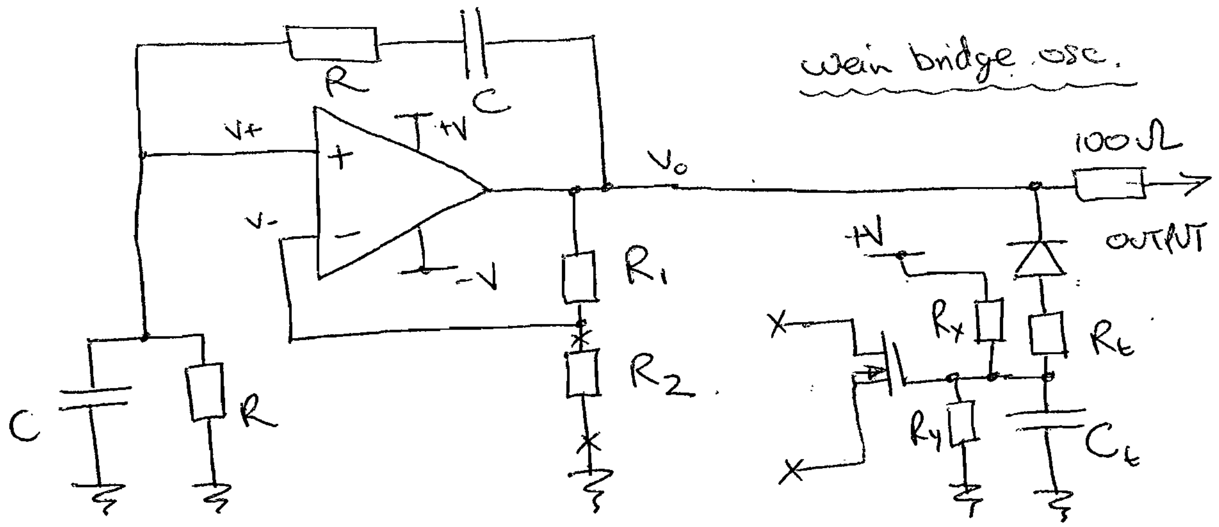
1 (a)

wein bridge osc.



$$V_+ = V_o \cdot \frac{\dfrac{R}{1+j\omega CR}}{\dfrac{R}{1+j\omega CR} + R + \dfrac{1}{j\omega C}} = \frac{1}{1 + \dfrac{R + 1/j\omega C}{\dfrac{R}{1+j\omega CR}}}$$

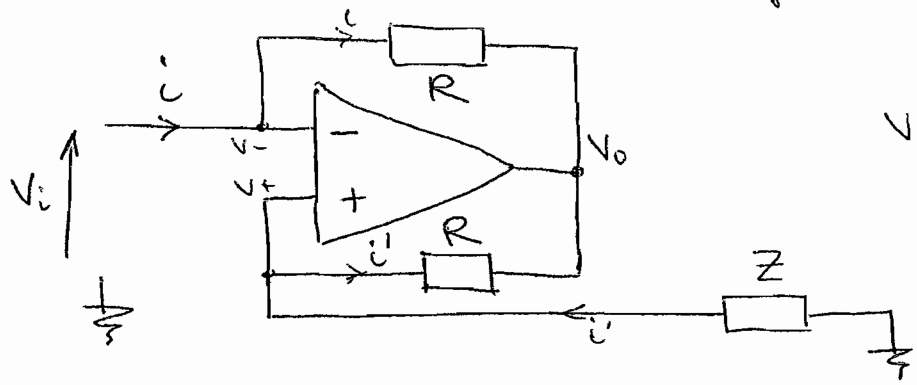
$$\therefore\ V_+ = \frac{V_o}{3 + j\omega CR + \dfrac{1}{j\omega CR}}$$; hence with $\omega = \frac{1}{CR}$: $V_+ = \frac{V_o}{3}$

Hence $1 + \frac{R_1}{R_2} = 3$ for stable oscillation and $f = \frac{1}{2\pi RC}$

We can stabilise the output using a ntc. thermistor as R_1 or by using a FET or MOSFET as a variable resistance to control the gain of the op-amp in response to the output amplitude. R_x and R_y are chosen to bias the FET to give a gain >3 for start-up. R_t and C_t are chosen to give a long time constant compared to the oscillations eg: $R_t = 100k\Omega$, $C_t = 100nF$ ($\tau = CR = 0.01s$). For 100kHz, let $R = 1k\Omega$ then $C = 1.59\ nF$. Supply rails $\pm V$ can be $\pm 5V$ for 5Vpp output.

