YEAR 2008 | PART (of Tripos) IIA | PAPER NUMBER & TITLE 3A3 – Fluid Mechanics II

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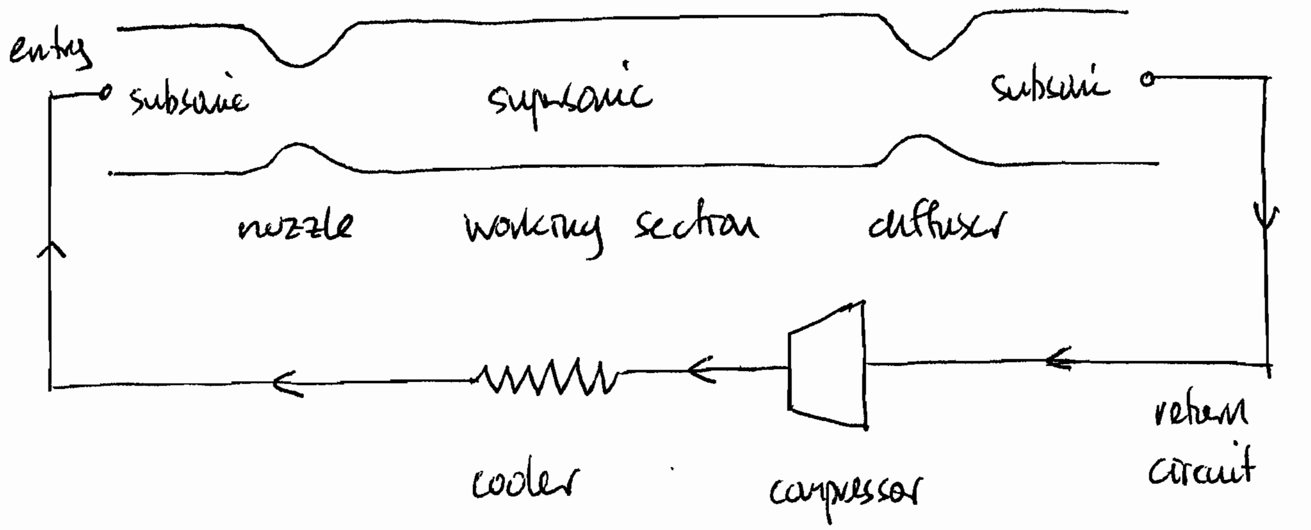
1. a)



The tunnel must accelerate the flow from subsonic through $M = 1.0$ at the nozzle throat up to $M = 3.0$ in the working section. It must then decelerate the flow back to subsonic after the diffuser throat. The compressor restores the stagnation pressure back to the value at entry while the cooler restores the stagnation temperature downstream of the compressor back to the value at entry.

Ideally, both throats would have the same area and the flow would be isentropic throughout (no losses). This is not achievable in practice due to frictional losses and the need to start the tunnel.

b) Design nozzle for $M = 3.0$ at end of divergence

At the nozzle throat $M = 1.0$ $\quad \dfrac{\dot{m}\sqrt{c_p T_0}}{A_n P_0} = 1.281$

At the end of divergence $M = 3.0$ $\quad \dfrac{\dot{m}\sqrt{c_p T_0}}{A_W P_0} = 0.3025$

but working section area $A_W = (0.25)^2 = 0.0625 m^2$

$\therefore$ nozzle area $A_n = \dfrac{0.3025}{1.281} A_W = \underline{0.01476 \, m^2}$

(12.15cm square)

c) Under starting conditions, have a $M = 3.0$ shock in the working section.

Tables at $M = 3.0$: $\quad \dfrac{P_{0s}}{P_0} = 0.3283$

At diffuser throat $\quad \dfrac{\dot{m}\sqrt{c_p T_0}}{A_D P_{0s}} \leq 1.281$ in order to swallow the shock.

$\therefore$ have $\dfrac{A_D}{A_n} \geq \dfrac{P_0}{P_{0s}}$; $A_D \geq \dfrac{0.01476}{0.3283} = \underline{0.04496 m^2}$

d) $P_{0s} = 33kPA$ at exit, T_0 unchanged at 300K.

Isentropic compressor: $\quad \dfrac{T_{exit}}{T_0} = \left(\dfrac{1.01}{0.33}\right)^{\frac{\gamma - 1}{\gamma}}$, $\gamma = 1.4$ for air

$= 1.3766 \quad \therefore T_{exit} = 413K.$

compressor work per kg $\left(\frac{W_c}{\dot{m}}\right) = c_p \Delta T = 1005 \times (413 - 300)$

