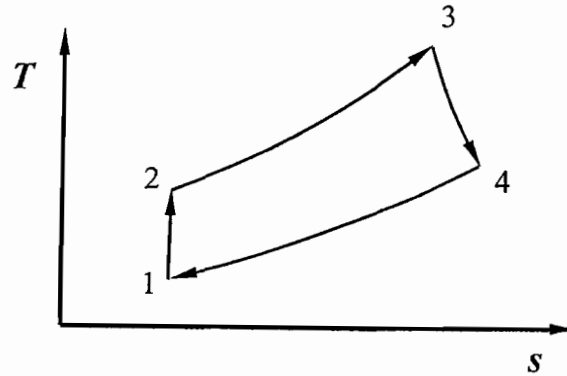# ENGINEERING TRIPOS   PART IIA   2008

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

SOLUTIONS TO TRIPOS QUESTIONS

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1   (a)

(i)   For helium, $R = 2.08$ kJ/kg K, $c_p = 5.19$ kJ/kg K, $\gamma = 1.67$

For convenience, set $h_0 = 0.0$ kJ/kg and $s_0 = 0.0$ kJ/kg K at the dead state (1 bar, 300 K).

For isentropic compression : $T_2 = T_1 \left(\dfrac{p_2}{p_1} \right)^{(\gamma-1)/\gamma}$ $= 690.9$ K

For isentropic expansion : $T_{4S} = T_3 \left(\dfrac{p_4}{p_3} \right)^{(\gamma-1)/\gamma}$ $= 651.3$ K

For the real expansion : $T_4 = T_3 - \eta_t (T_3 - T_{4S}) = 778.6$ K

Go round the cycle calculating $h = c_p(T - T_0)$, $s = c_p \ln(T/T_0) - R\ln(p/p_0)$ and $e = (h - T_0 s)$
Note : p in bar, T in K, h and e in kJ/kg, s in kJ/kg K :

State 1 : $p_1 = 1.0$ $T_1 = 300.0$ $h_1 = 0.0$ $s_1 = 0.0$ $e_1 = 0.0$
State 2 : $p_2 = 8.0$ $T_2 = 690.9$ $h_2 = 2029.0$ $s_2 = 0.0$ $e_2 = 2029.0$
State 3 : $p_3 = 8.0$ $T_3 = 1500.0$ $h_3 = 6228.0$ $s_3 = 4.0277$ $e_3 = 5019.7$
State 4 : $p_4 = 1.0$ $T_4 = 778.6$ $h_4 = 2483.9$ $s_4 = 4.9498$ $e_4 = 999.0$

Exergy supplied in heater $= (e_3 - e_2)$ $= 2990.7$ kJ/kg
Net work output $= (h_3 - h_4) - (h_2 - h_1) = 3744.1 - 2029.0 = 1715.1$ kJ/kg
Lost work in turbine $= (e_3 - e_4) - (h_3 - h_4) = 4020.7 - 3744.1 = 276.6$ kJ/kg
Lost work in cooling process $= (e_4 - e_1)$ $= 999.0$ kJ/kg   [55 %]

NOTE : It is also possible to calculate the 'lost work' terms as follows :

The turbine expansion is adiabatic, so all the lost work comes from the internal irreversibility
of the expansion. Hence,
Lost work in turbine $= T_0(s_4 - s_3) = 300 \times (4.9498 - 4.0277) = 276.6$ kJ/kg

The cooling process is at constant pressure, so all the lost work comes from the irreversible heat transfer to the environment (should really prove this). Hence :

Lost work in cooling process $= \int_4^1 \left(1 - \frac{T_0}{T}\right) dQ = -\int_4^1 \left(1 - \frac{T_0}{T}\right) c_p dT$

$$= c_p(T_4 - T_1) - c_p T_0 \ln\left(\frac{T_4}{T_1}\right) = 999.0 \text{ kJ/kg}$$

However, all this is unnecessary once the exergy values have been obtained.