[35%]

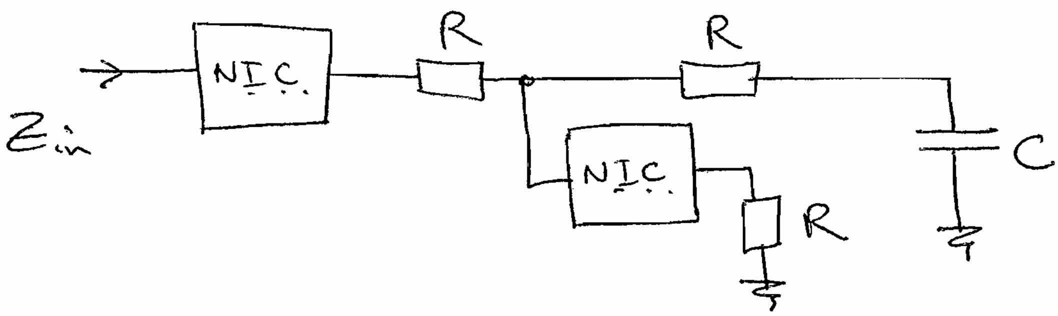
1b) Firstly we need a Negative Impedance Converter (NIC) :-



$V_- = V_i = V_+$

As $V_+ = V_-$ then the voltage across the impedance $Z = V_i$ and since $(V_o - V_i)$ appears across both + and − feedback resistors then $i' = i \Rightarrow V_+ = -iZ \therefore V_i = -iZ$, hence the input impedance $\frac{V_i}{i} = -Z$.

To synthesis an inductor, we cascade two NIC's with other passive components :-



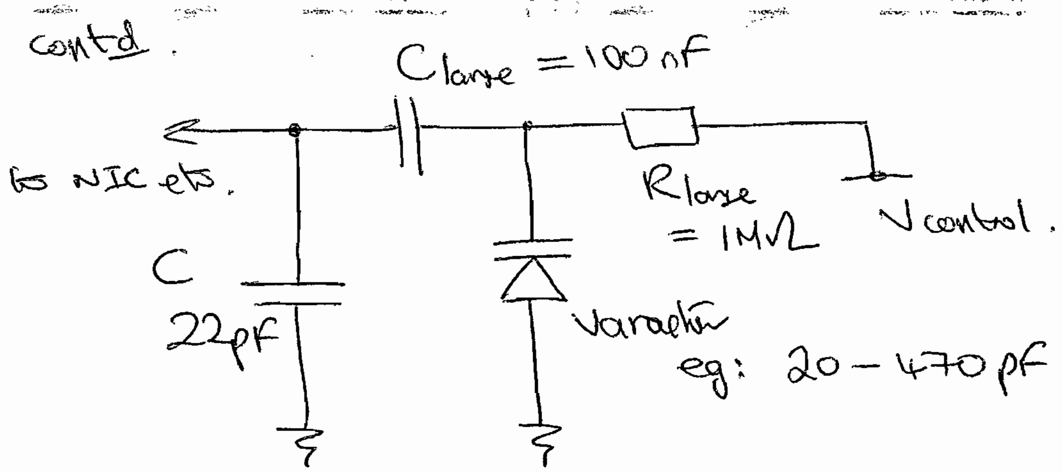
$$\therefore Z_{in} = -\left[R + -R /\!/ \left(R + \frac{1}{j\omega C}\right)\right]$$

$$= -\left[R + \frac{-R\left(R + \frac{1}{j\omega C}\right)}{\cancel{-R} + \cancel{R} + \frac{1}{j\omega C}}\right]$$

$$= -\left[\cancel{R} - R^2 j\omega C - \cancel{R}\right] = j\omega C R^2 \equiv j\omega L$$

Hence, synthesised inductor $L = CR^2$. To vary L electrically, we shunt C with a varactor diode.

1 b) contd.



Let's assume the varactor can vary 20 – 470pF, hence with C = 22pF, the total capacitance varies 42 – 492 pF ie: just over 10:1 range.