$= 113565 \text{ J/kg}$

In the nozzle $\frac{\dot{m}\sqrt{c_p T_0}}{A p_0} = 1.281$

$\therefore \dot{m} = 1.281 \frac{0.01476 \times 1.01 \times 10^5}{\sqrt{1005 \times 300}}$

ie $\dot{m} = 3.478 \text{ kg/s}$

$\therefore W_c = 3.478 \times 113565 = \underline{395 \text{kW}}$

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2 a) $\delta\left(F/A\right) = \delta\left(P + \rho V^2\right) = P(M^2 - 1)\frac{\delta V}{V}$; $\delta\left(\frac{F}{A}\right)$ always $-$ve due to friction

subsonic

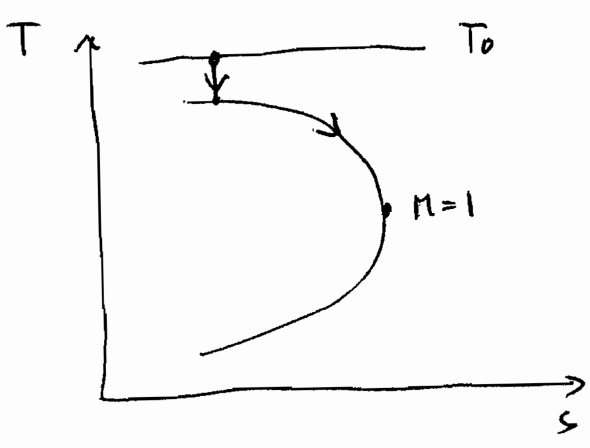
$M < 1$; $(M^2 - 1) < 0$

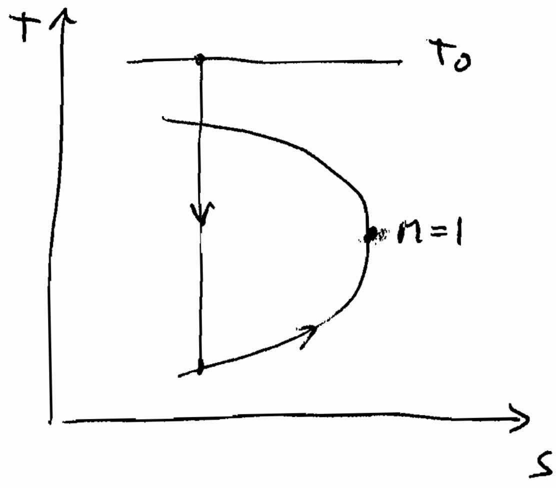
$\therefore \frac{\delta V}{V}$ +ve $\Rightarrow$ acceleration

supersonic

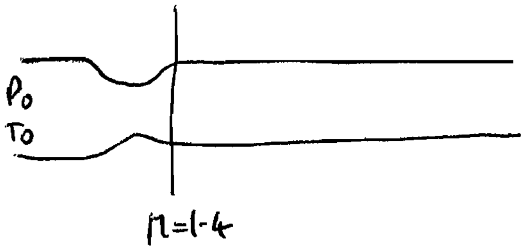
$M > 1$; $(M^2 - 1) > 0$

$\frac{\delta V}{V}$ $-$ve $\Rightarrow$ deceleration





b) i) tables at $M=1\cdot4$



$$\frac{\dot{m}\sqrt{c_pT_0}}{AP_0} = 1\cdot1490$$

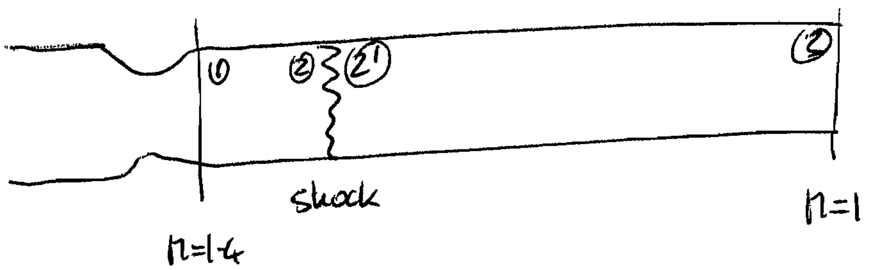
$$A = \frac{\dot{m}\sqrt{c_pT_0}}{1\cdot1490 \times P_0} = \frac{25\sqrt{1005\times288}}{1\cdot1490\times15\times10^5} = 0\cdot007803\,m^2$$

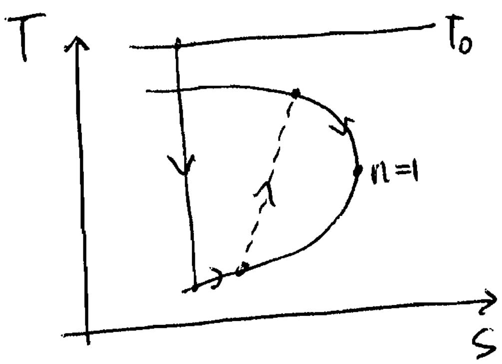
$A = \pi r^2$, $r = 0\cdot05m$ $\therefore$ diameter $= 0\cdot1m$

