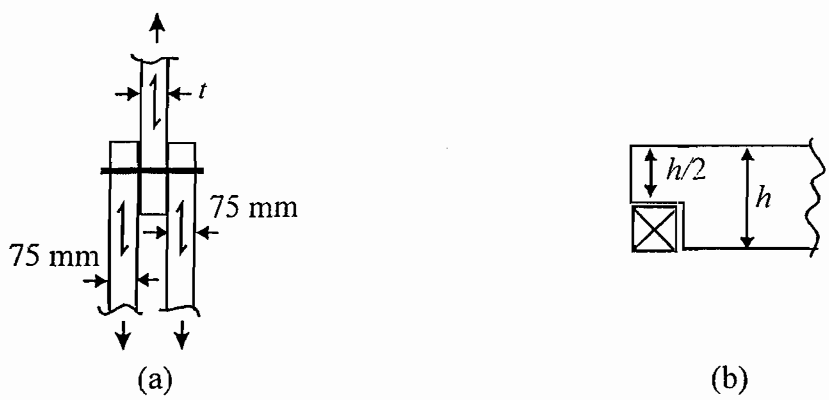


Fig. 3

END OF PAPER

Module 3D3 Selection of material and shape – design for bending

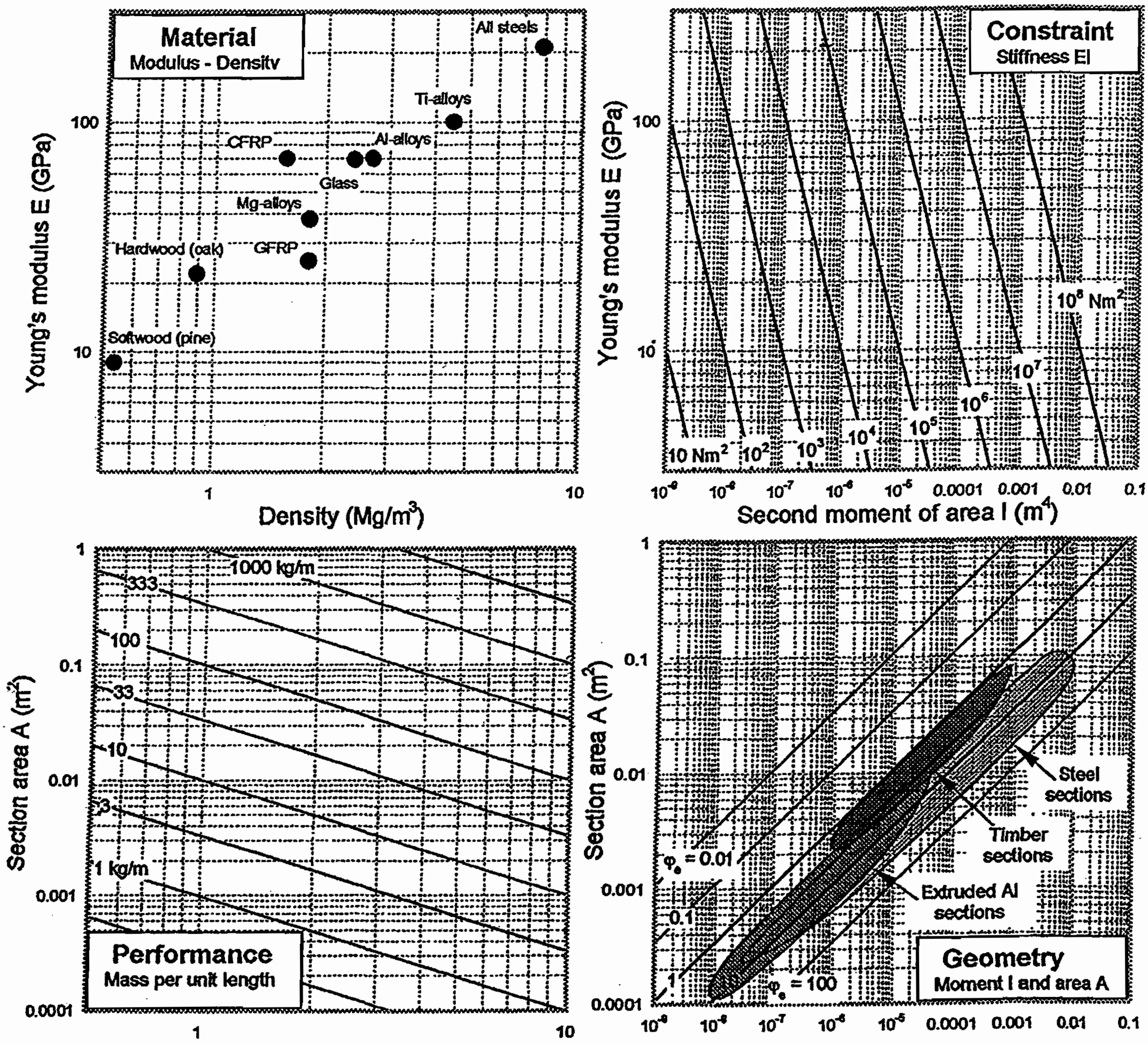


Figure 1. The chart-assembly for