So select 47pF ⇒ 1mH (470pF ⇒ 10mH)

$L = CR^2 = 10^{-3} = 47\times10^{-12}R^2 \quad \therefore \; R = 4.6\,k\Omega$

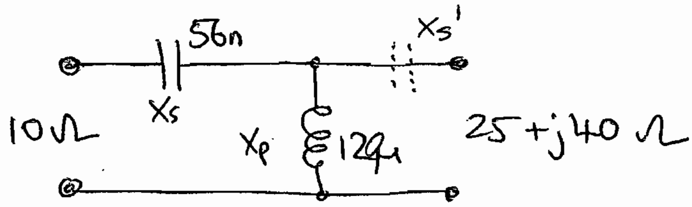
[30%]

c) Plot impedance point A, then susceptance B. A parallel inductor moves B → C $j(0.315 - 0.182) = -j0.133$. Point C is such that CO = OD, where D lies on the unit Re circle. A series capacitor $-j2.85$ matches point D to O. For 100kHz, $\omega = 2\pi\times10^5$ rad/s

$\therefore$ Inductor impedance (shunt) $= \dfrac{10}{-j0.133} = j752 = j\omega L$

$\therefore L = 120\mu H$

Series capacitor $= -j28.5 = \dfrac{1}{j\omega C}$ $\therefore C = 56nF$

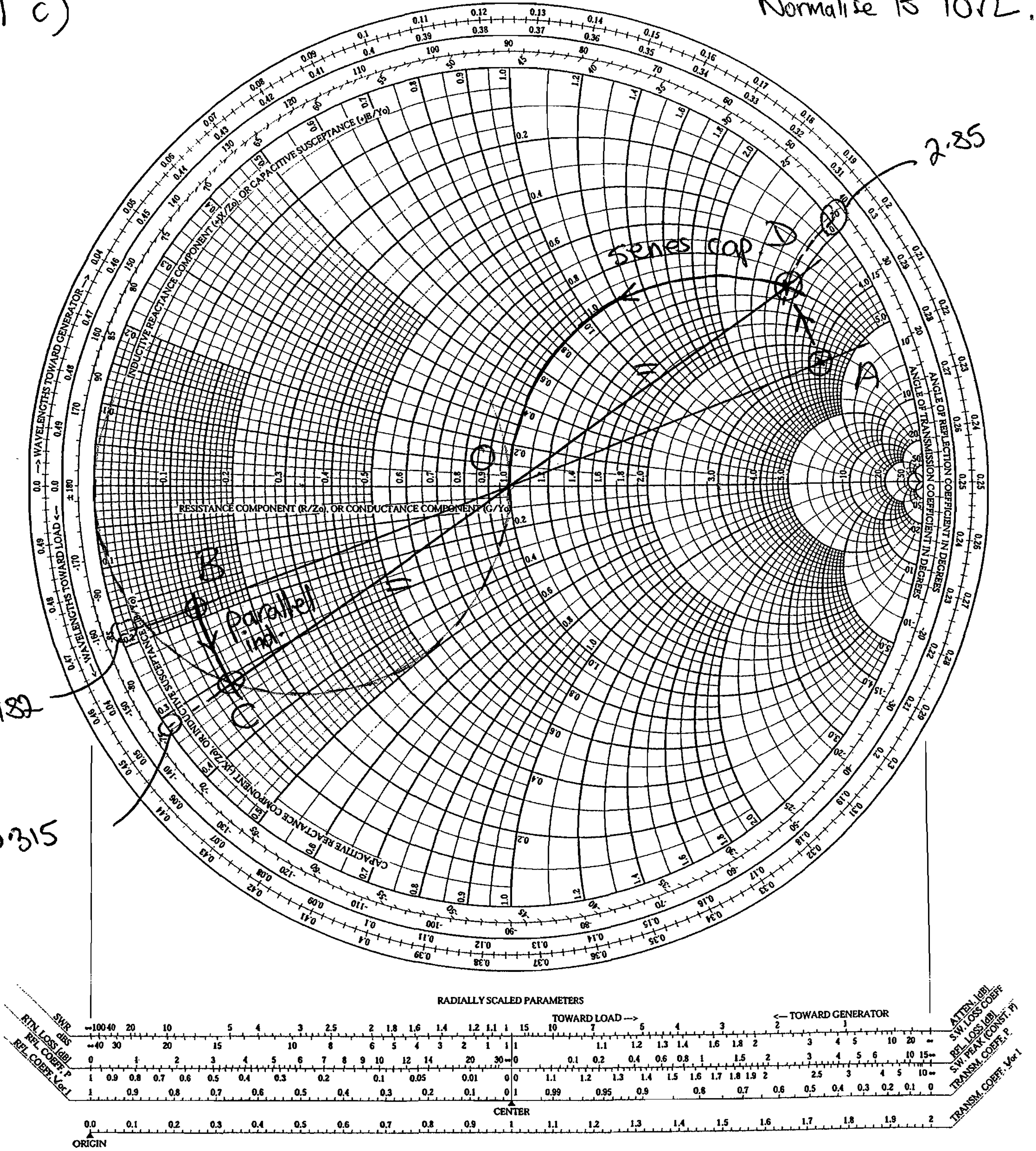


This can also be solved by calculation using $\dfrac{X_s}{10} = \dfrac{25}{X_p} = \sqrt{\dfrac{25}{10} - 1}$ but needs an additional X_s' ($-j40\Omega$) to cancel the inductive load term