ii) Tables $\frac{4c_fL_{max}}{D} = 0\cdot0997$ at $M=1\cdot4$

$$L_{max} = \frac{0\cdot0997\times0\cdot1}{4\times0\cdot0025} = 0\cdot997m$$

c)





$$\frac{4c_fL_{12}}{D} = \frac{4c_fL_{max}}{D} - \frac{4c_fL_{2max}}{D}; \quad \frac{4c_f}{D} = 0\cdot$$

but $L_{12} = 0\cdot035m$, $L_{max} = 0\cdot997m$

$$\therefore \frac{4c_fL_{2max}}{D} = 0\cdot0997 - 0\cdot0035$$

$$= 0\cdot0962$$

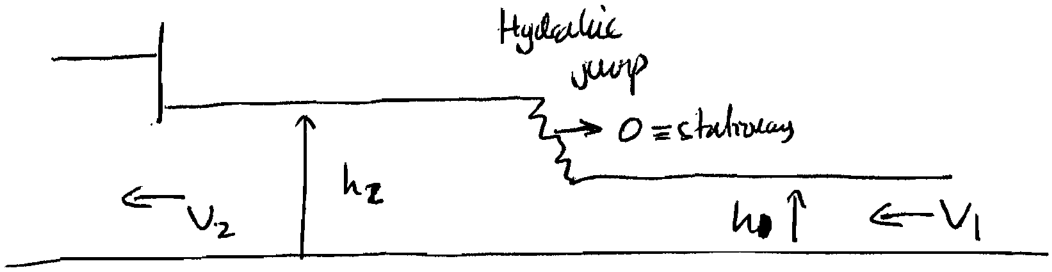
Tables: $M_2 = 1\cdot39$

tables again : $\eta_2' = 0.744$; $\frac{4c_f L_{2'max}}{D} = 0.1344$ tables (interpolated)
at $\eta = 1.39$

$$L_{2'max} = 1.344\,m$$

$$\therefore \text{total length} = L_{12} + L_{2'max} = \underline{1.379m}$$

3.
a)



continuity $\quad h_1 V_1 = h_2 V_2$

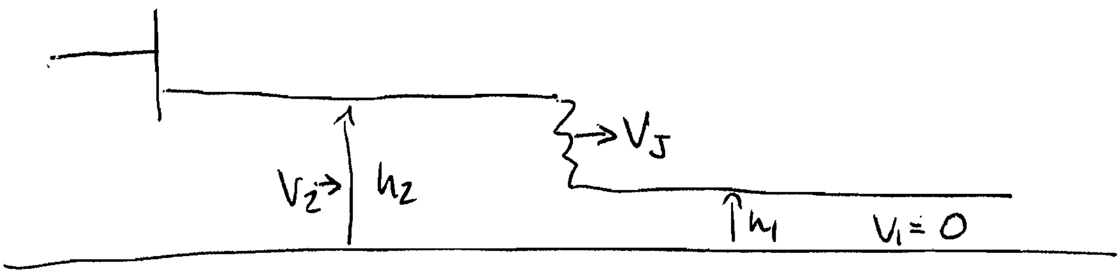
momentum $\quad (\tfrac{1}{2}\rho g h_1)h_1 - (\tfrac{1}{2}\rho g h_2)h_2 = \rho V_2^2 h_2 - \rho V_1^2 h_1$

combine: $\quad \tfrac{1}{2} g (h_1^2 - h_2^2) = V_2 V_1 h_1 - V_1 V_2 h_2$

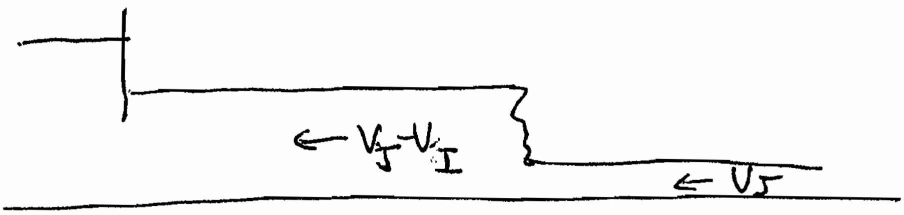
either $h_1 = h_2$, or $\underline{\tfrac{1}{2} g (h_1 + h_2) = V_1 V_2}$ as required

b)

i)



change frame of reference : subtract V_J :



$$\frac{V_J^2}{gh_1} = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1}+1\right) \quad ; \quad \frac{h_2}{h_1} = 1.2$$

$$\therefore \frac{V_J^2}{gh_1} = 1.32 \quad \therefore \underline{V_J = 1.149\sqrt{gh_1} = 1.149c_1}$$

