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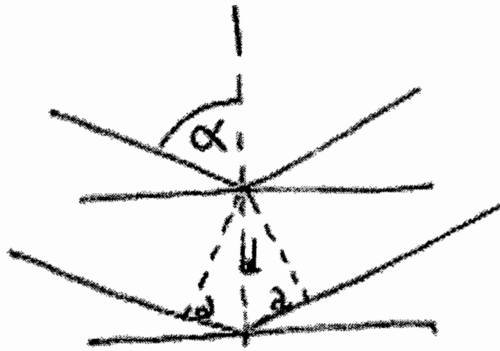
3B5 CRIB 2008

Q1

(a)



Phase difference $= 2a = 2d\cos\alpha$

(i) Eq. 1 $\quad 2d\cos\alpha = n\lambda \quad n = 1, 2 \ldots$

[20%]

Minimum voltage corresponds to $n = 1$ in Equation 1

$$eV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p} = 2d\cos\alpha$$

$$p^2 = \frac{h^2}{4d^2\cos^2\alpha} \qquad V = \frac{h^2}{8emd^2\cos^2\alpha}$$

(ii) $\quad$ **d= 2.10^{-10} m**

[30%]

b) $\psi(x) = A\sin kx + B\cos kx$

$$k^2 = \frac{2mE}{\hbar^2}$$

$\psi(a) = 0 \Longrightarrow A\sin ka = 0$

$ka = n\pi \qquad n = 1, 2, \cdots$

$$E_n = n^2\frac{\hbar^2\pi^2}{2ma^2}$$

$$\psi(x) = A\sin n\frac{\pi}{a}x$$

Determine A from normalization:

$$\int_0^a |\psi|^2 dx = |A|^2\int_0^a \sin^2 n\frac{\pi}{a}x \, dx = 1$$

(i) $A = \sqrt{\frac{2}{a}}$

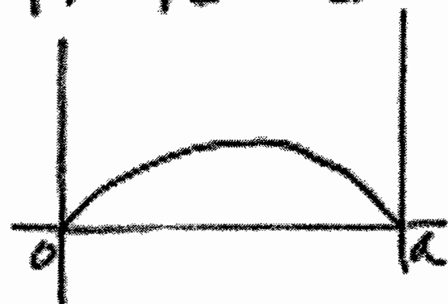
[30%]

(ii) For $n = 2$

$$\psi_2(x) = \sqrt{\frac{2}{a}}\sin\frac{2\pi}{a}x$$

For $n = 1$

$$\psi_1 = \sqrt{\frac{2}{a}}\sin\frac{\pi}{a}x$$



[20%]

Q2

a) [40%]

(i) n-type Silicon

Silicon is a Group IV element: $1s^2 2s^2 2p^6\ 3s^2 3p^2$

- The aim of doping the material n-type is to increase the number of electrons in the CB
- Therefore, we add an impurity from Group V
  - Arsenic
  - **Phosphorous:** $1s^2 2s^2 2p^6\ 3s^2 3p^3$
  - Antimony
- The dopant is added in a concentration of about 1 part in 10^6
- At such a low impurity level, to a first approximation the crystalline lattice of the semiconductor will not change
- This is known as ***substitutional doping***