Chart for question 4; to be detached and handed in with script.

1 c)

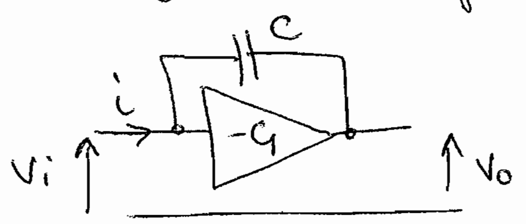
Normalise to 10Ω.



at 100 kHz, $\omega = 2\pi \times 10^5$ rad/s.

[35%]

2 a) The Miller Effect is apparent in electronic circuits which have gain and capacitance between input and output.



For example, summing the currents at the input :-

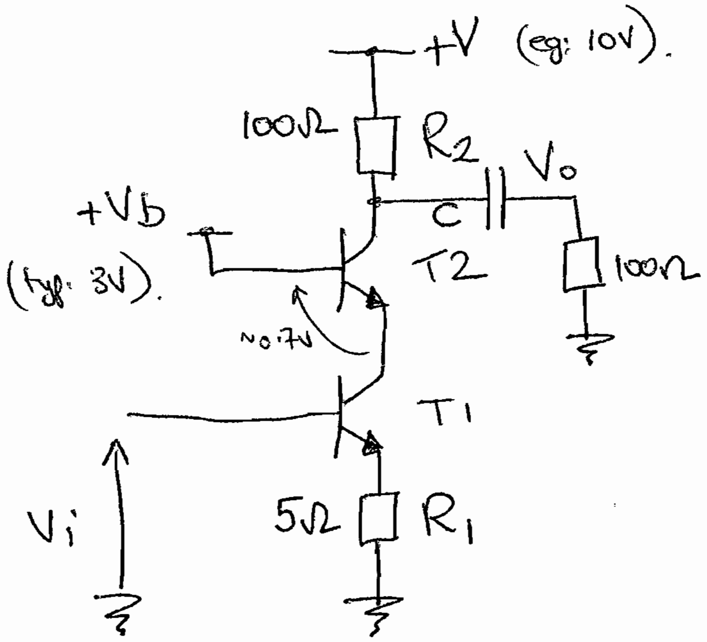
$$i = \frac{V_i - V_o}{\frac{1}{j\omega C}} \quad \text{where } V_o = -G V_i$$

$$\therefore \quad i = \frac{V_i + G V_i}{1/j\omega C} = \frac{V_i(1+G)}{\frac{1}{j\omega C}} \equiv \frac{V_i}{\frac{1}{j\omega(1+G)C}}$$

Hence, the equivalent value of capacitance seen at the input node is $(1+G)C$ and so the source impedance R will combine with this to give a -3dB roll-off at a frequency which decreases as the gain increases : $f_{-3dB} = \frac{1}{2\pi R(1+G)C}$

[10%]