exploring structural sections for stiffness limited design. Each chart shares its axes with its neighbours.

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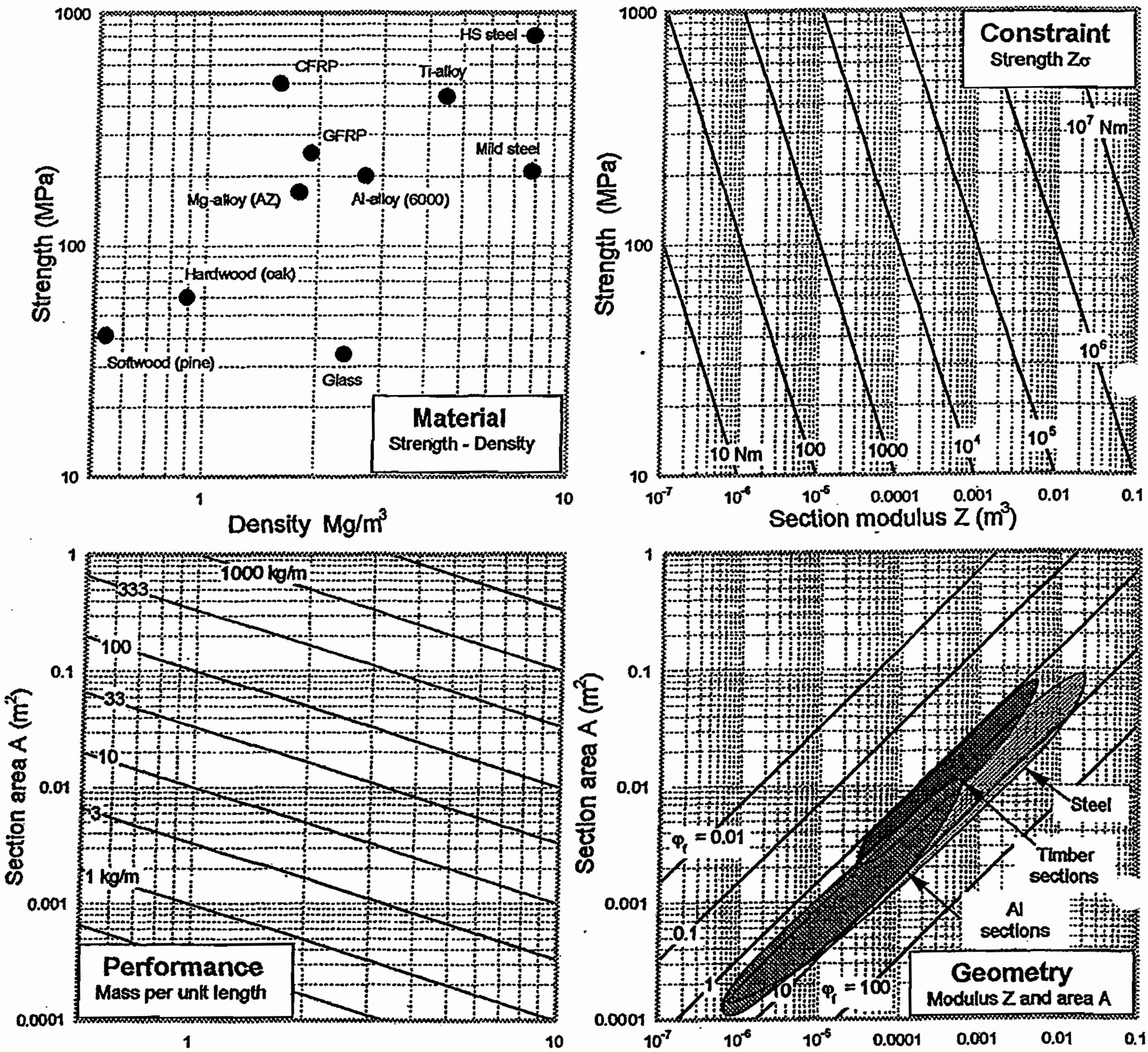


