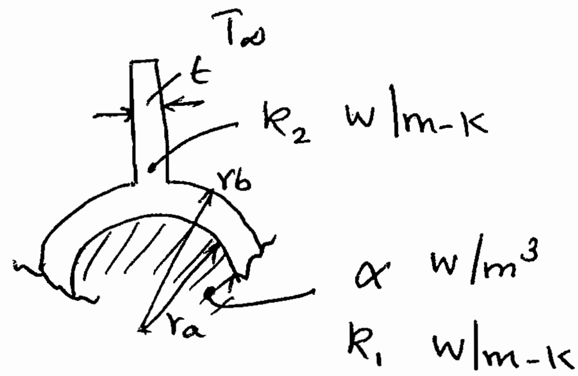
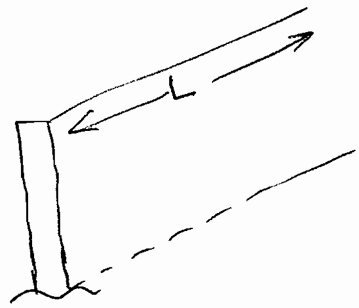
Dr Swaminathan (1)

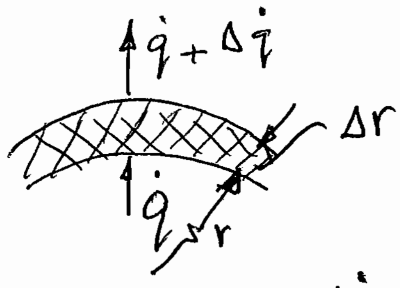
3A6 - 2008 - Cribs

(1)





(a)



$$\dot{q} = -k_1 A \frac{dT}{dr} \quad \text{(Fourier law)}$$

@ Steady state $\Delta\dot{q} = \alpha\, 2\pi r L\, \Delta r$

$$\frac{\Delta\dot{q}}{\Delta r} = \alpha\, 2\pi r L$$

as $\delta r \to 0$ $$\frac{d\dot{q}}{dr} = \alpha\, 2\pi r L$$

$$2\pi L k_1 \frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\alpha\, 2\pi r L \qquad \text{— (1)}$$

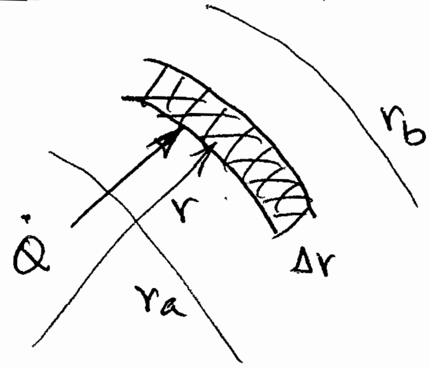
$$\Rightarrow r\frac{dT}{dr} = -\frac{\alpha r^2}{2k_1} + A \Rightarrow \frac{dT}{dr} = -\frac{\alpha r}{2k_1} + \frac{A}{r}$$

$\left.\frac{dT}{dr}\right|_{r=0} = 0$ because of symmetry in the fuel rod. $\Rightarrow A=0$

$$\Rightarrow \boxed{T(r) = -\frac{\alpha r^2}{4k_1} + T(0)}$$

$$T(r_a) = T(0) - \frac{\alpha r_a^2}{4k_1}$$

for the pipe:



Since there is no heat generation,

$$\dot{Q}_{in} = \dot{Q}_{out}$$

$$-k_2\, 2\pi r L \frac{dT}{dr} = \dot{Q}$$

$$\frac{dT}{dr} = -\frac{\dot{Q}}{2\pi k_2 L r}$$

$$\Rightarrow T(r) - T(r_a) = \frac{\dot{Q}}{2\pi k_2 L} \ln\left(\frac{r_a}{r}\right)$$

$$\therefore T(r_a) - T(r_b) = \frac{\dot{Q}}{2\pi k_2 L} \ln\left(\frac{r_b}{r_a}\right)$$