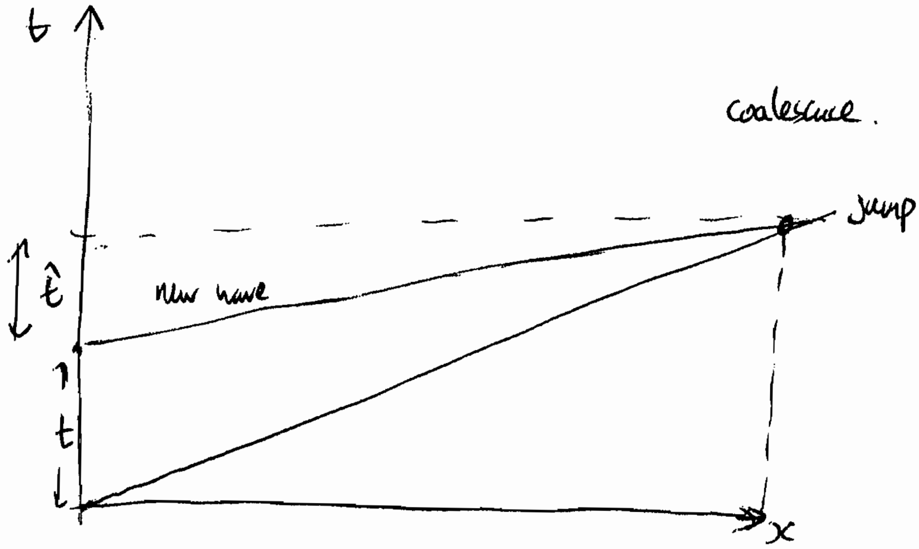
Therefore $V_J(V_J - V_2) = \frac{1}{2}g(h_1+h_2) = \frac{1}{2}gh_1(1+h_2/h_1)$

$$= 1.1gh_1 = 1.1c_1^2$$

$$\therefore V_J - V_2 = \frac{1.1c_1^2}{1.149c_1} = 0.957c_1$$

$\therefore$ Inflow velocity $= (1.149 - 0.957)c_1 = \underline{0.192c_1}$

ii)



The jump travels at a constant speed into the undisturbed channel. The new wave corresponds to a small increase in depth which travels at a higher speed than the jump due to the increased wave speed together with the non-zero flow velocity in the same direction. The new wave will catch up with the jump. Since it is a compression wave it will have a tendency to steepen and accelerate into a new jump. This is unlikely to happen within the short time available before the waves coalesce.

Head of compression wave moves at $V_c = V_2 + C_2$

$$\frac{c_2}{c_1} = \sqrt{\frac{h_2}{h_1}} = 1.095 \; ; \quad V_2 = 0.1924$$

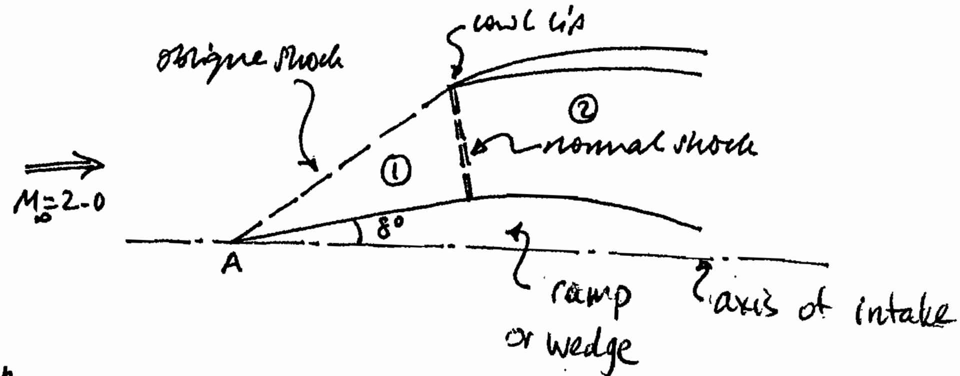
$$\therefore \; V_c = 1.287 c_1$$

Compression wave begins at time t

Time to catch up is $\hat{t}$ $\therefore$ $(t+\hat{t}) V_J = \hat{t} V_c$

$$\therefore \; \hat{t} = \frac{t V_J}{V_c - V_J} = \frac{1 \cdot 1.149}{1.287 - 1.149} = \underline{8.33\,s}$$

4 a)



$M_\infty = 2.0 \quad \delta = 8°$

$\frac{p_1}{p_\infty} = 1.541$

$M_1 = 1.713$

$\beta = 37.221°$

$\frac{p_2}{p_1} = 3.259 \Rightarrow \frac{p_2}{p_\infty} = 5.02$

b) $M_\infty = 2.5 \quad \frac{p_1}{p_\infty} = 1.6585 \quad M_1 = 2.167 \quad \frac{p_2}{p_1} = 5.316 \Rightarrow \frac{p_2}{p_\infty} = 8.82$

$\beta = 30.015°$

i.e. static pressure recovery nearly doubled from $M_\infty = 2.0$

distance to cowl lip $= 0.5m$ $\therefore$ point A is @ $x_{2.0}$ ahead of cowl lip @ $M_\infty = 2.0$

where $\tan(37.221) = \frac{0.5}{x_{2.0}} \Rightarrow x_{2.0} = 0.658m$

@ $M_\infty = 2.5$ A is at $x_{2.5}$ ahead of cowl lip

where $\tan(30.015) = \frac{0.5}{x_{2.5}} \Rightarrow x_{2.5} = 0.866m$

Shock angle β decreases as M_∞ goes from $2.0 \rightarrow 2.5$ hence ramp must translate left (i.e. emerge from the intake) by $\Delta x = 0.208m$ to maintain shock system focussed on cowl lip.