Figure 2 The chart-assembly for exploring structural sections for strength limited design. Like that for stiffness, each chart shares its axes with its neighbours.

Note: concept and charts are due to Prof. Michael Ashby, 2002
(see M.F. Ashby 'Materials selection in mechanical design' 2nd Edition, Butterworth 1999, JA163).

Materials available (see Structures Data Book 1999 pp. 1 and 11)

The two common structural steels BS EN – S275 and S355 have characteristic yield strengths σ_y of 275 and 355 MPa respectively. Both satisfy the usual criteria for plastic design (adequate ratio of UTS to yield strength; adequate elongation to fracture). In design, the calculated strength (e.g. buckling resistance) for either material would be divided by a specified partial safety factor γ_m, often 1.1. Strength design is at ULS, with specified partial safety factors γ_f on loads, often 1.4 on dead load, 1.6 on live.

Tension members (axial force only)

Gross area A; net area A_n is A minus hole(s). Effective section A_e is KA_n but not greater than A, where factor K is 1.2 for S275, 1.1 for S355.

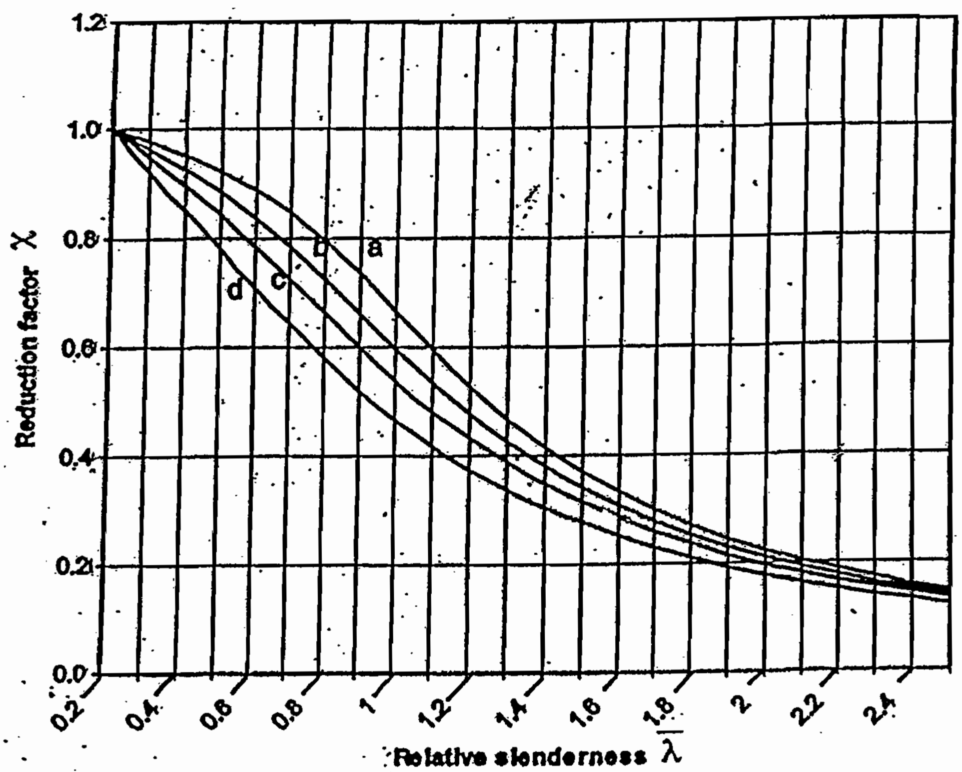
For eccentric connection, with area a_2 not connected at joint, effective area is often taken as $A_e - ca_2$, where factor c is 0.5 for bolted connections, 0.3 for welds.

Compression members (axial force only)

Joints do not normally control design, though criteria above should be checked. Base design against buckling on gross area A, yield strength σ_y, radius of gyration r, and column effective length L between points of contraflexure. Slenderness $\lambda = L/r$.