(ii)   Cycle thermal efficiency $= \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{1715.1}{4199.0} = 0.408$ [5 %]

(iii)   For a cycle with the same heater inlet and outlet conditions, e_2 and e_3 are the same. The maximum efficiency cycle is fully-reversible. It is therefore completed by steady-flow reversible processes connecting states 3 and 2 and exchanging heat reversibly with the environment. The maximum work output from this cycle is therefore $(e_3 - e_2)$. Hence,

Thermal efficiency of reversible cycle $= \frac{(e_3 - e_2)}{(h_3 - h_2)} = \frac{2990.7}{4199.0} = 0.712$

Rational efficiency of actual cycle $= \frac{(h_3 - h_4) - (h_2 - h_1)}{(e_3 - e_2)} = \frac{0.408}{0.712} = 0.573$ [10 %]

(b)   The given equation of state is

$$\frac{u}{u_0} = \left(\frac{v}{v_0}\right)^{-(R/c_v)} \exp\left(\frac{s - s_0}{c_v}\right)$$

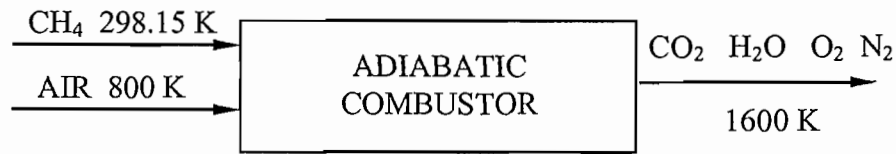From $du = Tds - pdv$ we have,

$$\left(\frac{\partial u}{\partial s}\right)_v = T = \frac{u_0}{c_v}\left(\frac{v}{v_0}\right)^{-(R/c_v)} \exp\left(\frac{s - s_0}{c_v}\right)$$

$$\left(\frac{\partial u}{\partial v}\right)_s = -p = u_0\left(-\frac{R}{c_v}\right)\left(\frac{1}{v}\right)\left(\frac{v}{v_0}\right)^{-(R/c_v)} \exp\left(\frac{s - s_0}{c_v}\right)$$
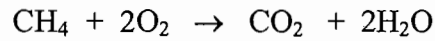
Dividing the second equation by the first gives,

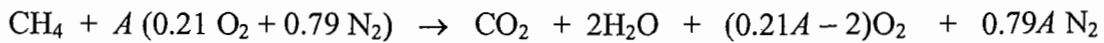$$\frac{p}{T} = \frac{R}{v} \quad \rightarrow \quad pv = RT$$ [30 %]

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2   (a)

For stoichiometric combustion of CH_4 with O_2 :

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

If A is the molar air-fuel ratio, the actual chemical reaction is :

$$CH_4 + A\,(0.21\,O_2 + 0.79\,N_2) \rightarrow CO_2 + 2H_2O + (0.21A - 2)O_2 + 0.79A\,N_2$$

Writing $T_0 = 298.15$ K, $T_A = 800$ K and $T_P = 1600$ K, the SFEE is,

$$\bar{h}_{CO2}(T_P) + 2\bar{h}_{H2O}(T_P) + (0.21A - 2)\bar{h}_{O2}(T_P) + 0.79A\,\bar{h}_{N2}(T_P)$$

$$= \bar{h}_{CH4}(T_0) + 0.21A\bar{h}_{O2}(T_A) + 0.79A\,\bar{h}_{N2}(T_A)$$

The LCV for CH_4 is 50 MJ/kg. Hence,

$$\Delta\bar{H}^0_{298} = \bar{h}_{CO2}(T_0) + 2\bar{h}_{H2O}(T_0) - \bar{h}_{CH4}(T_0) - 2\bar{h}_{O2}(T_0) = -16 \times 50 = -800 \text{ MJ/kmol}$$

Eliminating $\bar{h}_{CH4}(T_0)$ we obtain,

$$-\Delta\bar{H}^0_{298} = [\bar{h}_{CO2}(T_P) - \bar{h}_{CO2}(T_0)] + 2[\bar{h}_{H2O}(T_P) - \bar{h}_{H2O}(T_0)] - 2[\bar{h}_{O2}(T_P) - \bar{h}_{O2}(T_0)]$$

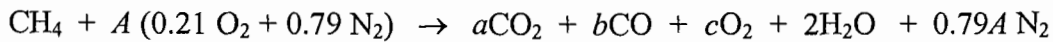$$+ 0.79A\,[\bar{h}_{N2}(T_P) - \bar{h}_{N2}(T_A)] + 0.21A\,[\bar{h}_{O2}(T_P) - \bar{h}_{O2}(T_A)]$$

Taking data from the molar enthalpy tables in the Data Book :

$$800.0 = (77.01 - 9.37) + 2(62.81 - 9.90) - 2(52.94 - 8.66)$$
$$+ 0.79A\,(50.59 - 23.72) + 0.21A\,(52.94 - 24.50)$$

$$= 67.64 + 105.82 - 88.56 + 21.23A + 5.97A$$

Hence, the molar air-fuel ratio is : $A = 26.29$ [50 %]