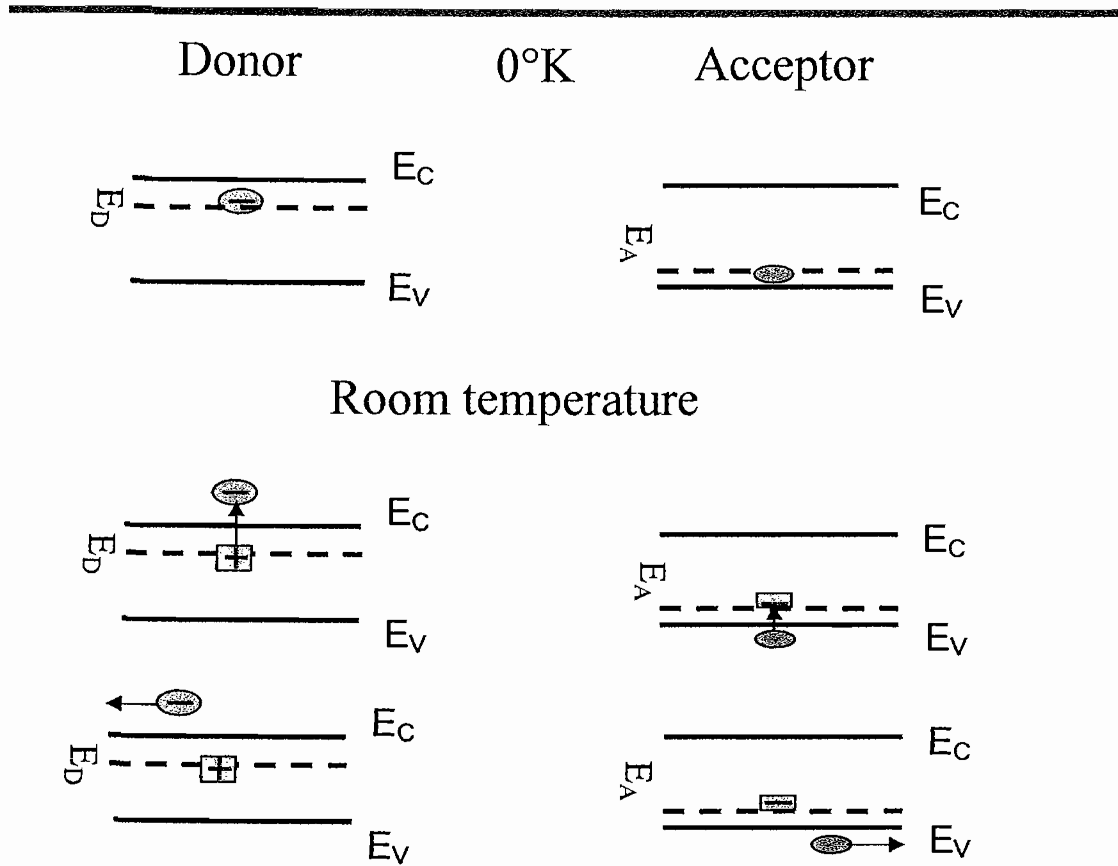
- In doing so, the Gp. V atom will use 4 of its 5 valence electrons to form covalent bonds with 4 neighbouring Si atoms
- There will now be one spare electron on the Gp. V atom
- The electron is only weakly bound
- In energy terms, it will form an energy level E_D only ~0.03eV below E_C which will be occupied at 0°K
- Electrons may be easily excited into the CB from the energy level E_D as temperature rises
- So, the addition of each Gp. V impurity leads to an extra available conduction electron ***without*** the creation of a hole in the VB to produce a ***n-type semiconductor.*** The Gp. V impurity is called a '***donor***'
- ***In a n-type semiconductor*** conduction is predominantly due to electrons in the conduction band which have been excited from the donor levels

p-type Silicon

To dope a Group IV semiconductor p-type, we add a Group III element to the material, such as Boron

- Boron $1s^2 2s^2 2p$ has only 3 valence electrons, but the surrounding silicon atoms provide 4 bonds
- 3 of these form covalent bonds with the 3 electrons of the Boron
- The 4th has only 1 electron, while we know it can take 2 electrons with opposite spin.
- The associated energy level E_A is located ~0.030 eV above E_V
- At 0°K this level is empty
- At higher temperatures (300°K) an electron can jump into the empty space from other bonds; another electron can jump into the newly vacated place and so on

• This is can be viewed more easily in terms of ***hole*** rather than ***electron***:
  - a ***hole*** is bound to the Boron atom at 0°K , but can move about ***at higher temperature***
  - Now there is an excess *–ve* charge *–e* on the Boron site due to the captured electron
  - The captured electron stays on the Boron site while the hole moves about
  - Such Gp.III impurity is called an '***acceptor***



•A donor is:
neutral when *occupied* by an ***electron*** (T=0°K)
positively charged when it ***looses*** the electron (T~300°K)

•An acceptor is:
neutral when *occupied* by a ***hole*** (T=0°K)
negatively charged when it looses the ***hole*** (T~300°K)