Define λ_0 as the slenderness at which the elastic critical stress for a perfect column equals the yield strength, so $\lambda_0 = \pi \{ E/\sigma_y \}^{1/2}$. Relative slenderness $\bar{\lambda} = \lambda/\lambda_0$

To allow for interaction between yield and buckling, use curves of reduction factor χ (on the full yield axial strength) plotted against the relative slenderness $\bar{\lambda}$. Typical curves are as shown below (these from the IStructE EC3 (Steel) Design Manual). Choice of empirical curve a to d depends on section type (extreme fibre distance y/r) and typical imperfection and residual stress magnitudes.



Beams (without axial force)

Moment - check maximum moment less than $\sigma_y Z_p$. Sections are Class 1 to 4, for ability to provide enough ductility for full plastic behaviour without local buckling.

Shear – yield strength q_w in shear often taken as $0.6\sigma_y$ for simplicity. But check for buckling in thin webs, for depth/thickness d/t and aspect ratio a/d, e.g. for a/d > 1, elastic critical average stress $q_{cr} = (0.75 + \{d/a\}^2)\,\{1000t/d\}^2$ in MPa.

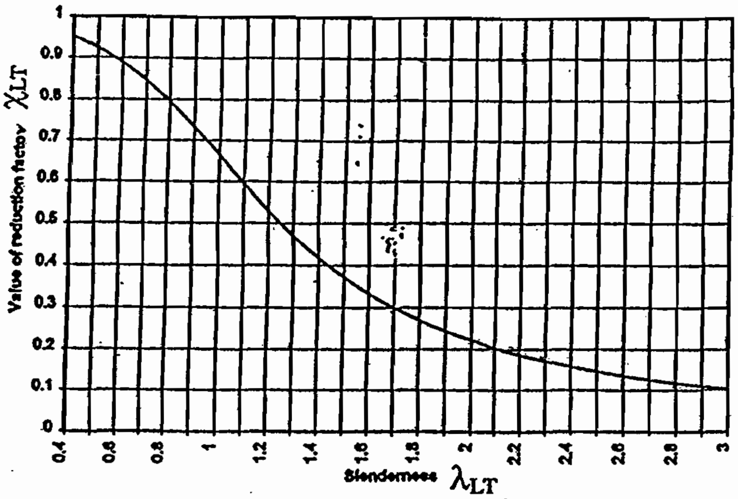
Bearing – check, using special formulae for the various failure modes, at supports and under high local loads, in case stiffener(s) are required.

Lateral-torsional buckling - for uniform bending moment M over a distance L between points where lateral displacement and rotation about beam axis are prevented, elastic critical value, with significant torsional stiffness due to restraint of warping,

$$M_c = \frac{\pi}{L}\sqrt{EI_{yy}\left\{GJ + \frac{\pi^2}{L^2}EC_w\right\}}.$$ For typical I-beam, basic torsion constant $J = \Sigma bt^3/3$

where b is the width of component plates of thickness t; and warping-restraint factor $C_w = D^2 I_{yy}/4$ where D is the distance between centres of flanges.

To allow for interaction between buckling and yield, again use empirical curves of the capacity reduction factor χ_{LT} against relative slenderness $\lambda_{LT} = \{M_p/M_{cr}\}^{1/2}$ (here M_{cr} is the hinge-position moment for elastic buckling).



Joints

Ductility of steel allows a reasonably simple equilibrium system to be envisaged for initial design – often with a transmitted force uniformly distributed across the various fasteners involved (particularly if they have similar properties). For a bolted joint in shear, a couple C about its centre can be taken simply by forces F_i on each bolt, perpendicular to the line to the centre and proportional to the distance d_i from the centre, so that $F_i = Cd_i/\Sigma d_i^2$, but other equilibrium systems can be envisaged.

Applied shear forces F on bolts are commonly checked against the shear strength (say $0.6\sigma_y$) of the bolt, depending on the number of shear planes activated; and against the bearing strength $\sigma_b dt$ in each plate (d is bolt diameter, t is plate thickness), despite the fact that bolts may not actually transmit force in this way. For M20 bolts, σ_y is typically 600 MPa, and in Codes nominal σ_b is typically of order 400 MPa in S275 steel.