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(b)   With dissociation of CO_2 the actual chemical reaction is :

$$CH_4 + A\,(0.21\,O_2 + 0.79\,N_2) \rightarrow aCO_2 + bCO + cO_2 + 2H_2O + 0.79A\,N_2$$

Conservation of atomic C : $a + b = 1$

Conservation of atomic O : $2a + b + 2c + 2 = 0.42A$

Number of moles of products : $a + b + c + 2 + 0.79A = n$

Working in terms of b which is the 'degree of dissociation' of CO_2 :

$$a = 1 - b\,; \qquad c = 0.21A - 2 + 0.5b\,; \qquad n = 1 + A + 0.5b$$

$-\,CO - \dfrac{1}{2}O_2 + CO_2 = 0$ is Data Book reaction (7). The equilibrium equation is :

$$\left(\frac{p_{CO}}{p_0}\right)^{-1}\left(\frac{p_{O2}}{p_0}\right)^{-0.5}\left(\frac{p_{CO2}}{p_0}\right) = K_{p7}$$

From the Data Book at 2400 K, $\ln(K_{p7}) = 3.859 \;\rightarrow\; K_{p7} = 47.418$

The partial pressures are given by the mole fractions multiplied by the sample pressure p :

$$p_{CO2} = \frac{a}{n}\,p\,; \qquad p_{CO} = \frac{b}{n}\,p\,; \qquad p_{O2} = \frac{c}{n}\,p$$

Substituting into the equilibrium equation and rearranging gives :

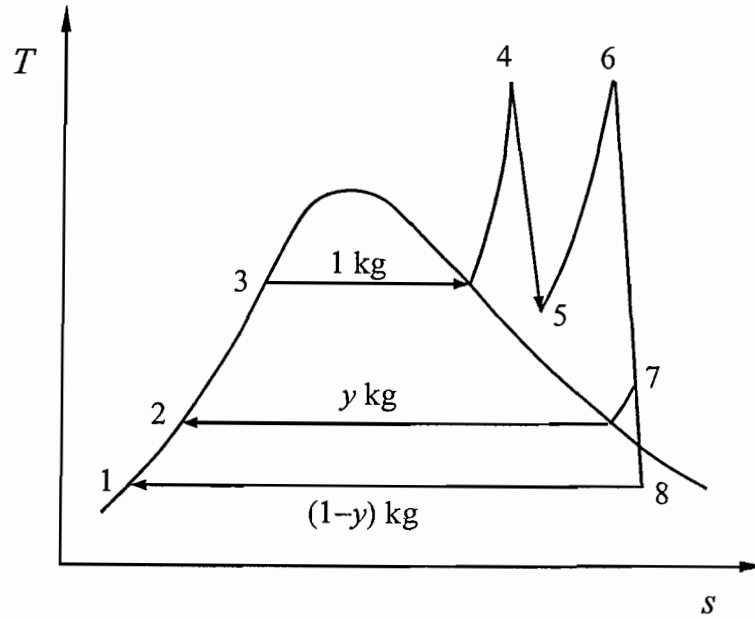$$\left(\frac{p}{p_0}\right)^{0.5} = \frac{1}{K_{p7}}\left(\frac{a}{b}\right)\left(\frac{n}{c}\right)^{0.5}$$

Hence, with $A = 26.29$ as calculated above and $b = 0.05$ as given :

$$\frac{p}{p_0} = \frac{1}{K_{p7}^2}\left(\frac{1-b}{b}\right)^2\left(\frac{1+A+0.5b}{0.21A-2+0.5b}\right) = \frac{1}{47.418^2}\left(\frac{1-0.05}{0.05}\right)^2\left(\frac{1+26.29+0.5\times0.05}{0.21\times26.29-2+0.5\times0.05}\right)$$

With standard pressure $p_0 = 1$ bar : $p = 1.237$ bar.   [50 %]

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3   (a)   (i)

[15 %]

(ii)   HP turbine work output $= w_{HPT} = (h_4 - h_5)$

From Table 1, $h_4 = 3439.5$ kJ/kg, $s_4 = 6.486$ kJ/kg K

For an isentropic expansion, $h_{5S} = 3033.2$ kJ/kg   (To be estimated from the steam chart)

Hence, $h_5 = h_4 - \eta_{HPT}(h_4 - h_{5S}) = 3439.5 - 0.90 \times (3439.5 - 3033.2) = 3073.8$ kJ/kg

Hence, $w_{HPT} = 3439.5 - 3073.8 = 365.7$ kJ/kg   [15 %]

(iii)   Ignoring the feed pump work, the SFEE for the feed heater is,

$$(1-y)h_1 + yh_7 = h_2 \qquad \rightarrow \qquad y = \frac{(h_2 - h_1)}{(h_7 - h_1)}$$