(ii) GaAs
Group VI S donor
Group II Zn acceptor
Group IV Si can be donor or acceptor

(b) [40%]

$$n = \int_{E_C}^{\infty} g_c(E) f(E) dE$$

$$g_C(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} (E - E_C)^{1/2} = A(E - E_C)^{1/2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \cong \exp\left(\frac{E_F - E}{kT}\right)$$

$(for\ E - E_F >> kT)$

$$n = A \exp\left(\frac{E_F - E_C}{kT}\right) \int_{E_C}^{\infty} \exp\left(\frac{E_C - E}{kT}\right) (E - E_C)^{1/2} dE$$

$$= A \exp\left(\frac{E_F - E_C}{kT}\right) (kT)^{3/2} \int_0^{\infty} x^{+1/2} e^{-x} dx = A \frac{\sqrt{\pi}}{2} (kT)^{3/2} \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$n = N_C . \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$N_c = \frac{2}{h^3} (2\pi m^* kT)^{3/2}$$

c) [20%]

$$+\text{ve} \;\; \text{charge} = -\text{ve} \;\; \text{charge}$$

$$p + N_D^+ = n + N_A^- \qquad (1)$$

For $E_C \gg E_F$ and $N_D \gg N_A$:

$$p \approx 0$$

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$n \approx N_D$$

$$E_C - E_F = kT \ln \frac{N_C}{N_D} = 0.198eV$$

3 (a) A schematic diagram of the MESFET is shown in Fig. 1. It consists of a thin layer of a doped semiconductor, in this case n-type GaAs, on a semi-insulating substrate. Ohmic source and drain contacts are made onto the n-type GaAs which becomes the channel. The gate is made from a metal that will form a Schottky Barrier with the GaAs, so that a depletion region forms which extends into the channel semiconductor. The operation of the device is very similar to that of the junction field effect transistor (JFET). Application of a negative bias to the gate puts the Schottky barrier into reverse bias and causes the depletion region to extend into the channel. The undepleted area of the channel is reduced, and hence the resistivity of the channel is increased thereby reducing the current flow between the source and drain for a given bias between these two contacts, as shown in Fig. 2. As with the JFET, increasing the drain source voltage will also widen the depletion region at the drain end of the channel. This will cause the channel to pinch off and the source-drain current to saturate (see Fig. 2).