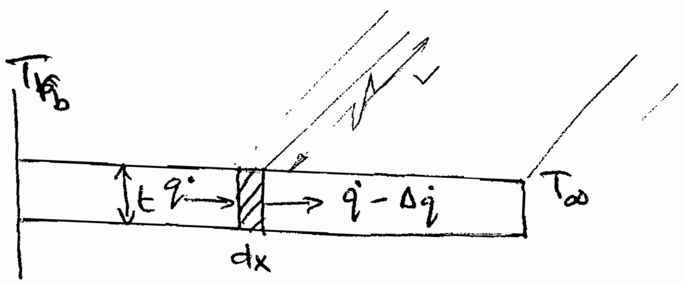
$$\Rightarrow \dot{Q} = \frac{2\pi L k_2 (T_{r_a} - T_{r_b})}{\ln(r_b/r_a)} = \alpha(\pi r_a^2 L)$$

$$\Rightarrow T_{r_a} - T_{r_b} = \frac{\alpha r_a^2}{2k_2} \ln\left(\frac{r_b}{r_a}\right)$$

But $T_{r_a} = T_0 - \dfrac{\alpha r_a^2}{4k_1}$

$$\Rightarrow \boxed{(T_0 - T_{r_b}) = \frac{\alpha r_a^2}{2}\left[\frac{1}{2k_1} + \frac{1}{k_2}\ln\left(\frac{r_b}{r_a}\right)\right]}$$

(b)



$$-\Delta\dot{q} = h A_s (T - T_\infty)$$

$$d\left(k_2 t L \frac{dT}{dx}\right) = h\, 2 * L * dx\, (T - T_\infty)$$

$$\Rightarrow \quad \frac{d^2T}{dx^2} = \left(\frac{2h}{k_2 t}\right)(T - T_\infty)$$

B.C: $T = T_\infty$ as $x \to \infty$

$T = T_b$ @ $x = 0$.

Now: $$\frac{d^2(T - T_\infty)}{dx^2} = m^2 (T - T_\infty) \qquad m^2 \equiv \left(\frac{2h}{k_2 t}\right)$$

The general solution is

$$(T - T_\infty) = c_1 e^{-mx} + c_2 e^{+mx}$$

Since $(T - T_\infty) \to 0$ as $x \to \infty$ $\quad c_2 = 0$.

$$\therefore \quad T - T_\infty = c_1 e^{-mx}$$

@ $x = 0$ $\quad T - T_\infty = (T_b - T_\infty)$

$$\therefore \quad \boxed{T - T_\infty = (T_{rb} - T_\infty)\, e^{-mx}}$$

~~heat~~
amount of heat coming into each fin

@ the base is

$$\Rightarrow -k_2 t L \left.\frac{dT}{dx}\right|_{x=0} = \left(\frac{\dot{Q}}{6}\right) = \frac{\pi r_a^2 L \alpha}{6}.$$

But $$\left.\frac{dT}{dx}\right|_{x=0} = -m\left(T_{r_b} - T_\infty\right) = -\frac{\pi r_a^2 \alpha}{6 k_2 t}$$

$$\therefore T_{r_b} - T_\infty = \frac{\pi \alpha r_a^2}{6 k_2 t m}$$

$$(T_{r_b} - T_\infty) = (T_0 - T_\infty) + (T_{r_b} - T_0)$$

$$\therefore T_0 - T_\infty = \overbrace{(T_0 - T_{r_b})}^{\text{from (a)}} + (T_{r_b} - T_\infty)$$