(information mainly from Eurocode 3) CTM Jan. 2003

3D3 – Structural Materials and Design – Concrete Datasheet

Structural system	Span/effective depth ratio	
	EC2*	
	high	light
1. Simply supported beam, one-way or two-way spanning simply supported slab	14	20
2. End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	18	26
3. Interior span of beam or one-way or two-way spanning slab	20	30
4. Slab supported on columns without beams (flat slab), based on longer span	17	24
5. Cantilever	6	8

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 7.4N, NA.5 [10.2]

Table 10.1 Span versus depth ratio

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width cover	240 70	200 50	200 45
Slabs with plain soffit	thickness cover	170 45	125 35	100 35

Extracts from Table 4.1 [10.1]

Table 10.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

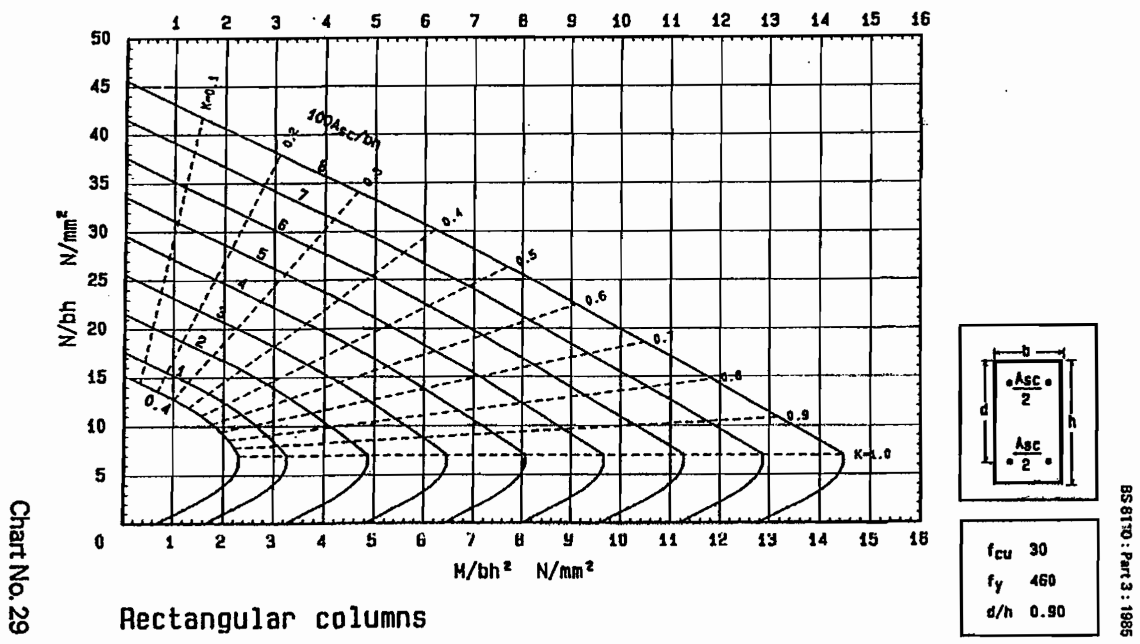


Fig 10.1 Interaction diagram from [10.3]

[10.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[10.2] Eurocode 2: Design of concrete structures, EN 1992-1-1:2004, UK National Annex –NA to BS EN 1992-1-1:2004
[10.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure

Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1-0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if $x/d = 0.5$

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Bond anchorage

$$l_b = \frac{f_y \phi}{4(2.25\eta_1\eta_2 f_{ctd})}$$

where: η_1 is 1.0 for good bond, 0.7 otherwise
η_2 is 1.0 for $\phi \leq 32$

Cracking

$$w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff}$$

where: k_1 is 0.8 for high bond, 1.6 for plain bars
k_2 is 1.0 for pure tension, 0.5 for bending
k_3, k_4 factors (in UK k_3=3.4 and k_4=0.425)
$\rho_{p,eff}$ is the effective steel ratio A_s/A_{ceff}

Shear

Without internal stirrups

$$V_{Rd,c} = [\frac{0.18}{\gamma_c} k(100\rho_1 f_{ck})^{1/3}] b_w d \geq (0.035 k^{3/2} f_{ck}{}^{1/2}) b_w d$$

where: f_{ck} is the characteristic concrete compressive cylinder strength (MPa).
$k = 1 + \sqrt{200/d} \leq 2.0$ (d in mm)
$\rho_l = A_s/b_w d \leq 0.02$