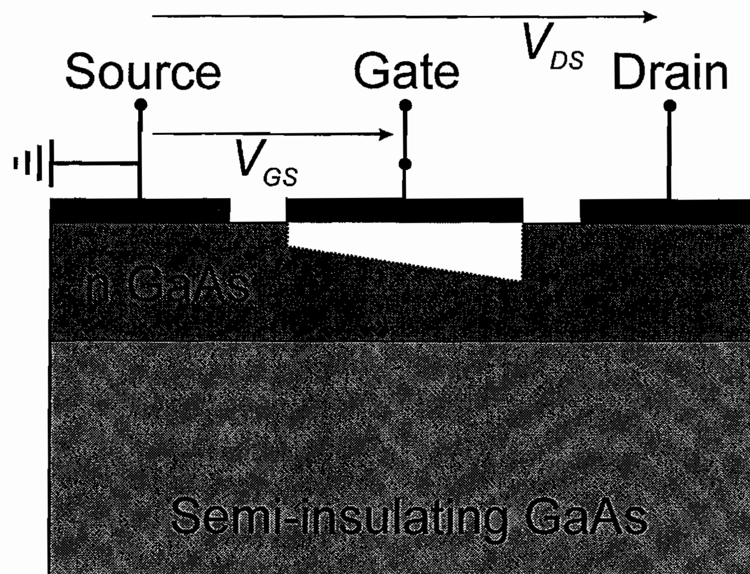


Fig. 1

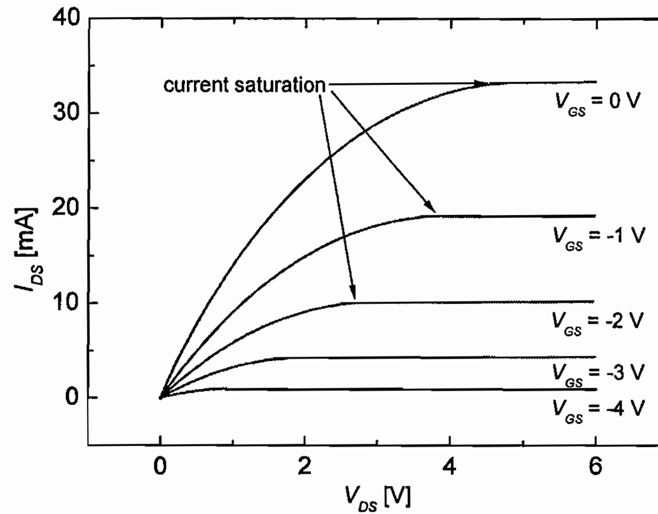


Fig. 2 [35%]

(b) The carrier density in a semiconductor is given from the data book by

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right).$$

Assuming that a all donors are ionized, then $n \approx N_D$, and so this becomes

$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right) = 0.862 \times 10^{-4}.298 \ln\left(\frac{2 \times 10^{22}}{4.7 \times 10^{23}}\right) = -0.08\, eV \ .$$

However, the work function ϕ of a semiconductor is related to the electron affinity χ by

$$\phi = \chi - (E_F - E_C) = 4.07 + 0.08 = 4.15\, eV \ .$$

[20%]

(c) Gold has a greater work function than the n-type GaAs. The band diagram for this junction is shown in Fig. 3. An Ohmic contact is formed as there is no barrier to electron flow for either sense of applied bias. Therefore, gold should be used for the source and drain contacts, as current needs to be injected to and from the channel at these points.

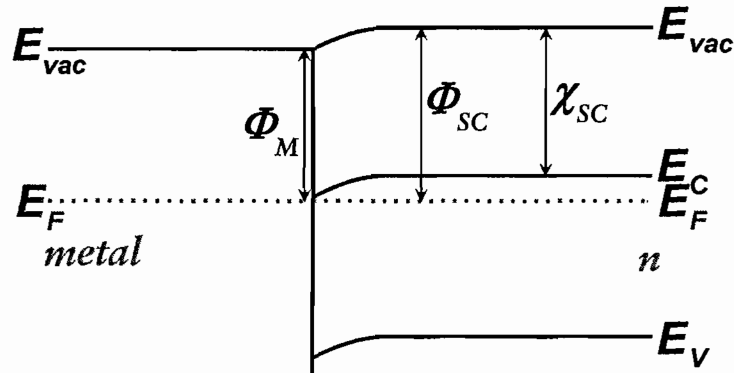


Fig. 3

Aluminium, on the other hand, forms a Schottky barrier with the n-type GaAs as it has a smaller work function. A barrier exists to electron flow from the metal into the semiconductor, as shown in the band diagram of Fig. 4. Therefore, aluminium should be used for the gate contact, as this will form a depletion region into the channel, as required.

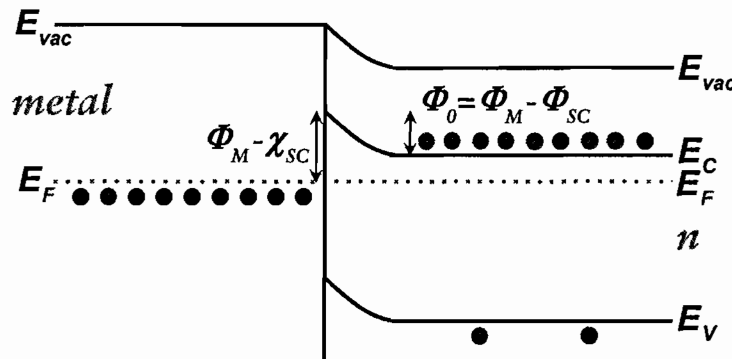


Fig. 4

[30%]

(d) GaAs has a much greater electron mobility than silicon. The electron mobility determines the velocity of carriers under the application of an electric field. The depletion region in the channel forms because of the penetration of an electric field from the gate into the semiconductor. The rate at which the depletion region can form is therefore governed by the electron mobility. Therefore, GaAs MESFETs can be operated at significantly higher frequencies than those based on Si. However, GaAs is far more expensive than silicon, and therefore these devices are more expensive. [15%]