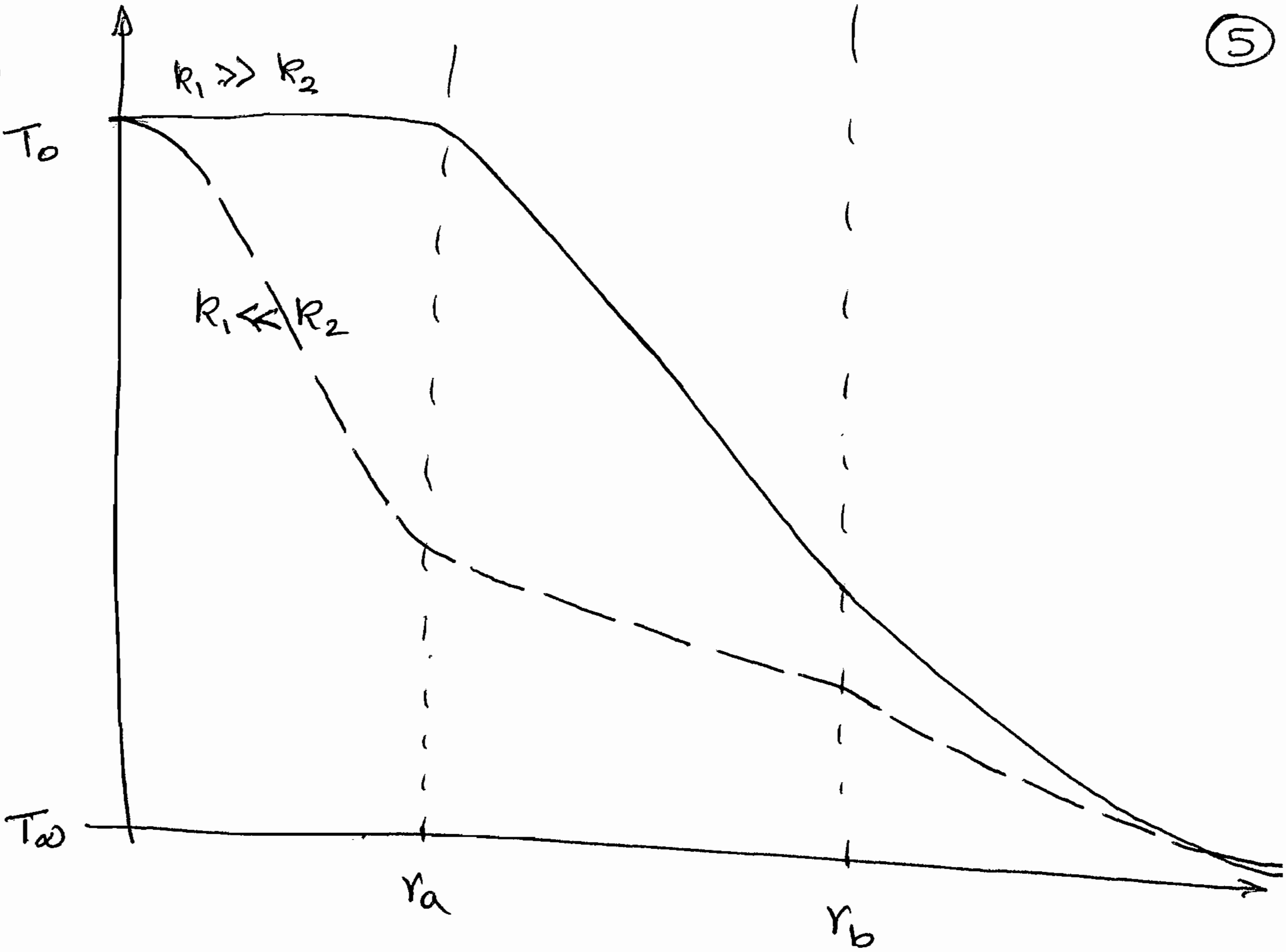
$$= \frac{\alpha r_a^2}{2}\left[\frac{1}{2k_1} + \frac{1}{k_2}\ln\left(\frac{r_b}{r_a}\right)\right]$$

$$+ \frac{\pi \alpha r_a^2}{6 k_2 t m}$$

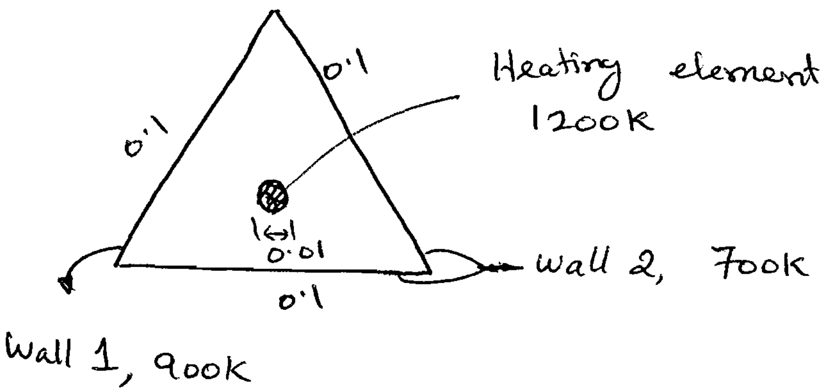
$$\boxed{\therefore (T_0 - T_\infty) = \frac{\alpha r_a^2}{2}\left[\frac{1}{2k_1} + \frac{1}{k_2}\ln\left(\frac{r_b}{r_a}\right) + \frac{\pi}{3k_2 m t}\right]}$$