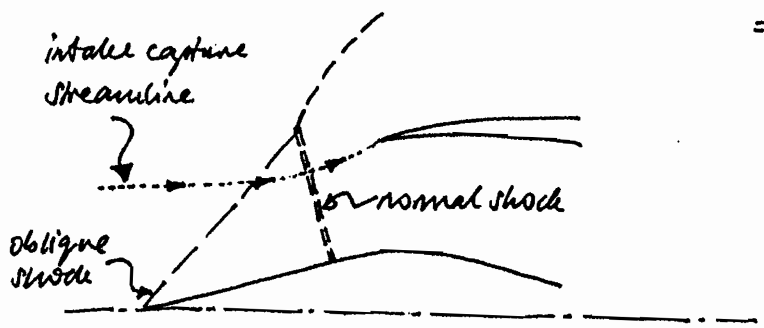
c) Let shock system attain focus at Mach number m

$\therefore x_m = 0.762 \Rightarrow \beta_m = 33.27°$. From tables: $\delta = 8°$ $M = 2.20$ $\beta = 33.84°$

$M = 2.25$ $\beta = 33.11°$

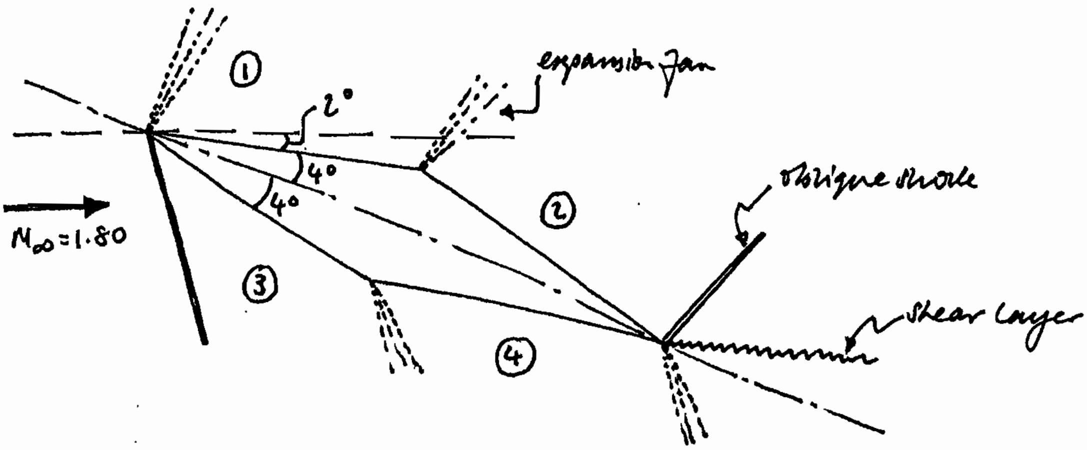
$\Rightarrow m = 2.24$

d)



In this configuration, the shock system does not focus on the cowl lip. The normal shock stands off, ahead of the cowl lip. Since the capture streamline still passes wholly through the 2 shock system, the static pressure recovery is essentially that in part (a) and inlet distortion to the engines should be low $\Rightarrow$ engines unaffected. However stand-off normal shock $\Rightarrow$ increase in flow ...

5 a)



$\theta_\infty = 0°$

$\nu_\infty = 20.68$

$\frac{p_{0\infty}}{p_\infty} = 5.751$

$\theta_1 = -2°$

$\theta_2 = -10°$

$\theta_3 = -10°$

$\theta_4 = -2°$

b) $\nu_\infty + \theta_\infty = \nu_1 + \theta_1 \quad \Rightarrow \nu_1 = 22.68 \quad \Rightarrow M_1 = 1.870$

$p_\infty = 2\ N/m^2 \qquad \frac{p_{01}}{p_1} = 6.402 \qquad p_{01} = p_{0\infty} \Rightarrow p_1 = 1.797\ N/m^2$

$\nu_1 + \theta_1 = \nu_2 + \theta_2 \quad \Rightarrow \nu_2 = 30.68 \quad \Rightarrow M_2 = 2.163$

$\frac{p_{02}}{p_2} = 10.089 \qquad \Rightarrow p_2 = 1.140\ N/m^2$

$M_\infty = 1.80 \quad \delta_{\infty \to 3} = 10° \quad \Rightarrow \frac{p_3}{p_\infty} = 1.663 \Rightarrow M_3 = 1.449 \qquad \nu_3 = 10.39 \qquad \frac{p_{03}}{p_3} = 3.$