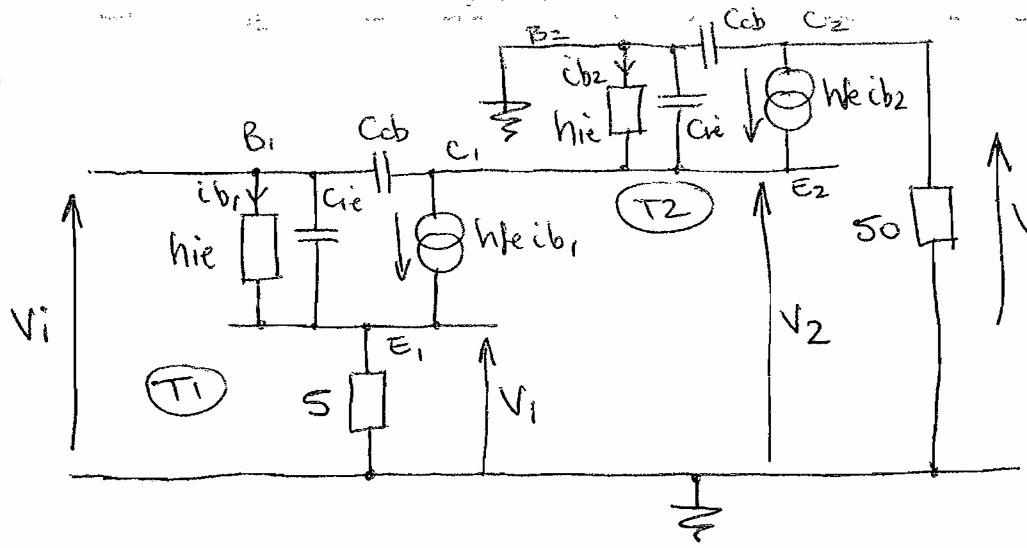
b) Cascode circuit



$$\frac{V_o}{V_i} = \frac{-100/2}{5} = -10.$$

T1 is the input transistor, it has current gain but not voltage gain - hence the Miller effect is small, i.e. the collector voltage is kept nearly constant by the B-E diode of T2. T2 provides the voltage gain but since the base is connected to a low impedance source, the CR time constant remains small - so maintaining the bandwidth. [25%]

2c)



$h_f = 500$

$f_T = 250$ MHz

$C_{cb} = 5$ pF

$r_e = \frac{V_T}{I_c}$

$h_{ie} = h_{fe}\, r_e$

$f_T = \frac{1}{2\pi r_e C_{ie}}$

with DC bias current of 50 mA, $r_e = \frac{0.025}{0.05} = 0.5\,\Omega$

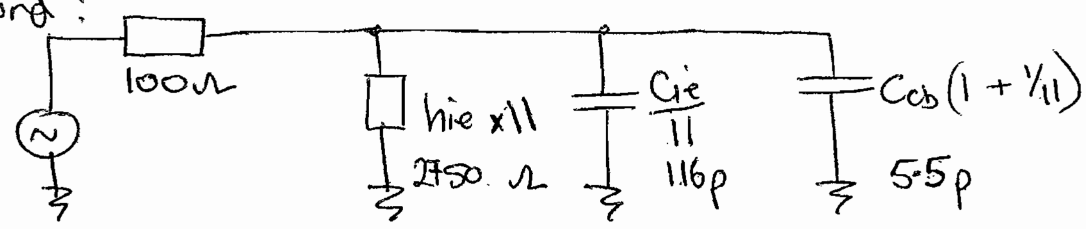
$\therefore h_{ie} = 250\,\Omega$ and $C_{ie} = 1275$ pF

$V_1 = \frac{5}{5+r_e} V_i = \frac{10}{11} V_i$, $V_2 = -i_{b2} h_{ie}$ where $i_{b1} \approx i_{b2}$

(from summing currents at V_2 node) and $i_{b1} = \frac{V_i - V_1}{h_{ie}} = \frac{V_i}{2750} = \frac{V_i}{11\, h_{ie}}$

$\therefore V_2 = \frac{-V_i \cdot \cancel{h_{ie}}}{11\, \cancel{h_{ie}}} = \frac{-V_i}{11}$

Hence, looking at the input circuit for T1 and referring R_s' and C_s' to ground:



$\therefore R = 100 \| 2750\ \Omega$ $C = 116 + 6$ pF

$\therefore f_{-3dB} = \frac{1}{2\pi \cdot 96 \cdot 122 \times 10^{-12}} = \underline{13.6 \text{ MHz}}$

Without the Cascode's second transistor, Ccb would be multiplied by 11 = 55 pF and the roll-off would be ≈ 9.7 MHz.