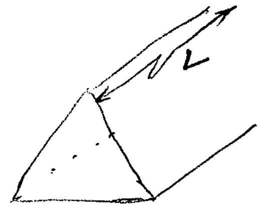
where $m = \left(\dfrac{2h}{k_2 t}\right)^{1/2}$

(c)



2





(a) The view factor, F_{ij}, between surfaces 'i' & 'j' is the fraction of radiation leaving surface 'i', which is intercepted by the surface j.

Key properties

1) Sum of the view factors should be one

2) Obeys the reciprocity theorem

$$A_i F_{ij} = A_j F_{ji}$$

ie between two surfaces 1 & 2

$$A_1 F_{12} = A_2 F_{21}$$

(i) $F_{1e} = ?$ $\quad F_{e1} + F_{e2} = 1$

By inspection of the Geometry & noting the Symmetry

$$F_{e1} = 1/3, \quad F_{e2} = 2/3$$

$$A_1 F_{1e} = A_e F_{e1}; \quad F_{1e} = \frac{\pi * 0.01 \times L}{0.1 \times L} \times 1/3$$

$$\boxed{F_{1e} = \pi/30}$$

(ii) $A_2 F_{2e} = A_e F_{e2}$

$$F_{2e} = \frac{\pi D L}{2 * 0.1 \times L} F_{e2} = \frac{\pi \times 0.01}{0.2} \times \frac{2}{3}$$

$$\boxed{F_{2e} = \frac{\pi}{30}}$$

(iii) $F_{1e} + F_{12} = 1$

$$\Rightarrow F_{12} = 1 - F_{1e} = 1 - \pi/30 = 0.895$$

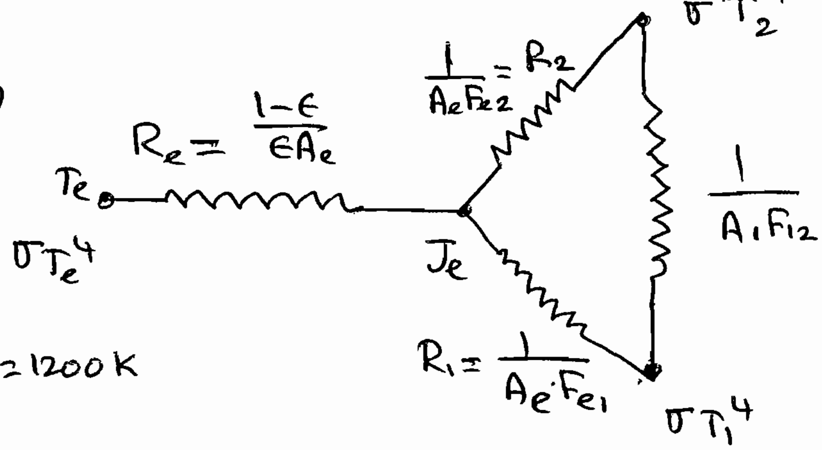
$$\boxed{F_{12} = 0.895}$$

(b) A black body absorbs all the radiation incident upon it, and emit with an emissive power of σT^4

A Gray body does not absorb all the radiation incident upon it, It can absorb a part of it and emit the rest. The emissive power of a gray body is $\epsilon \sigma T^4$, where ϵ is the emissivity. For gray body the emissivity & absorptivity are equal, constant and donot ~~dependent~~ depend on wave length.