$\Rightarrow p_3 = 3.326\ N/m^2$

$\nu_3 - \theta_3 = \nu_4 - \theta_4 \quad \Rightarrow \nu_4 = 18.39 \quad \Rightarrow M_4 = 1.721 \qquad \frac{p_{04}}{p_4} = 5.101 \Rightarrow p_4 = 2.2$

	p	α	$\cos\alpha$	$R(\uparrow)$	$\sin\alpha$	$R(\rightarrow)$
1	1.797	2°	0.999	−1.795	0.035	−0.063
2	1.140	10°	0.985	−1.229	0.174	−0.198
3	3.326	10°	0.985	+3.276	0.174	+0.579
4	2.227	2°	0.999	+2.225	0.035	+0.078
				+2.477		+0.396

$q_\infty = \left(\frac{\gamma}{2}\right) p_\infty M_\infty^2 \quad \Rightarrow q_\infty = 4.536\ N/m^2 \qquad \text{chord} = 1.998\ m$

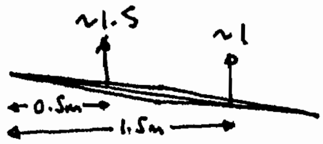
$\Rightarrow C_L = 2.477/(4.536 \times 1.998) = 0.273$

$C_{D_p} = 0.396/(4.536 \times 1.998) = 0.044$

$\Rightarrow L/D_p = 6.14$ i.e. much lower than subsonic even without C_{D_v}.

c) same for equilibrium assuming small angles

→ centre of lift ~0.9m from l/e ⇒ 0.45 chord



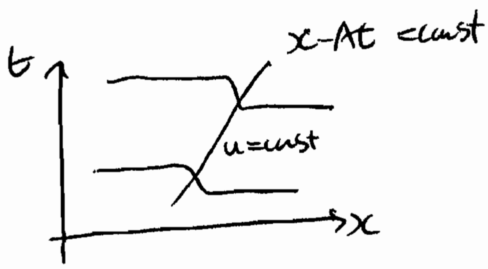
subsonic expect c. of lift @ ~0.25 chord ⇒ KUTTA condition only necessarily met subsonic, not supersonic.

6 a) i) Hyperbolic $\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$

ii) Elliptic $\nabla^2 u = 0$

iii) Parabolic $\frac{\partial u}{\partial t} = A\frac{\partial^2 u}{\partial x^2}$

b) Hyperbolic $\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0 \Rightarrow u = f(x-At)$ for const. A
(f arbitrary)



A profile of shape f is convected without change of shape to later times.

Characteristic lines $x-At$ = const ; u = const.

Information flows along characteristics in a wave-like manner

BCs : initial value problem and inflow conditions for each characteristic.

Application : steady fully supersonic flow, eg aerofoil nozzle.

Elliptic $\nabla^2 u = 0$: Solution at any point depends on the solution at every other point and on all boundary points. No characteristic lines.

BCs must be supplied on a closed curve (or surface) surrounding the domain of interest. Values are required at all points.

Examples : steady incompressible flow or steady heat conduction

<u>Parabolic</u> $$\frac{\partial u}{\partial t} = A\frac{\partial^2 u}{\partial x^2}$$

Initial and boundary value problem. Solution spreads out in space with increasing time. No characteristics, but a definite direction of information flow does exist.

BCs must be specified on the boundary and initial conditions are required at the start.

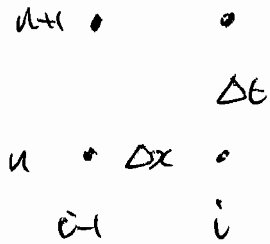
Examples: steady boundary layer or unsteady heat conduction

S12

7a) $$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0 \;;\quad A \text{ +ve}.$$

Discretise $$\frac{u_i^{n+1} - u_i^n}{\Delta t} + A\,\frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

1st order forward (time) | 1st order upwind (space)



"Upwind" differencing allows information to travel from upstream to downstream, mimicking the physics of the information flow.

$$u_i^{n+1} = u_i^n - \frac{A\Delta t}{\Delta x}\left(u_i^n - u_{i-1}^n\right)$$

use discrete perturbation: raise a sawtooth disturbance $+\varepsilon$ at i, $-\varepsilon$ at $i-1$

$$u_i^{n+1} = \varepsilon - \frac{A\Delta t}{\Delta x}\left(\varepsilon - (-\varepsilon)\right)$$