From Table 1, $h_1 = 121.4$ kJ/kg, $h_2 = 640.1$ kJ/kg

Assuming the LPT expansion line is straight on the Mollier diagram, $h_7 \approx 3110$ kJ/kg

$$y = \frac{(640.1 - 121.4)}{(3110 - 121.4)} = 0.1736$$ [15 %]

(iv)   Ignoring the feed pump work, the cycle efficiency is given by,

$$\eta_{CYCLE} = \frac{w_{HPT} + w_{LPT}}{q_{Boiler} + q_{Reheater}} = \frac{(h_4 - h_5) + (h_6 - h_7) + (1-y)(h_7 - h_8)}{(h_4 - h_2) + (h_6 - h_5)}$$

From Table 1, $h_8 = 0.05 \times 121.4 + 0.95 \times 2553.7 = 2432.1$ kJ/kg

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$$\eta_{CYCLE} = \frac{(3439.5 - 3073.8) + (3560.3 - 3110) + (1 - 0.1736)(3110 - 2432.1)}{(3439.5 - 640.1) + (3560.3 - 3073.8)}$$

$$= \frac{365.7 + 450.3 + 560.2}{2799.4 + 486.5} = 0.4188 \qquad [15\%]$$

(b) Environmental impact can be reduced in the following ways (four required) :

(i) Improve the cycle efficiency. This could be done by including further feedheaters and improving the LP turbine performance. Increasing the boiler pressure and superheat temperature would also help, but these possibilities are less practical as major modifications would be required. Any changes in the plant would be costly and would only give a limited emissions reduction.

(ii) Install flue gas desulpherisation. This is now law in the UK for all new coal-fired plant. It is expensive and plant efficiency is reduced by 1-2 percentage points.

(iii) Capture the CO_2 from the exhaust using, for example, an amine absorption tower. The CO_2 could be piped out to the North Sea and stored in an undersea saline aquifer. This is technically possible but there are currently no large plants equipped with amine towers, partly because the government has put no legislation in place, and partly because the efficiency of capture from the exhaust, where the CO_2 concentration is low, is not good.

(iv) Change to a coal gasification system which is cleaner than using pulverised coal. However, gasification plants are expensive and only a few demonstrator units are currently operating world-wide. Although there is a large penalty in the overall power plant efficiency, this is probably the future for large coal-fired plant.

(v) Introduce the use of biofuels. This is more sustainable, but could not replace all the fuel requirement of a large power station and would require a biomass boiler.

(vi) Use the heat rejected in the condenser for district space heating. The current condenser temperature is probably too low for this to be effective. Significant new plant and a demand for local space heating would be needed.

(vii) Change from coal to natural gas or light fuel oil. This would only be done if the coal-fired plant were to be replaced by a CCGT. However, the existing plant could be repowered with a GT, a popular option with older coal plants. The power output would be increased and there can be a small increase in efficiency. [40%]

Examiner's note: Part (b) was poorly answered because most candidates simply listed a few vaguely-relevant points half-remembered from lectures, and did not focus their discussion on the power plant in the question.

$$p_3 = p_2$$
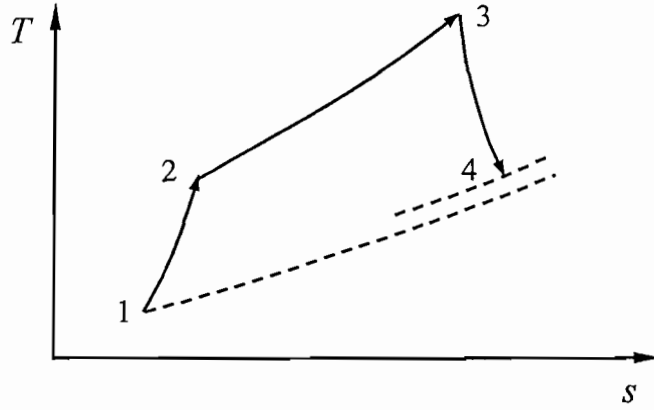
$$\alpha = \frac{p_4}{p_1}$$

$$\theta = \frac{T_3}{T_1}$$

(a) Compressor isentropic temperature ratio $= r_t = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

Compressor actual temperature ratio $= \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma \eta_c}} = r_t^{1/\eta_c}$