(c)

(i)



$T_2 = 700K$

$T_e = 1200K$

$T_1 = 900K$

$\epsilon = 0.8$

$\sigma = 5.67 \times 10^{-8}$

(ii) Total Power radiated

$$P = \frac{(\sigma T_e^4 - J_e)}{R_e}$$

$$R_e = \frac{0.2}{0.8 \pi D L} = \frac{7.96}{L} \Rightarrow \boxed{\left(\frac{P}{L}\right) = \frac{(\sigma T_e^4 - J_e)}{7.96}}$$

$$R_1 = \frac{3}{\pi D L} = \frac{95.49}{L}$$

$$R_2 = \frac{3}{2\pi D L} = \frac{47.75}{L}$$

Balance of current @ J_e:

$$\frac{(\sigma T_e^4 - J_e)}{R_e} = \frac{(J_e - \sigma T_2^4)}{R_2} + \frac{(J_e - \sigma T_1^4)}{R_1}$$

$$\Rightarrow J_e\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_e}\right) = \sigma\left(\frac{T_e^4}{R_e} + \frac{T_1^4}{R_1} + \frac{T_2^4}{R_2}\right)$$

Substituting the values

$$J_e = \sigma * 1.735 \times 10^{12}$$

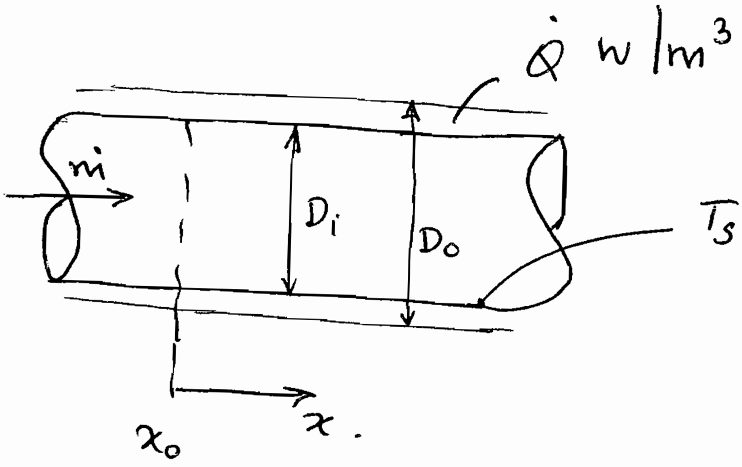
$$\Rightarrow \quad J_e = 98.375 \times 10^{3}$$

$$\therefore \left(\frac{P}{L}\right) = \frac{5.67 \times 10^{-8} * 1200^{4} - 98.375 \times 10^{3}}{7.96}$$

$$= \frac{(5.67 * 1.2^{4} - 9.8375) \times 10^{4}}{7.96}$$

$$\boxed{\left(\frac{P}{L}\right) = 2.412 \ kW/m.}$$

3)



* Constant surface heat flux

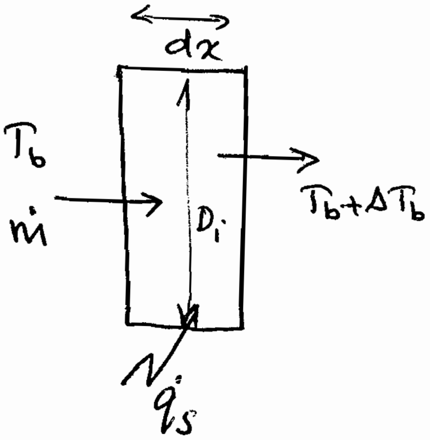
$$-\dot{q}_s = h\left(T_b - T_s\right) \Rightarrow \left(T_s - T_b\right) = \left(\frac{\dot{q}_s}{h}\right) = \text{Const}$$

* The flow is fully developed from x_o

⇒ Bulk mean temperature "profile" does not vary with x.

⇒ Neglect the axial conduction in the analysis.

(a)



$$\dot{m}\, c_p\, \Delta T_b = \pi D_i\, \Delta x\, \dot{q}_s$$

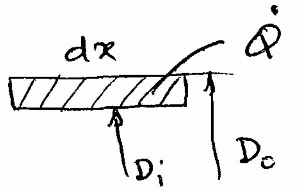
$$\Rightarrow \boxed{\frac{dT_b}{dx} = \left(\frac{\pi D_i}{\dot{m}\, c_p}\right)\dot{q}_s}$$

integrate:

$$\boxed{T_b(x) = T_{b,o} + \left(\frac{\pi D_i}{\dot{m}\, c_p}\right)\dot{q}_s\,(x - x_o)}$$

since $\dot{q}_s = \text{const}$

(b)