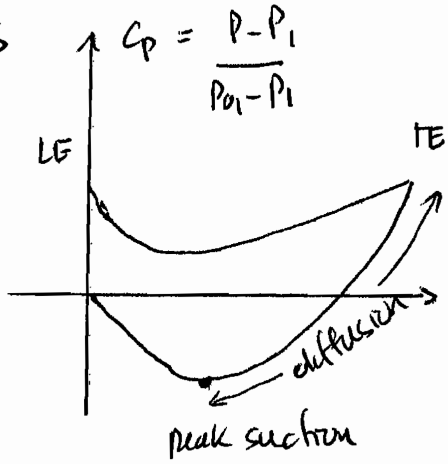
$$\frac{u_i^{n+1}}{\varepsilon} = 1 - 2\,\frac{A\Delta t}{\Delta x}$$

Stable provided that $\left|\frac{u_i^{n+1}}{\varepsilon}\right| \le 1$; $\left.\begin{array}{l} -1 \le 1 - 2\frac{A\Delta t}{\Delta x} \\ 1 - 2\frac{A\Delta t}{\Delta x} \le 1 \end{array}\right\}$

$$\therefore \frac{A\Delta t}{\Delta x} \le 1$$

7b)

i) Suction surface boundary layer experiences a large diffusion from peak suction to the trailing edge. Consequently expect rapid boundary layer growth and thus larger momentum thickness.



ii) Viscous mixing, probably turbulent.

iii) Momentum conservation in axial direction:
momentum in $\dot{m}U - \rho\theta U^2$ and pressure is uniform at both ends. No applied shear stresses, steady flow:

$$\dot{m}V_{te} - \rho\theta_{ss}V_{te}^2 - \rho\theta_{ps}V_{te}^2 + sP_{te} = \dot{m}V_2 + sP_2$$

$$\underline{\dot{m}V_{te} - \rho(\theta_{ss}+\theta_{ps})V_{te}^2 + sP_{te} = \dot{m}V_2 + sP_2}$$

iv) $V_2 \approx V_{te} \Rightarrow \dot{m}V_{te} \approx \dot{m}V_2$

$$P_{te} = P_{01} - \tfrac{1}{2}\rho V_{te}^2 \qquad P_2 = P_{02} - \tfrac{1}{2}\rho V_2^2$$

$$\Rightarrow -\rho(\theta_{ss}+\theta_{ps})V_2^2 + s\left(P_{01} - \tfrac{1}{2}\rho V_2^2\right) = s\left(P_{02} - \tfrac{1}{2}\rho V_2^2\right)$$

$$\Rightarrow \quad s(P_{01}-P_{02}) = \rho(\theta_{ss}+\theta_{ps})V_2^2$$

$$\Rightarrow \quad \frac{P_{01}-P_{02}}{\frac{1}{2}\rho V_1^2} = \frac{2(\theta_{ss}+\theta_{ps})}{s}\left(\frac{V_2}{V_1}\right)^2$$

Mass conservation $V_1 \cos\alpha_1 = V_2$

$$\Rightarrow \quad \frac{P_{01} - P_{02}}{\frac{1}{2}\rho V_1^2} = Y_p = \frac{2(\theta_{ss} + \theta_{ps})}{s} \cos^2\alpha_1$$

$V_2 \approx V_{te}$ provided that the boundary layers are thin,

ie $\delta^*_{ss} + \delta^*_{ps} \ll s$.

[result can be modified to account for δ^* effects]

8(a) TURBINE: $Y_P = \dfrac{P_{01} - P_{02}}{P_{02} - P_2} = \dfrac{P_{01}/P_{02} - 1}{1 - P_2/P_{02}}$

$$\frac{P_2}{P_{02}} = \frac{1}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}} = \frac{1}{\left(1+0.2\times 1^2\right)^{3.5}} = 0.5283$$

$$\Rightarrow \quad Y_P = \frac{12.4/12.0 - 1}{1 - 0.5283} = \underline{\underline{0.071}}$$

TORQUE $= \dot{m}\Delta(rV_\theta)$ $\quad V_{\theta 1} = 0 \quad \Rightarrow$ TORQUE $= \dot{m} r_2 V_{\theta 2}$

$T_2 = T_{02}/\left(1+\frac{\gamma-1}{2}M_2^2\right) = 1400/(1+0.2\times1^2) = \underline{1166.7\ K}$

$V_2 = M_2\sqrt{\gamma R T_2} = 1\times\sqrt{1.4\times 287\times 1166.7} = \underline{684.7\ m/s}$