Note: for T2 input cct. the grounded base gives a very small time constant with f_T roll-off.

[65%]

3 a) Power density from ideal dipole $= \dfrac{1.5\ P_t}{2\pi R^2}$

$\therefore$ at 100km, $P_r = \dfrac{1.5 \times 15 \times 10^3}{2\pi\ (10^5)^2} = 3.58 \times 10^{-7}\ W/m^2$

at 350km, $P_r = \dfrac{1}{3.5^2} \cdot 3.58\times10^{-7} = 2.92 \times 10^{-8}\ W/m^2$

(8.2% of higher value)

$P = \frac{1}{2}\eta H^2 = \frac{1}{2}\dfrac{E^2}{\eta}$ where $\eta = 120\pi$ = impedance of free space

$\therefore$ $\underline{H = 1.2 \times 10^{-5}\ A/m}$ $\underline{E = 4.7\ mV/m}$ [20%]

b) $A_e = \dfrac{\pi L^2}{4} = 78.5\ m^2$ with $L = 10\,m$

$\therefore$ Received power $= 78.5 \times 2.92 \times 10^{-8}\ W = 2.29\ \mu W$

into a matched load = 4kΩ gives

$2.29 \times 10^{-6} = \dfrac{V_r^2}{4000}$ $\therefore$ $V_r = 96\ mV$ rms OR

$\underline{0.27\ V_{pp}}$

$G = \dfrac{4\pi A_e}{\lambda^2}$ with $\lambda = 5000\ m$ @ 60 kHz

$\therefore$ $G = 3.94\times10^{-5}$ $\underline{\equiv -44\ dB}$

$R_r = 20\pi^2\left(\dfrac{L}{\lambda}\right)^2$ assuming a linear current distribution

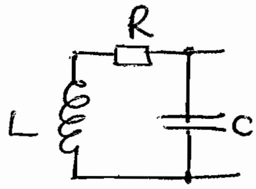
$\underline{= \cancel{2.51} \times 10^{-4}\ \Omega}$

7.89

[35%]

3 c) $\delta = \sqrt{\dfrac{2}{\omega\mu\sigma}} = \sqrt{\dfrac{1}{\pi f \mu \sigma}} = 0{\cdot}325$ mm @ 60kHz

$\therefore$ assuming 500 turns of 0·325 mm ϕ wire around 5cm diameter,
the resistance $R_1 = \dfrac{\rho l}{a} = \dfrac{N\pi D}{\sigma \frac{\pi d^2}{4}} = \dfrac{4ND}{\sigma d^2}$



$$\therefore \quad R = \frac{4 \,.\, 500 \,.\, 0{\cdot}05}{4\times10^{7} \,.\, (0{\cdot}325\times10^{-3})^2} = 23{\cdot}7\,\Omega.$$

If the coil is resonant at 60kHz with C=660pF, then

$60\times10^{3} = \dfrac{1}{2\pi\sqrt{LC}}$ and $L = 10{\cdot}6$ mH

and at 60kHz, $\omega L = 4{\cdot}0\,\text{k}\Omega$ hence the Q-factor is given by :-

$Q = \dfrac{4000}{23{\cdot}7} = 169$ [assuming the capacitor is ideal - lossless]

[25%]

? d) $B = \mu_0 H \quad \therefore \quad B = 4\pi\times10^{-7} \times 1{\cdot}2\times10^{-5} = 16$ pT

$V_{induced} = NBA\,2\pi f$ $\leftarrow$ (from $V = N\frac{d\phi}{dt}$, $\phi = BA$)

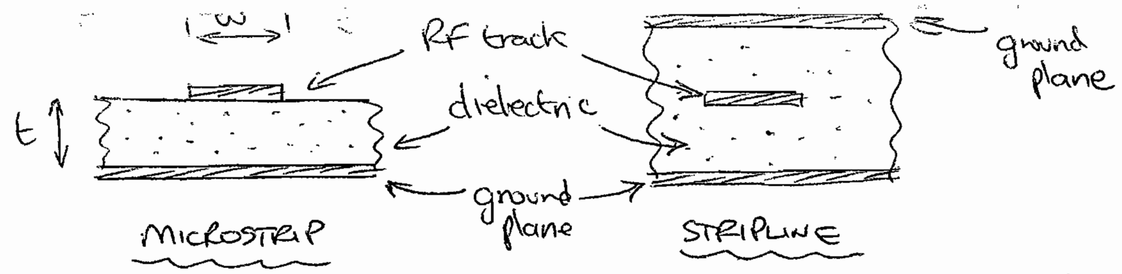
$A = \pi r^2 = \dfrac{\pi}{4}0{\cdot}05^2 = 1{\cdot}96\times10^{-3}\,\text{m}^2$