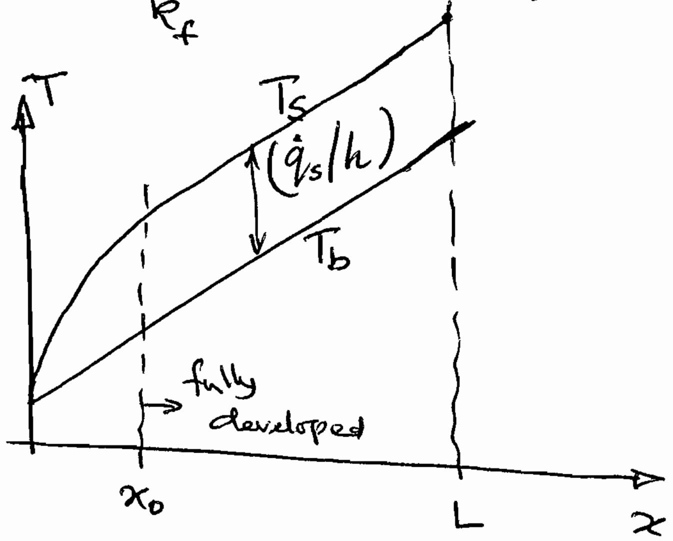


$$\dot{Q}\,\frac{\pi}{4}\left(D_o^2 - D_i^2\right) dx = \dot{q}_s \,\pi D_i \,dx$$

$$\boxed{\dot{q}_s = \frac{\dot{Q}}{4}\, D_i \left[\left(\frac{D_o}{D_i}\right)^2 - 1\right]}$$

(c) $Nu_{D_i} = \dfrac{h D_i}{k_f} = 4.4 \Rightarrow h = \left(\dfrac{4.4\, k_f}{D_i}\right)$

(i)



(ii) The maximum T_s occurs @ $x = L$

$$\Rightarrow \dot{q}_s = -h\left(T_{b_L} - T_{s_L}\right)$$

$$= -\frac{4.4 * 0.682}{0.03} * (353 - 400)$$

$$\boxed{\dot{q}_s = 4.7 \ \text{kW/m}^2}$$

(iii) from (a)

$$T_{b,L} = T_{b,o} + \left(\frac{\pi D_i}{\dot{m} C_p}\right) \dot{q}_s * L$$

$$\Rightarrow L = (T_{b,L} - T_{b,o}) \frac{\dot{m} C_p}{\pi D_i q_s}$$

$$= (353 - 298) \frac{0.05 * 4.7 \times 10^3}{\pi \times 0.03 \times 4.7 \times 10^3}$$

$$\boxed{L = 29.18 \text{ m.}}$$

(iv) $\dot{m} C_p \Delta T = P$

$0.05 \times 4.7 \times 10^3 \times 55 = 12.925$ kw

alternatively:

$$\dot{q}_s = 0.25 \dot{Q} D_i \left[\left(\frac{D_o}{D_i}\right)^2 - 1\right]$$

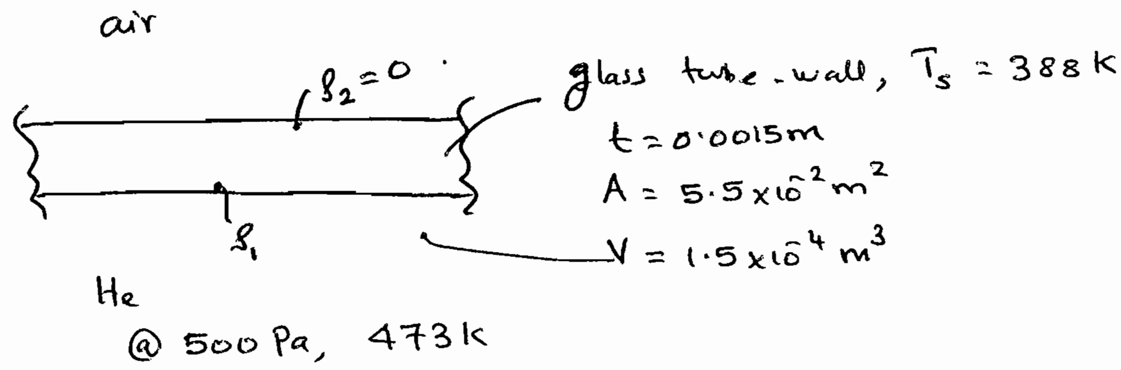
$$\Rightarrow \dot{Q} = 208.889 \text{ kw/m}^3.$$

$$V = \frac{\pi}{4}(D_o^2 - D_i^2) * L = 0.0619 \text{ m}^3$$

$$\Rightarrow P = 12.930 \text{ kw}$$

or $\dot{q}_s \pi D_i L = P$

4)

