ENGINEERING TRIPOS PART IIA

Thursday 3 May 2007 9 to 10.30

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) What are the objectives of equalisation in baseband transmission systems? Briefly explain the principles of operation of the following:

(i) a Zero Forcing (ZF) equaliser;
(ii) a Minimum Mean Squared Error (MMSE) equaliser;
(iii) a Decision Feedback (DF) equaliser.

[30%]

(b) A certain channel has a sampled response to a unit pulse given by

$$p(n) = 1,\ 0.5,\ 0,\ 0,\ 0,\ \ldots$$

Show the form of filter that would be required to implement an ideal ZF equaliser. What practical problems exist with this form of filter? [20%]

(c) The actual equaliser is to be implemented using a 4-coefficient Finite Impulse Response (FIR) filter.

(i) Determine suitable coefficients for this filter using the ZF criterion.

(ii) For a binary unipolar transmission scheme having the pulse response in (b) , determine the worst-case Bit Error Rate (BER) performance with and without the 4-coefficient FIR equaliser if the channel noise is white, Gaussian, zero mean, with a standard deviation of 0.2 V .

[50%]

Note the Gaussian error integral approximation is

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

2 (a) Describe:

(i) nearest neighbour decoding;
(ii) syndrome decoding;

as applied to linear binary block codes. [30%]

(b) A systematic $(6, 3)$ block code C is constructed as follows. To any 3-bit data word (d_1, d_2, d_3) a 6-bit codeword $(c_1, c_2, c_3, c_4, c_5, c_6)$ is constructed such that $(c_1 = d_1, c_2 = d_2, c_3 = d_3)$ and, using modulo-2 arithmetic:

$$c_4 = d_1 + d_3,$$
$$c_5 = d_1 + d_2,$$
$$c_6 = d_2 + d_3.$$

(i) Show that C is a linear block code.

(ii) Find the generator matrix and parity check matrix for C . [40%]

(c) Syndrome decoding is used to decode the code C . The minimum Hamming distance for the code is 3 . If the Bit Error Rate (BER) for the channel is 0.02 , estimate the BER after decoding. State any assumptions made. [30%]

3 (a) The expression for a modulated signal is given by $s(t) = \mathrm{Re}\left\{ p(t)\, e^{j\omega_C t} \right\}$, where $p(t)$ is the modulation phasor and ω_C is the carrier frequency. Derive expressions for the signals obtained by multiplying $s(t)$ by $2\cos(\omega_C t)$ and $-2\sin(\omega_C t)$. Hence explain how the real and imaginary parts of $p(t)$ can be recovered from $s(t)$. [25%]

(b) For the following modulation schemes:

Binary Phase-Shift Keying (BPSK);
Quadrature Phase-Shift Keying (QPSK);
16-level Quadrature Amplitude Modulation (16-QAM).

(i) Sketch the constellations of phasor values. [15%]

(ii) Assuming the presence of Additive White Gaussian Noise (AWGN), but without detailed derivation, describe approximately how the Bit Error Rate (BER) is related to the signal-to-noise ratio (energy per bit, E_b, divided by the noise power spectral density, N_0) in each case. [20%]

(iii) Explain how the bandwidth of these modulation schemes is related to the symbol period T_s in each case, and hence how the transmitted bandwidth is related to the bit period T_b, assuming rectangular symbols. [20%]

(c) Briefly discuss the trade-offs available to a designer when choosing between the modulation schemes in (b) for a given application. [20%]

4 In Digital Audio Broadcasting (DAB), a multiplexed block of typically 6 digital audio channels has a data bit rate of 1.23 Mbit/s , and with rate $1/2$ error correction coding, has a coded bit rate of 2.46 Mbit/s . Multiple transmission paths via different transmitters can occur with path delay differences of up to 0.24 ms .

(a) Explain why the presence of these multiple paths at any particular time would cause problems if the coded bits were transmitted using conventional Quadrature Phase Shift Keying (QPSK) modulation of a single carrier wave. [20%]

(b) In practice, Orthogonal Frequency-Division Multiplexing (OFDM) with QPSK modulation is used to overcome such multipath problems. Describe OFDM and show how it is able to achieve this. [35%]

(c) In the DAB system used in the United Kingdom, there are 1536 OFDM carriers, the receiver analysis period is 1 ms , and the guard period is 0.246 ms . Show that these parameters are able to support the previously specified coded bit rate, and estimate the bandwidth of the complete OFDM signal. [25%]

(d) Compare the relative spectral efficiencies of DAB and conventional analogue Frequency Modulation (FM) broadcasting, in which each audio channel requires a bandwidth of 300 kHz . How is this comparison affected by the possible need for multiple frequency allocations in order to cover a large geographical area such as the United Kingdom? [20%]

END OF PAPER

Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2007- Answers

1. Attempted by all candidates and was generally well answered. The most common mistakes were to incorrectly calculate the eye height or to neglect to calculate the noise power after the equaliser when evaluating the equalised Bit Error Rate (BER).
a) See notes.
b)

$$H_E(z) = \frac{1}{P(z)} = \frac{1}{1+0.5z^{-1}}$$

Stability not guaranteed.
Numerical instability.
Adaptive methods hard to derive.
c)
(i) 1, -0.5, 0.25, -0.125
(ii) Without equaliser, BER = Q (1.25) = 0.1058. With equaliser, BER = Q (2.03) = 0.0212.

2. This was the second most popular question and was in general answered quite well. A surprising number of candidates could not correctly perform the BER calculation in part (c).
a) (i) and (ii) see notes.
b)
(i) See notes.
(ii)

$$\mathrm{H} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \mathrm{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

c) $2.923*10^{-3}$

3. This question was in general very well answered. Surprisingly, part (a) proved to be troublesome for some of the candidates even though the solution is in the course notes.
a)

$$i'(t) = i(t) + i(t)\cos(2\omega_C t) - q(t)\sin(2\omega_C t)$$

$$q'(t) = q(t) - q(t)\cos(2\omega_C t) - i(t)\sin(2\omega_C t)$$

Low pass filtering is required to reject the high frequency terms.
b) (i) and (ii) see notes.
(iii) BPSK: $1/T_b$ Hz
QPSK: $1/(2T_b)$ Hz
16-QAM: $1/(4T_b)$ Hz
The widths of the main lobes of the spectra to the first zeros are twice these values.
c) See notes

4. This question was the least popular and also the least well answered. The descriptions of OFDM required in part (b) proved to be quite variable in detail and accuracy. In addition, part (d) proved to be problematic for a number of candidates.
a) See notes.
b) See notes.
c) 802.57 sym/s, total bit rate = 2.4655 Mbit/s, bandwidth = 1.536 MHz, including frequency guard bands gives a total bandwidth = 1.736 MHz, yielding a required bandwidth per audio channel of 289.3 kHz.
d) See notes.

I. J. Wassell May 2007