With internal stirrups

-Concrete resistance

$$V_{Rd,\max} = f_{c,\max}(b_w 0.9d)/(\cot\theta + \tan\theta)$$

where: $f_{c,\max} = 0.6(1 - f_{ck}/250) f_{cd}$

-Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d)(\cot\theta)/(s\gamma_s)$$

Columns – axial loading only

$$\sigma_u = 0.6\frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm) - 6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 – Structural Materials and Design – Timber Datasheet

			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens ‖	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens ⊥	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp ‖	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp ⊥	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod ‖	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod ⊥	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	Service class 2	Service class 3
Solid and glued laminated timber and plywood			
Permanent	0.60	0.60	0.50
Long-term	0.70	0.70	0.55
Medium-term	0.80	0.80	0.65
Short-term	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – Design shear stress

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – Design bearing stress

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$\lambda_{rel,m} = \sqrt{f_{m,k} / \sigma_{m,crit}}$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e)"

$$k_{crit} = \begin{cases} 1 & \text{for} \quad \lambda_{rel,m} \le 0.75 & (5.2.2c) \\ 1.56 - 0.75\lambda_{rel,m} & \text{for} \quad 0.75 < \lambda_{rel,m} \le 1.4 & (5.2.2d) \\ 1/\lambda_{rel,m}^2 & \text{for} \quad 1.4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm^2

For bolts up to 30 mm diameter at an angle α to the grain:

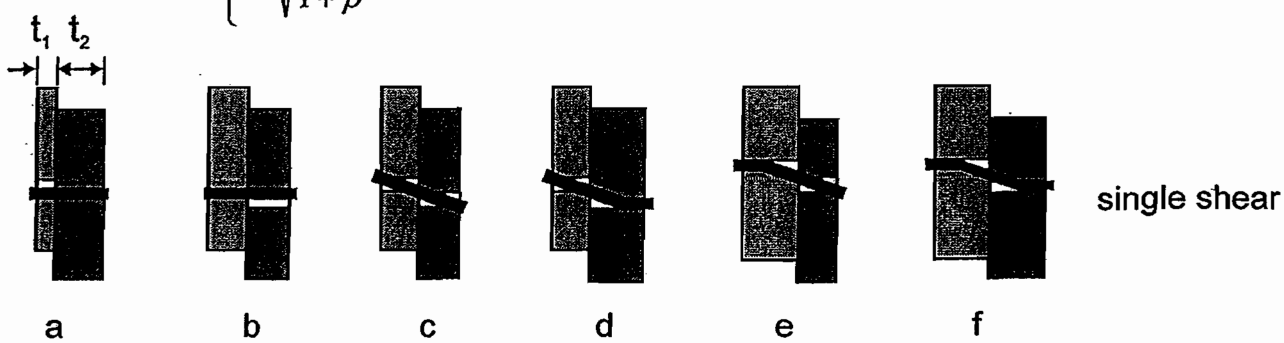
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear

$$R_d = \min. \begin{cases} f_{h,1,d}\, t_1 d & (6.2.1a) \\ f_{h,1,d}\, t_2 d\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}\, t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \dfrac{t_2}{t_1} + \left(\dfrac{t_2}{t_1}\right)^2\right] + \beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1 + \dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1.1\dfrac{f_{h,1,d}\, t_1 d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}\, d t_1^2}} - \beta\right] & (6.2.1d) \\ 1.1\dfrac{f_{h,1,d}\, t_2 d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}\, d t_2^2}} - \beta\right] & (6.2.1e) \\ 1.1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d} f_{h,1,d}\, d} & (6.2.1f) \end{cases}$$