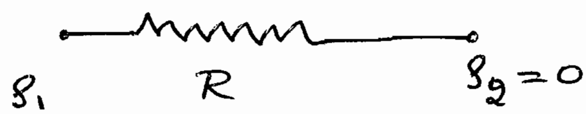


$$\rho_1 = S\rho \qquad S = -1.19\times10^{-3} + 3\times10^{-5}\,T_s$$
$$= 0.01045$$

Helium as ideal gas

$$\Rightarrow \rho = \frac{PM}{RT} = \frac{500\times4}{8.314\times10^{3}\times473} = 5.086\times10^{-4}\,\frac{kg}{m^3}$$

(a)



$$A\,G_{dif} = A\mathcal{D}\frac{(\rho_1-\rho_2)}{t}$$
$$= A\mathcal{D}\frac{\rho_1}{t}.$$

$$R = \frac{t}{A\mathcal{D}}$$

$$\mathcal{D} = 1.4\times10^{-8}\exp\left(-\frac{3280}{T_s}\right)$$
$$= 2.984\times10^{-12}\ m^2/sec.$$

$$\therefore R = \frac{0.0015}{5.5\times10^{-2}\times2.984\times10^{-12}}$$

$$= \frac{1.5}{5.5\times2.984}\ 10^{11}$$

$$\boxed{R = 9.1397\times10^{9}\ s/m^3}$$

(b) Leak rate = diffusion rate

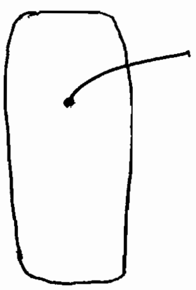
$$= A\, G_{diff} = \frac{(\rho_1 - \rho_2)}{R}$$

$$= \frac{\Delta \rho}{R} = \frac{0.01045 \times 5.086 \times 10^{-4}}{9.1397 \times 10^{+9}}$$

$$= \frac{1.045 \times 5.086}{9.1397} \times 10^{-15}$$

$$\boxed{\text{leak rate} = 5.815 \times 10^{-16} \text{ kg/sec.}}$$

(c)



He, M_o - initial mass. $= \rho V$

$$\frac{dm}{dt} = -(A\, G_{diff}) = -\text{leak rate.}$$

Since He inside the lamp is replenished (topped up) from the reservoir, "Pressure" remains const.

$\Rightarrow \rho = \text{Const} \Rightarrow$ leak rate is also Const.

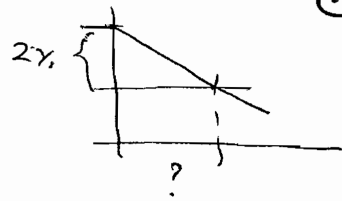
$$\Rightarrow \boxed{m(t) = m_o - (\text{leak rate}) * t.}$$

$\Rightarrow \Delta m = [m(t) - m(o)]$ is linear with t.

$\Delta m = 2\%$ of m_o $= 0.02 \times 5.086 \times 10^{-4} \times 1.5 \times 10^{-4}$

$= 1.526 \times 10^{-9}$ kg

$$\Delta m = 1.526 \times 10^{-9} \text{ kg}$$



$$t = \frac{\Delta m}{\text{diffusion rate}} = \frac{1.526 \times 10^{-9}}{5.815 \times 10^{-16}}$$

$$\boxed{t = 2.624 \times 10^{6} \text{ sec.}} = 728.89 \text{ hrs.}$$

$\Rightarrow$ 30.37 days.

(d) The leak rate can be reduced by increasing the resistance 'R'

$$R = \frac{t}{AD}$$

1) increase the glass wall thickness 't'

2) Decrease Area, 'A', available for diffusion

3) change the diffusivity D (decrease)

a) by reducing T_s

b) changing the glass composition

– This will change D & S.