ENGINEERING TRIPOS PART IIA

Wednesday 2 May 2007 2.30 to 4

Module 3E8

MODELLING DATA AND DYNAMICS IN MANAGEMENT

Answer one of the first two questions and one of the last two questions

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There is one attachment.

ATTACHMENT
Statistical tables (*three pages*)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A young couple have just moved into a large house in the country. On the first night they hear a mouse scratching in the loft. They choose to ignore it, turn over and go back to sleep. Unfortunately, they are unaware of the breeding cycle for mice.

Here are the facts: house mice reach sexual maturity in 8 weeks from birth; gestation is 20 days and the litter size is 7 baby mice (on average). One female can have 10 litters a year. Mice living in a house have a life expectancy of 2 years but (these mice) are not sexually active after one year.

As a friend of the young couple, you wish to demonstrate what will happen to the mouse population in the house by building a simple model. For simplicity, assume that mice breed in pairs and equal numbers of males and females are born. Assume also that there are at the outset just two eight-week-old mice, one male and one female; that there is plenty of food available and no mice die except from old age.

Your model will contain the following elements:

- Deaths
- Births
- Time for old mice before death
- Time to get old
- Time to mature
- Baby mice per breeding pair
- Young mice
- Mature mice
- Total mice
- Mice too old to breed
- Maturing mice
- Mice becoming old

(i) Identify each element as a stock, flow, or constant, and label its units. For each stock, determine its inflows and outflows. [20%]

(ii) Combine the elements to represent the structure of the system. Please include the model diagram. [25%]

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(iii) Using the description of the system provided above, define the equations for all model elements. [10%]

(iv) Sketch reference modes for all stocks in the model. [10%]

(b) Mice are the second most populous animal on Earth and it is believed that domestic cats were bred originally to control them. You estimate that a typical cat will catch on average five mice per week. Accordingly, you advise your friends in the large house to obtain a cat. They take this advice and buy a kitten the day after they first heard the mice. It takes exactly seven weeks for the kitten to grow into a typical mouse-catcher but it does not catch a single mouse before this time. Your friends report that ¾ of the mice caught are very young.

Will the cat manage to control the mouse population? Explain your reasoning and show any workings. [20%]

(c) Populations of mice, even in the absence of predators, are unlikely to go on growing for ever. Using an example, explain in dynamic terms why this might be so. [15%]

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2 (a) You are an expert criminologist who has been asked by the United Nations to give an opinion on the crime levels in two Polynesian islands, Tofu and Sushi. The islands have identical populations of 100,000 people but the crime level, measured by number of criminals per head of population, has been consistently higher in the poorer of the two islands, Tofu. However, there is concern at the UN that crime in both islands is rising and there is a question as to which island should receive priority from the Regional Law Enforcement Budget for 2010 (the budget has already been set from 2007 to 2009).

As a result of your research, you discover that today (2007) in Tofu the crime level is 0.015 and in Sushi it is 0.010; furthermore, in Tofu for every 100 criminals, 5 formerly law-abiding people are recruited to criminality each year whereas in Sushi, approximately 20 people per 100 criminals are similarly corrupted each year. Interestingly, there is no jail system on either island and criminals have no incentive to become law-abiding citizens.

(i) Which island should receive priority from the UN budget in 2010? Explain your logic. [20%]

(ii) Build a model of the system and use the different parameter values to study the number of criminals on each island. Include the model diagram. [15%]

(iii) Provide the documented equations, and graphs of expected model behaviour. [15%]

(b) As a criminal expert, you find it strange that neither island society has an effective penal system. Based on your experience, you want to make a recommendation to the UN for investment in a prison and rehabilitation service. Such a service will include a prison to hold criminals once they are arrested and sentenced. You estimate that the proportion of criminals arrested each year will be 15% and the average length of sentence will be 2 years with no parole. In prison, attempts will be made to reform criminals and return them at the end of their sentence as law-abiding citizens. You estimate that 75% of people leaving prison will never commit a crime again but 25% will return to crime.