Extract from [11.2] – Timber-to-timber and panel-to-timber joints

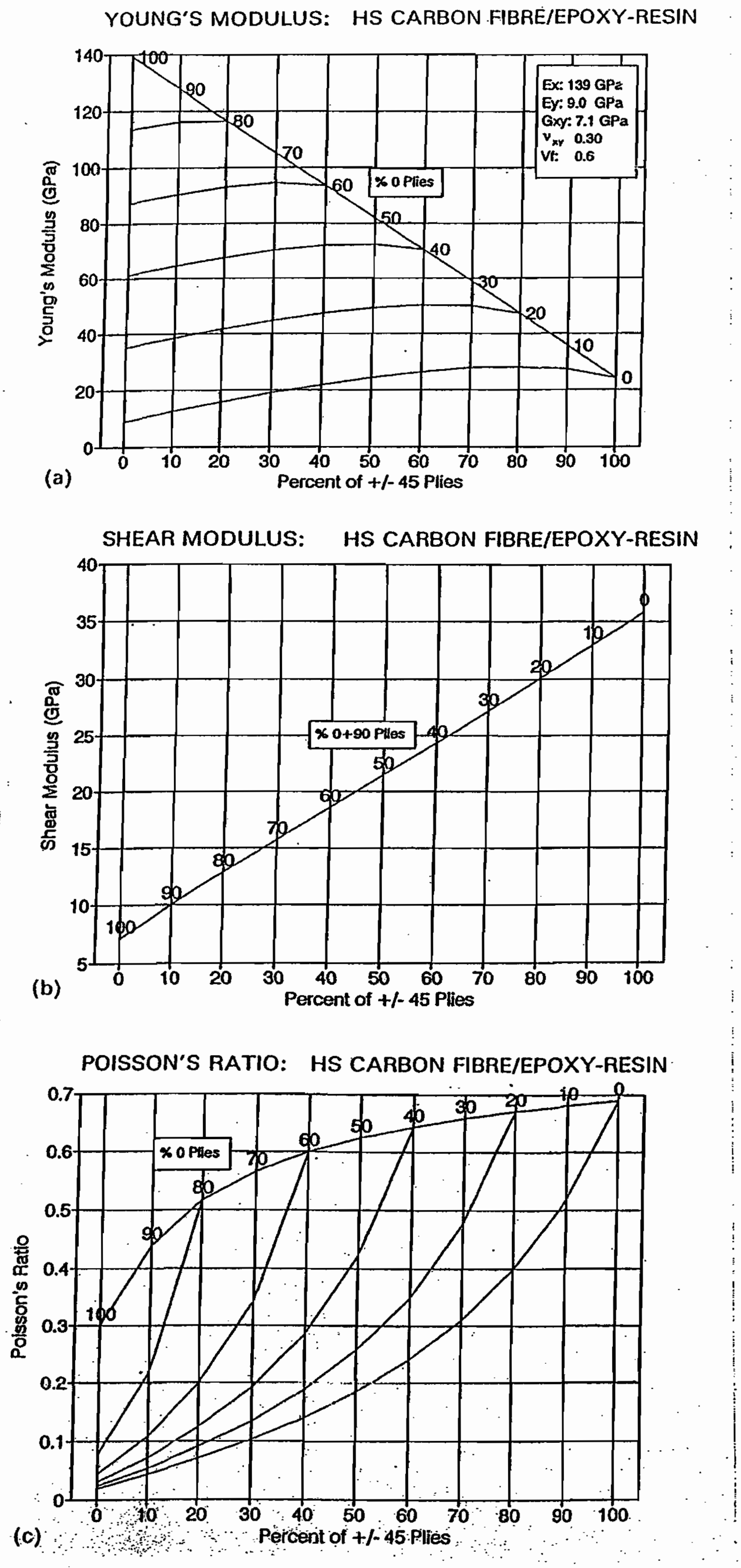


Figure 1. HS carbon/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

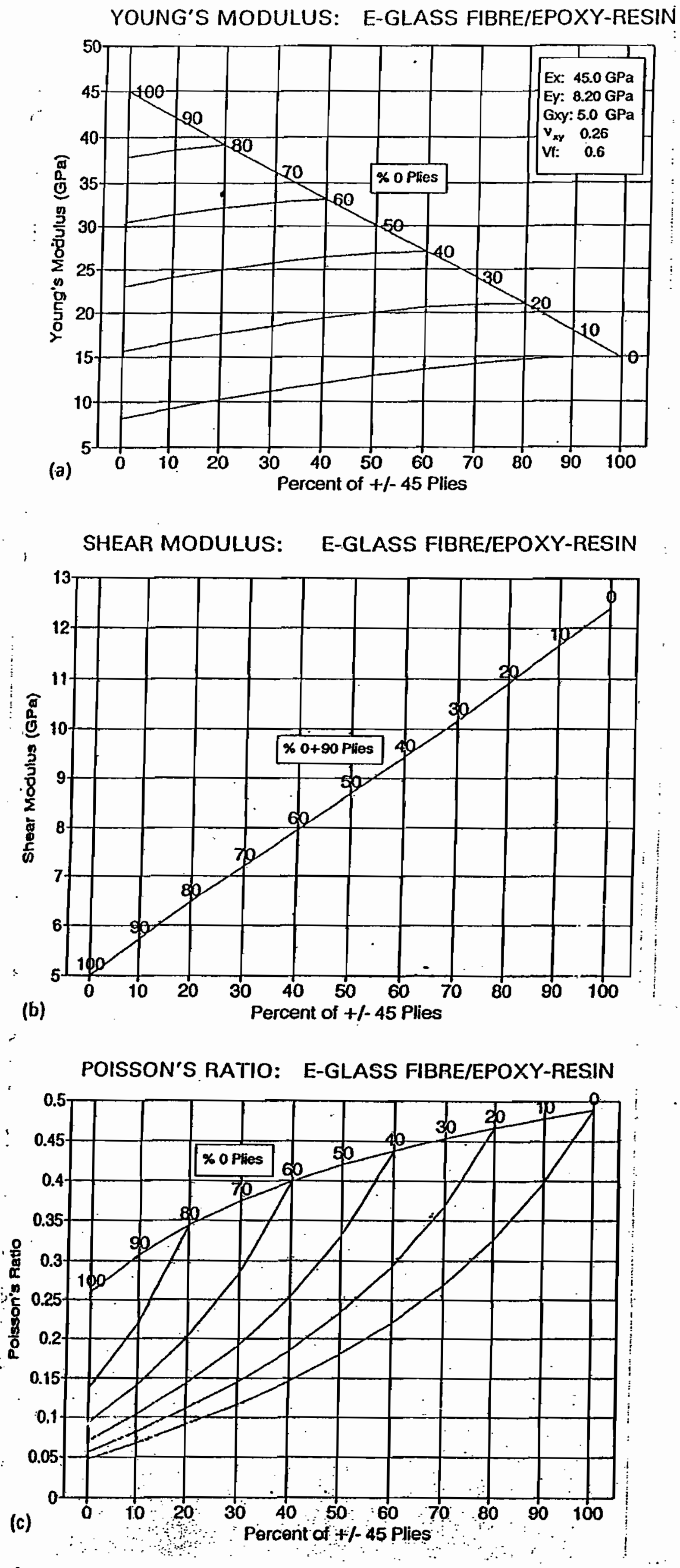


Figure 2. E-glass/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

[12.3] M.G. Bader, Materials selection, preliminary design and sizing for composite laminates. Journal Paper: Composites Part A, 27A, (1996), pp 65-70.

Engineering Tripos Part IIA, 2007

Paper 3D3 Structural Materials and Design

Answers

1. (b) (i) secondary beam V_{max} (bearing) = 501 kN, V_{max} (shear) = 942 kN
so bearing controls and maximum force is 501 kN
primary beam - $V_{applied}$ = 62.6 kN, $T_{applied}$ (top bolt) = 35.9 kN
connection is adequate

(ii) M_{max} = 22.5 kNm, $V_{applied}$ = 43.8 kN, $T_{applied}$ (top bolt) = 57.1 kN

2. (a) (i) L_{max} (strength) = 21.5m, L_{max} (deflection) = 27.3m
$\therefore$ strength controls – L_{max} = 21.5 m

(ii) L_{max} (strength) = 13.1m, L_{max} (deflection) = 14.3m
$\therefore$ strength controls – L_{max} = 13.1 m

(b) (ii) max spacing of restraints ≈ 3.5 m

3. (a) P = 61.7 kN

(b) P = 71.0 kN

(c) $V_{Rd,maxc}$ = 187 kN, $V_{Rd,s}$ = 200 kN $\therefore$ concrete capacity controls – V_{allow} = 187 kN

4. (b) (i) failure in C24 – $R = f_{h,1,d} \cdot t_1 \cdot d$, failure in C16 - $R = 2 f_{h,2,d} \cdot t_2 \cdot d$

(ii) t_1 = 133 mm, R_{max} = 23.7 kN

(c) (i) h_{shear} = 241 mm, $h_{flexure}$ = 255 mm, k_{crit} likely to be < 1 but not significantly so

J.L. June, 2007