Turbine actual temperature ratio $= \dfrac{T_4}{T_3} = \left(\dfrac{p_4}{p_3}\right)^{\frac{\eta_t(\gamma-1)}{\gamma}} = \left(\dfrac{p_4}{p_1}\dfrac{p_1}{p_3}\right)^{\frac{\eta_t(\gamma-1)}{\gamma}} = \left(\dfrac{\alpha^{(\gamma-1)/\gamma}}{r_t}\right)^{\eta_t}$

Temperature ratio $\dfrac{T_4}{T_1} = \dfrac{T_4}{T_3}\dfrac{T_3}{T_1} = \theta\left(\dfrac{\alpha^{(\gamma-1)/\gamma}}{r_t}\right)^{\eta_t}$ [15 %]

Compressor work input $= w_c = c_p(T_2 - T_1) = c_p T_1\left(\dfrac{T_2}{T_1} - 1\right) = c_p T_1(r_t^{1/\eta_c} - 1)$
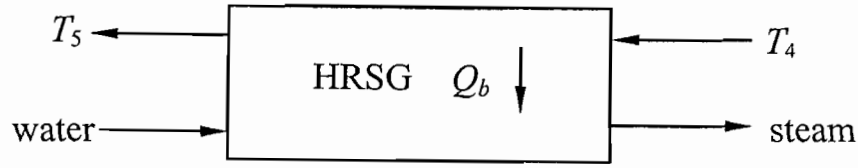
Turbine work output $= w_t = c_p(T_3 - T_4) = c_p T_1\left(\dfrac{T_3}{T_1} - \dfrac{T_4}{T_1}\right) = c_p T_1\left(\theta - \dfrac{T_4}{T_1}\right)$

Heat input $= q_{in} = c_p(T_3 - T_2) = c_p T_1\left(\dfrac{T_3}{T_1} - \dfrac{T_2}{T_1}\right) = c_p T_1(\theta - r_t^{1/\eta_c})$

GT thermal efficiency $= \eta_{gt} = \dfrac{w_t - w_c}{q_{in}} = \dfrac{(\theta - T_4/T_1) - (r_t^{1/\eta_c} - 1)}{(\theta - r_t^{1/\eta_c})}$

$$= 1 - \left(\dfrac{T_4/T_1 - 1}{\theta - r_t^{1/\eta_c}}\right)$$ [25 %]

(b) (i)

Maximum heat is transferred when exit temperature is $T_1 = 300$ K. Hence,

$$\text{HRSG efficiency} \quad = \quad \eta_b \quad = \quad \frac{T_4 - T_5}{T_4 - T_1}$$

For a compressor pressure ratio of 20, $\gamma = 1.35$, $\theta = 1500/300 = 5$, $\alpha = 1.1$, and $\eta_t = 0.9$,

$$r_t = 20^{0.35/1.35} = 2.174$$

$$\frac{T_4}{T_1} = \theta \left(\frac{\alpha^{(\gamma-1)/\gamma}}{r_t} \right)^{\eta_t} = 5 \times \left(\frac{1.1^{0.35/1.35}}{2.174} \right)^{0.9} = 2.542 \quad \rightarrow \quad T_4 = 762.5 \text{ K}$$

For a stack temperature of 120 °C = 393.15 K, the HRSG efficiency is,

$$\eta_b = \frac{762.5 - 393.15}{762.5 - 300.0} = 0.799 \qquad \text{[20\%]}$$

(ii) GT efficiency $= \eta_{gt} = 1 - \left(\frac{T_4/T_1 - 1}{\theta - r_t^{1/\eta_c}} \right) = 1 - \left(\frac{762.5/300 - 1}{5 - 2.174^{1/0.9}} \right) = 0.414$

Steam turbine work output $= w_{st} = \eta_b \, \eta_{st} \, (q_{in} - w_{gt})$

CCGT overall efficiency $\eta_{ccgt} = \dfrac{w_{gt} + w_{st}}{q_{in}} = \eta_{gt} + \eta_b \eta_{st} (1 - \eta_{gt})$

$$= 0.414 + 0.799 \times 0.4 \times (1 - 0.414) = 0.601 \qquad \text{[20\%]}$$

(c) The HRSG performance can be improved by (i) lowering the stack temperature using a preheating loop (subject to the limitations given below), (ii) minimising the pinch-point temperature difference (this requires a larger HRSG) or (iii) by using a dual or triple pressure level steam cycle to reduce the heat transfer exergy loss. [10 %]

Condensation in the stack limits the exhaust gas temperature. If the fuel is sulphur-free natural gas the dewpoint temperature of the pure H_2O condensate may be as low as 80 °C. If the fuel is a light oil containing sulphur, the dewpoint temperature of the H_2O-H_2SO_4 condensate may be as high as 135 °C. [10 %]