(i) Build a model to estimate the reduction in the level of crime on the island of Sushi. [15%]

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(ii) Provide documented equations for your model including units of measurement. You are not expected to calculate the outcome precisely but you should explain what you anticipate will happen to the level of crime from 2007 through to 2010 if a penal system is adopted. [20%]

(c) On another island, Ramen, which does have a penal system, there is a problem with prison overcrowding. There are 1000 prisoners but space for only 250. The authorities estimate that 5% of the prison population can be reduced each month, starting with the lesser criminals. If the reduction scheme started on 1st January 2007, when, to the nearest month, will the prison population reach the nominal capacity? [15%]

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3 (a) Define confidence interval and confidence level. [15%]

(b) Bob, the manager of the pub El Bodegon, wants to know how many pints are sold per person on average on Friday night. Over the month he randomly selected 100 people. Bob found the average number of pints consumed per person on a Friday night was 2.84, and the standard deviation was 0.8. What is the point estimate of the population mean? Develop a 95% confidence interval for this mean. [25%]

(c) A year ago, beer sales at El Bodegon were only two pints per person on average. To boost sales, Bob introduced a policy of giving free snacks to the customers. Has the policy significantly increased beer sales? [30%]

(d) Bob remunerates suppliers according to the quality of the beer he receives, which is measured in AA degrees (the higher the AA the better the quality). The price he pays increases by £0.15 per AA degree. Elena, a local town supplier producing a low quality beer, claims that this price differential is much higher than what the market usually charges to her. She wants the price differential to be reduced to be between £0.03 and £0.05 per AA degree. You computed the following regression analysis with data obtained from the market.

SUMMARY OUTPUT:

Y = PRICE

Regression Statistics	
R Square	0.922949536
Adjusted R Square	0.915944948
Standard Error	0.13248961
Observations	13

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.312912	2.312912	131.7635787	1.83362E-07
Residual	11	0.193088	0.017553		
Total	12	2.506			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	9.444774028	0.288548	32.73208	2.58116E-12
AA	0.094927623	0.00827	11.47883	1.83362E-07

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(i) Calculate the confidence interval for the coefficient corresponding to variable AA. [15%]

(ii) How would Bob justify the price differential? How could Elena justify her request for a lower price differential? [15%]

4 The motorway demand from Sant Cugat to Barcelona in Spain is composed mainly of commuters to the city, who otherwise could drive for free through a mountain road. Consider the following variables: log_Dd = log(demand for motorways); log_DPI = log(disposable personal income of consumers); log_price = log(tolls charged). trend = a time trend starting at the beginning of the period. Finally,
Dlog_Dd = log_Dd(t)-log_Dd(t-1)

(a) Define *non-stationary series* and *weak stationarity*. What are the potential effects of estimating linear regression models with non-stationary series? [15%]

(b) The results of a Unit Roots Analysis are presented below as graphs and Dickey-Fuller analyses for the series log_Dd. Are the tests presented appropriately performed? Why? Find whether log_Dd I(1) is I(1)? [25%]

(c) Assume that log_Price and log_DPI are I(1). The estimation of the model is as below. t-values are shown in brackets. Comment on the results. [30%]

$$
\begin{aligned}
D\log\_Dd_t = & -0.151\left[\log\_Dd_{t-1} + 0.102\log\_Price_{t-1} - 1.332\log\_DPI_{t-1} - 0.002\cdot t_{t-1} - 0.03\right] + \\
& \quad (-6.71) \qquad\qquad (4.22) \qquad\qquad (-3.15) \qquad\qquad (-1.22) \qquad (-3.88) \\
& -0.05\cdot D\log\_Price_t + 0.05\cdot D\log\_DPI_t \\
& \quad (-3.44) \qquad\qquad (5.12)
\end{aligned}
$$

(d) Use the information provided below to suggest some improvements to the model in the form of new variables to be added and the signs expected for the parameters associated with these variables.