$V_{r2} = V_2\cos\alpha_2 = 684.7\cos 70^\circ = \underline{234.2\ m/s}$

$V_{\theta 2} = V_2 \sin\alpha_2 = 684.7 \sin 70^\circ = \underline{643.4\ m/s}$

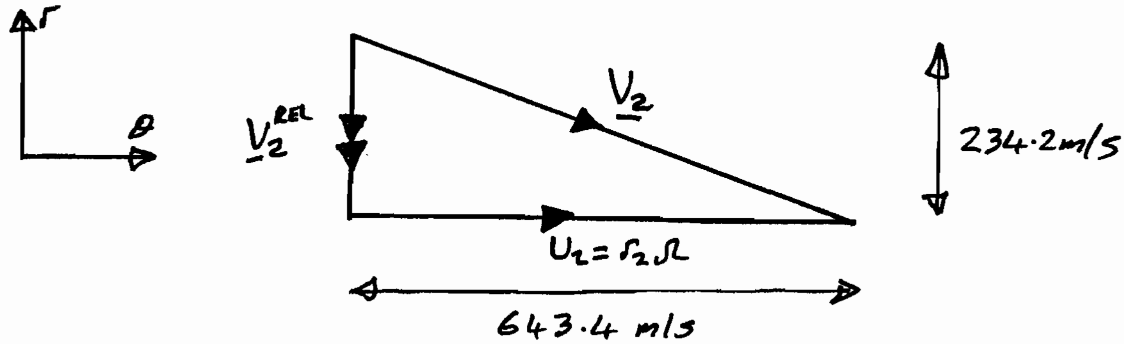
TORQUE $= \dot{m} r_2 V_{\theta 2} = 1.3\times 0.08 \times 643.4 = \underline{\underline{66.9\ Nm}}$

(b) HIGH STRESS & HIGH TEMPERATURE $\Rightarrow$ RADIAL BLADES AT OUTER DIAMETER.

NO RELATIVE INCIDENCE $\Rightarrow V_{\theta 2}^{REL} = 0$

$V_{\theta 2}^{REL} = V_{\theta 2} - U_2 = V_{\theta 2} - r_2\Omega = 0$

$\Rightarrow \Omega = V_{\theta 2}/r_2 = 643.4/0.08 = \underline{\underline{8042.5\ \text{RAD/s}\ (= 76800\ \text{rpm})}}$

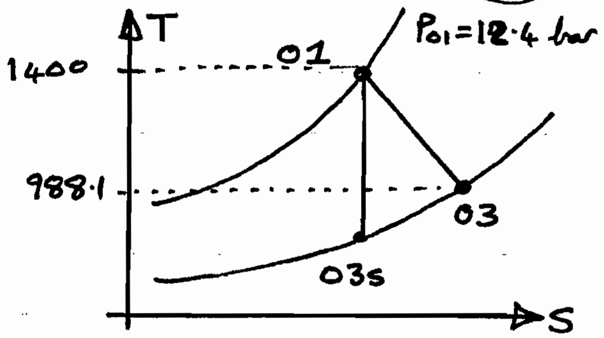


(c)(i) ZERO EXIT SWIRL $\Rightarrow$ LOW EXIT KE $\Rightarrow$ HIGH TOTAL-STATIC EFFICIENCY.

$h_{02} - h_{03} = U_2 V_{\theta 2} - U_3 V_{\theta 3} = U_2 V_{\theta 2} = 643.4^2$

$T_{03} = T_{02} - U_2 V_{\theta 2}/c_p = 1400 - 643.4^2/1005 = \underline{988.1\ K}$

POWER $= \dot{m}\Delta h_0 = 1.3\times 643.4^2 = \underline{\underline{538.2\ kW}}$



(ii)

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03s}}$$

$$T_{03s} = T_{01} - (T_{01} - T_{03})/\eta_{tt}$$

$$= 1400 - (1400 - 988.1)/0.9$$

$$T_{03s} = \underline{942.3\ K}$$

$$P_{03} = P_{01}\left(\frac{T_{03s}}{T_{01}}\right)^{\gamma/\gamma-1} = 12.4\left(\frac{942.3}{1400}\right)^{3.5} = \underline{\underline{3.102\ bar}}$$

(d) AT EXIT $\dfrac{\dot{m}\sqrt{C_p T_{03}}}{P_{03} A_3 \cos\alpha_3} = f_n(M_3)$ $\qquad \alpha_3 = 0$ (NO EXIT SWIRL)

$$A_3 = \pi(0.04^2 - 0.012^2) = \underline{4.5742 \times 10^{-3}\ m^2}$$

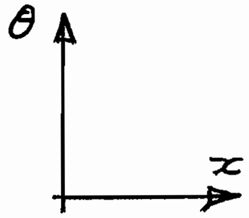
$$f_n(M_3) = \frac{1.3\sqrt{1005 \times 988.1}}{3.102 \times 10^5 \times 4.5742 \times 10^{-3}} = 0.9130$$

$\dot{m}\sqrt{C_p T_0}/AP_0 = 0.9130$, TABLES $\Rightarrow$ $\underline{M_3 = 0.4695}$

AXIAL FLOW AT EXIT (ABSOLUTE FRAME)

$M_3 = 0.4695$, TABLES $\Rightarrow$ $V/\sqrt{C_p T_0} = \underline{0.2906}$

$$V_3 = 0.2906\sqrt{1005 \times 988.1} = \underline{\underline{289.6\ m/s}}$$



$$U_3 = r_3\Omega = 0.026 \times 8042.5$$

$$= 209.1\ m/s$$

(AT MID HEIGHT)