$\therefore V_i = 500 \,.\, 16\times10^{-12} \,.\, 1{\cdot}96\times10^{-3} \,.\, 2\pi \,.\, 60\times10^{3} = 5{\cdot}91\,\mu\text{V}$ [not resonant]

$\therefore V_i' = 169 \times 5{\cdot}91 = 1{\cdot}0\,\text{mV}$ [resonant]

[20%]

4 a)



- easy to fabricate
- easy for surface mount components / connections
- higher radiation losses

- more complex fabrication
- vias required for connections
- low loss

[10%]

b) Taking $\epsilon_r = 3$ $\quad t = 0.25\,mm$ $\quad Z_0 = 50\,\Omega$

$$C \text{ per unit length} = \frac{A\epsilon_0\epsilon_r}{t} = \frac{(w+2t)\epsilon_0\epsilon_r}{t}, \text{ assuming}$$

the effective track width to be t each side of w.

$$\text{Speed of light in polymer, } v = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{LC}} = \sqrt{3}\times10^8\,m/s$$

$$\text{Characteristic impedance, } Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{vC}$$

$$\therefore Z_0 = \frac{t}{(w+2t)\epsilon_0\epsilon_r v} = 50\,\Omega = \frac{0.25\times10^{-3}}{\sqrt{3}\times10^8 \cdot (w+0.5\times10^{-3}) \cdot 8.854\times10^{-12} \cdot 3}$$

$$\therefore w = 0.587\ mm$$

$$Z_0 \text{ range} = \sqrt{\frac{3}{3\pm0.5}} \cdot 50\,\Omega = 46 \rightarrow 55\,\Omega$$

$9\,\Omega$ pp

i.e. $\pm 8\%$ approx

[40%]

c) $\underline{\text{Max.}}$ speed of light $= \dfrac{3\times10^8}{\sqrt{2.5}} = 1.90\times10^8\ m/s$

$\underline{\text{Min.}} = \dfrac{3\times10^8}{\sqrt{3.5}} = 1.60\times10^8\ m/s$

$\therefore$ wavelength @ 6 GHz $= 31.7\ mm$ $\quad = 26.7\ mm$

4c) contd.

$$\text{Hence relative phase shift over } 50\text{ mm} = 360^\circ \times \left(\overbrace{\frac{50}{26.7} - \frac{50}{31.7}}^{\Delta N\lambda \text{ in } 50\text{ mm}}\right)$$

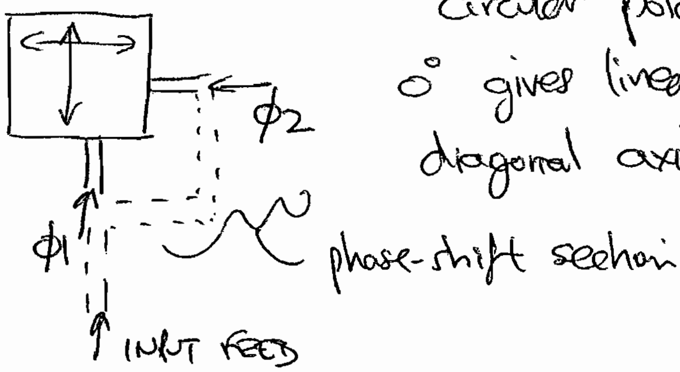
$$= 360^\circ \times 0.295$$

$$= \underline{106^\circ}$$

[30%]

d) To resonate, the patches should be $\lambda/2$ across. With $\epsilon_r = 3$
$v = \sqrt{3} \times 10^8$ m/s and $\lambda = 28.9$ mm @ 6 GHz
Hence the patches will be 14.4 mm square. (v. small)

To control polarisation, the 2 edges of the square can be driven with different phases: $\pm 90^\circ$ gives right or left hand circular polarisation
0° gives linear polarisation along diagonal axis.



[20%]