(i) The price of cars decreased during this period, especially after 1999 [15%]

(ii) Trains from San Cugat to Barcelona were heavily subsidised at the beginning of the period under analysis, but then the subsidies were dramatically reduced towards the end of the sample period. [15%]

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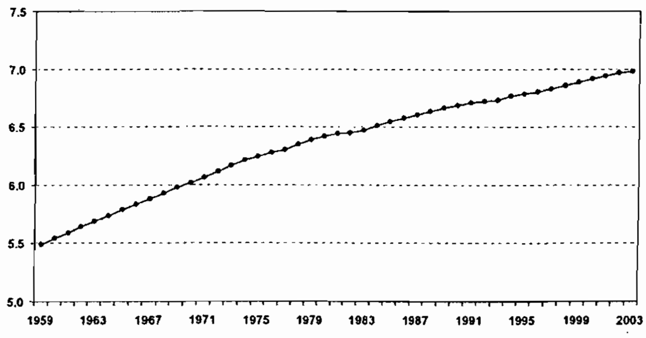
Log Demand for motorways



Augmented Dickey-Fuller Unit Root Test on log_Dd

Augmented Dickey-Fuller test statistic		-1.691709	0.7378
Test critical values	1% level	-4.186481	
	5% level	-3.518090	
	10% level	-3.189732	

Dependent Variable: D(log_Dd)
Method: Least Squares Sample(adjusted): 1961 2003

Variable	Coefficient	Std. Error	t-Statistic	Prob.
log_Dd(-1)	-0.034904	0.020632	-1.691709	0.0987
D(log_Dd(-1))	0.274772	0.149339	1.839923	0.0734
C	0.232945	0.117491	1.982662	0.0545
@TREND(1959)	0.000576	0.000672	0.855999	0.3972

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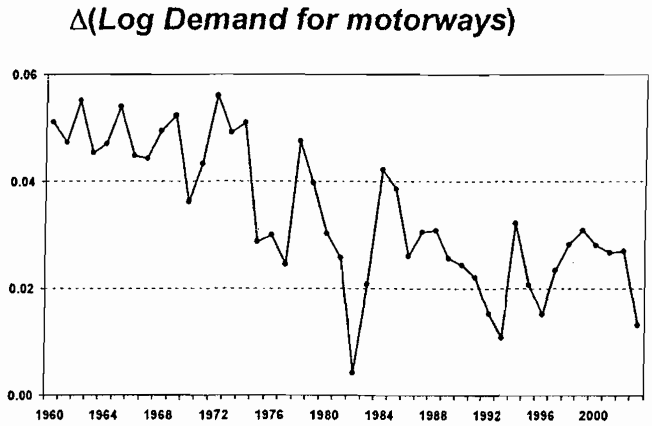

Augmented Dickey-Fuller Unit Root Test on Dlog_Dd

Augmented Dickey-Fuller test statistic	-4.518902	0.0042
Test critical values1% level	-4.192337	
5% level	-3.520787	
10% level	-3.191277	

Dependent Variable: D(Dlog_Dd)
Method: Least Squares   Sample(adjusted): 1962 2003

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Dlog_Dd (-1)	-0.833121	0.184363	-4.518902	0.0001
D(Dlog_Dd (-1))	0.232715	0.161176	1.443855	0.1570
C	0.043288	0.010121	4.277022	0.0001
@TREND(1959)	-0.000668	0.000181	-3.691100	0.0007

END OF PAPER

Statistical Tables

Table 1 **Area Under the Standard Normal Distribution**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Source: This table was generated using the SAS® function PROBNORM.

Table 2 **Right-Tail Critical Values for the *t*-distribution**

DF	α = .10	α = .05	α = .025	α = .01	α = .005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

Source: This table was generated using the SAS® function TINV.

Table 3 Right-Tail Critical Values for the *F*-Distribution

$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
									Upper 5% Points										
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.1	251.14	252.2	253.25	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	301	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.95	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: This table was generated using the SAS® function FINV. v_1 = numerator degrees of freedom; v_2 = denominator degrees of freedom.