ENGINEERING TRIPOS PART IIA

Wednesday 9 May 2007 2.30 to 4

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Data Sheet for Question 4.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A thin-walled angle section is made from sheet of uniform thickness t, density ρ, elastic modulus E, shear modulus G and has the cross-section shown in Fig. 1.

(a) Determine the principal second moments of area. Describe why the shear centre is located at the intersection of the two legs of the angle. [40%]

(b) The angle section is mounted as a horizontal cantilever of length L with its longer side pointing upwards and is loaded by its own weight.

(i) Determine the magnitude of the deflection of the centroid at the tip. [25%]

(ii) Determine the twist at the tip. [35%]

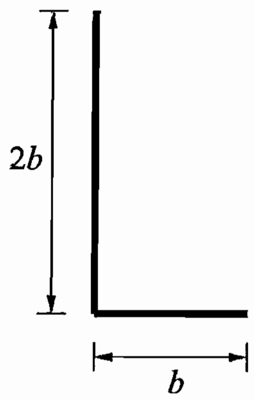


Fig. 1

2 Figure 2 shows the side view of a continuous beam bridge with uniform flexural rigidity EI and two unequal spans. The beam is simply supported at its two ends and continuous over the middle span.

(a) Sketch the influence line for the shear force at point X. [20%]

(b) Find a mathematical expression for the influence line for the shear and determine its value at the supports, at the centre of each span and at X. [50%]

(c) Discuss qualitatively how a bridge designer would use this influence line to calculate maximum and minimum shear forces at point X when the bridge is loaded by:

(i) a single concentrated force representing a heavy vehicle; [15%]

(ii) a distributed load of any length. [15%]

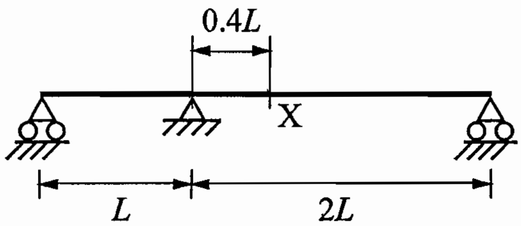


Fig. 2

3 Figure 3 shows a two-dimensional structure consisting of straight rods of equal length L, connected by frictionless pin joints. In the unloaded configuration members AB and BC form an angle of 30° with the horizontal, and member CD is horizontal. Member CD is axially linear-elastic with stiffness k, whereas the other members are effectively rigid. All members can be regarded as infinitely rigid in bending.

(a) Set up an expression for the total potential energy of the structure, $\Pi(\theta)$. [30%]

(b) Find an expression for P in terms of θ and plot a graph of $P(\theta)$ marking salient values. [30%]

(c) Find the maximum value of P prior to snap-through. [30%]

(d) Analyse the stability of the equilibrium path $P(\theta)$ and mark stable and unstable regions on your graph. [10%]

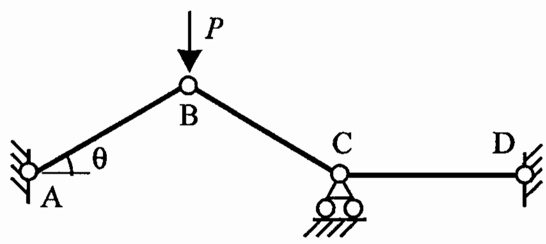


Fig. 3

4 (a) Figure 4(a) shows a beam-column of length L and bending stiffness EI loaded by a compressive axial force P. End A of the beam-column is connected by a frictionless pin to a rigid foundation.

Determine the relationship between the end couple M_B and the corresponding rotation θ_B, in terms of the stability functions s and c given in the attached data sheet. Show that for $P = 0$ this relationship agrees with the deflection coefficient given in the Structures Data Book. [30%]

(b) Figure 4(b) shows a two-dimensional structure consisting of three linear-elastic members of equal length L and bending stiffness EI. The members are rigidly jointed at B. Member AB is fully connected to a rigid foundation at A and member BD is pinned to a rigid foundation at D. Forces of magnitude $P/2$ and P are applied at joints B and C in the directions shown.

(i) Set up the 2×2 stiffness matrix relating the rotations θ_B and θ_C to corresponding couples, in terms of the stability functions for the members of the frame. [40%]

(ii) What is the critical value of P? [30%]

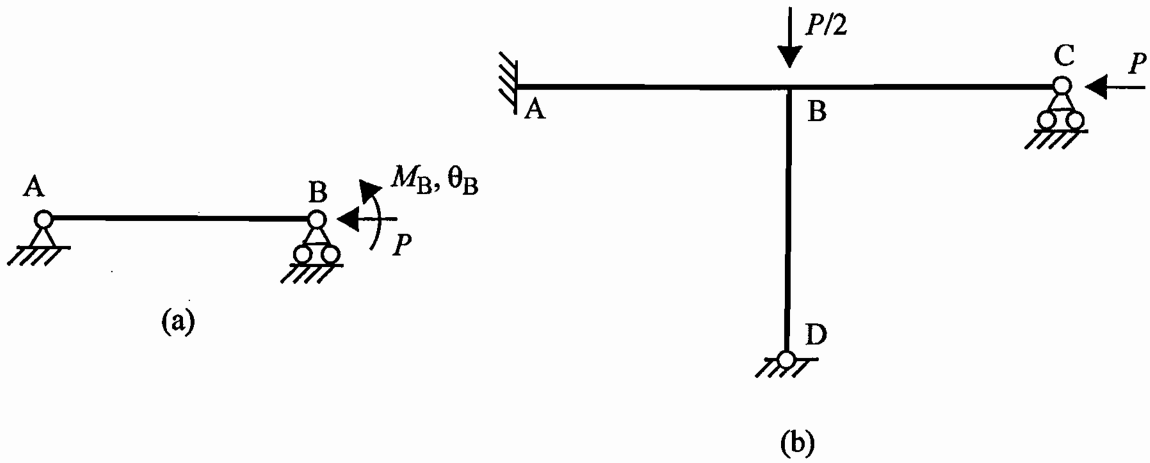
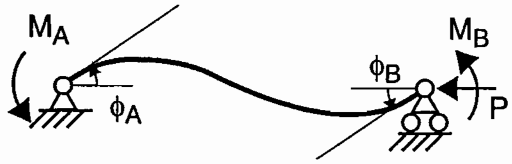


Fig. 4

END OF PAPER

Data Sheet for Question 4: Stability Functions.



For a beam of length L, as shown in the figure above, the following stiffness relationships apply:

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s & sc \\ sc & s \end{bmatrix} \begin{bmatrix} \phi_A \\ \phi_B \end{bmatrix}$$

where E is the Young's modulus and I the second moment of area for in-plane bending. The stability functions s and c are tabulated below for a member with Euler buckling load P_E.

P/P_E	s	c
0.0	4.00	0.50
1.0	2.47	1.00
1.1	2.28	1.11
1.2	2.09	1.25
1.3	1.89	1.42
1.4	1.68	1.66
1.5	1.46	1.97
1.6	1.22	2.43
1.7	0.98	3.17
1.8	0.72	4.50
1.9	0.44	7.66

P/P_E	s	c
2.0	0.14	24.68
2.1	-0.18	-21.07
2.2	-0.52	-7.51
2.3	-0.89	-4.62
2.4	-1.30	-3.37
2.5	-1.75	-2.67
2.6	-2.25	-2.23
2.7	-2.81	-1.93
2.8	-3.44	-1.71
2.9	-4.18	-1.54
3.0	-5.03	-1.42