4 (a) From the data book, for a p-type semiconductor, where E_V is defined as 0 eV,

$$p = N_V \exp\left(\frac{-E_{Fp}}{kT}\right), \quad (1)$$

where E_{Fp} is the Fermi energy in the p-type semiconductor. Similarly, for an n-type semiconductor,

$$n = N_C \exp\left(\frac{E_{Fn} - E_C}{kT}\right). \quad (2)$$

Assuming that all donors and acceptors are ionized, then $n \approx N_C$ and $p \approx N_V$. Therefore, rearranging these equations to make the Fermi energies the subject gives

$$E_{Fp} = kT \ln\left(\frac{N_V}{N_A}\right); \; E_{Fn} = E_C - kT \ln\left(\frac{N_V}{N_A}\right).$$

Hence, the contact potential is

$$eV_0 = E_{Fn} - E_{Fp} = E_C - kT\left[\ln\left(\frac{N_C}{N_D}\right) + \ln\left(\frac{N_V}{N_A}\right)\right]. \quad (3)$$

Furthermore, from the Law of Mass Action, formed by multiplying equations (1) and (2),

$$n_i^2 = N_C N_V \exp\left(\frac{-E_C}{kT}\right)$$

$$\therefore \ln(N_C N_V) = \ln(n_i^2) + \frac{E_C}{kT} \quad (4)$$

Substituting (4) into (3) gives the required result that

$$V_0 = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

[30%]

(b) From the information provided, it is clear that this is a p^+n junction, as $N_A >> N_D$. Therefore, the depletion may be considered to be in the n-type semiconductor alone. The Poisson equation (the Gauss Law expressed in terms of voltage) states that

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0 \varepsilon_r},$$

where ρ is the charge density due to ionised donors, which is eN_D. Therefore, given that V only varies in the x direction (across the junction), this becomes

$$\varepsilon_0\varepsilon_r \frac{d^2V}{dx^2} = -eN_D \ .$$

Integrating with the boundary condition that $E = 0$ at $x = w$ (the width of the depletion region) by assuming that there are no electric fields outside the depletion region, gives

$$\frac{dV}{dx} = \frac{eN_D}{\varepsilon_0\varepsilon_r}(w - x).$$

Integrating again with the boundary condition that $V = 0$ at $x = 0$ gives

$$V = \frac{eN_D}{\varepsilon_0\varepsilon_r}\left(wx - \frac{x^2}{2}\right),$$

which may be evaluated at $x = w$ to find the give the contact potential,

$$V_0 = \frac{eN_D w^2}{2\varepsilon_0\varepsilon_r}.$$

Rearranging this gives the required result for the depletion width,

$$w = \left(\frac{2\varepsilon_0\varepsilon_r V_0}{eN_D}\right)^{1/2}.$$

[40%]

(c) In order to find the capacitance of the junction per unit area, the charge stored on one side of the junction per unit area Q must first be determined. This is the charge per unit area in the depletion region, which is

$$Q = eN_D w = (2eN_D\varepsilon_0\varepsilon_r V_0)^{1/2} \ .$$

The capacitance per unit area is then

$$C = \frac{dQ}{dV} = eN_D w = \left(\frac{eN_D\varepsilon_0\varepsilon_r}{2V_0}\right)^{1/2} . \qquad (5)$$

To calculate this, we must first evaluate the contact potential using the equation from part (a),

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.862\times10^{-4}.298\ln\left(\frac{5\times10^{22}.5\times10^{20}}{\left(1.5\times10^{16}\right)^2}\right) = 0.653\,V \ .$$

The capacitance is then given by equation (5),

$$C = \left(\frac{eN_D\varepsilon_0\varepsilon_r}{2V_0}\right)^{1/2} = \left(\frac{1.602\times10^{-19}.5\times10^{20}.8.854\times10^{-12}.11.8}{2.0.653}\right)^{1/2} = 80\ \mu F\ m^{-2}$$ [30%]