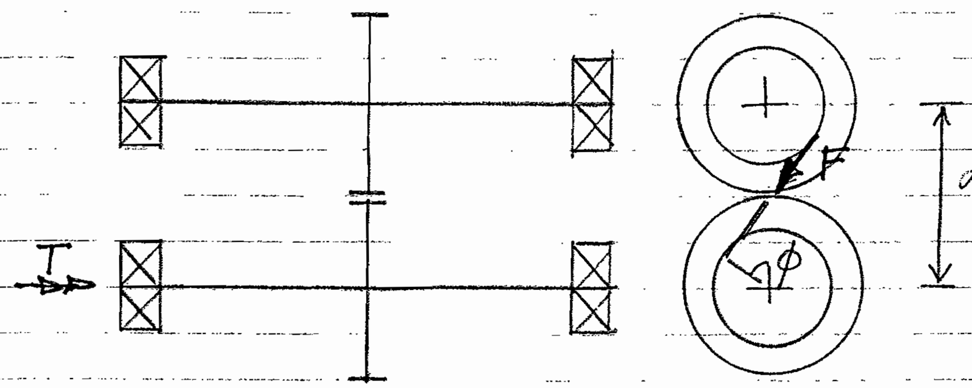
Prof J A Williams

Part IIA - 2008 - 3C8 - Solutions

1 (a)



$$T = F \cdot \frac{d}{2} \cos\phi$$

$$\therefore \; F = \frac{100}{0.05 \cos 20} = \underline{2128\,N}$$

From symmetry of FBD, radial bearing forces are $\frac{F}{2} = \underline{1064\,N}$

$$L = a_1 a_{23} \left(\frac{C}{P}\right)^n \qquad n = 3 \text{ for ball brg}$$

$$L = \frac{7000 \times 60 \times 100}{10^6} = 42 \quad \text{(millions of cycles)}$$

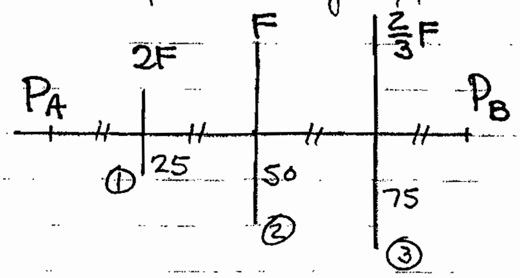
$a_1 = 0.21$ (99% reliability) $\quad a_{23} = 1$

$$\therefore \; 42 = 0.21 \left(\frac{C}{1064}\right)^3$$

hence $\underline{C = 6.22\,kN}$

From SKF data sheet $d = 35\,mm$, choose $D = 55\,mm$
$C = 9.56\,kN$ brg $\underline{61907}$

(b) Consider equm of i/put shaft : Since torque const $\underline{F \propto \frac{1}{r}}$



contact on gear 1 ... $P_A.1 = P_B.3$; $P_A + P_B = 2F$

$\therefore$ $P_A = 1.5F$; $P_B = 0.5F$

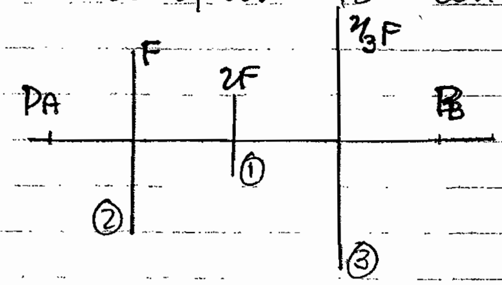
" " " 3 $P_A.3 = P_B.1$ $P_A + P_B = 2/3 F$

$\therefore$ $P_A = 1/6 F$ $P_B = 1/2 F$

" " " 2 $P_A = P_B = F/2$

hence 1 gives highest load on A.

(c) move 0.33 gears to central position



contact on gear 1 $P_A = P_B$ $P_A + P_B = 2F$

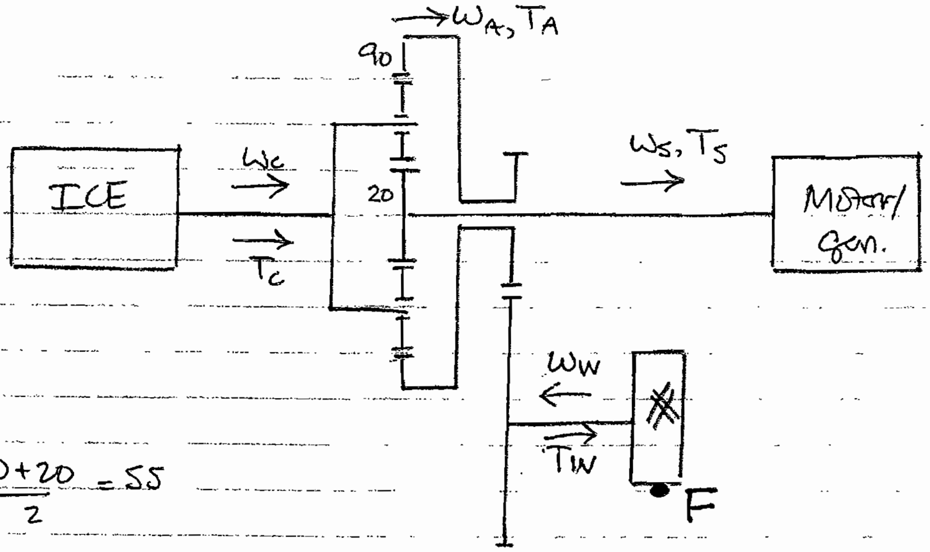
$P_A = P_B = F$

contact on gear 2 $P_A.1 = P_B.3$ $P_A + P_B = F$

$P_A = \frac{3}{4}F$; $P_B = \frac{1}{4}F$

So 33% reduction in max load on A (from 1.5F to F)

Popular and generally well done. A common error in (a) was to forget the effect of the pressure angle on calculating the bearing forces although this did not change the recommended SKF component. Not many candidates actually evaluated the change in bearing loads brought about by rearranging the gears in part (c).



$\frac{90+20}{2} = 55$

Kinematics $\quad 55\omega_c = \frac{1}{2}(20\omega_s + 90\omega_A)$

Eqm $\quad T_A + T_c + T_s = 0$

Virtual Power $\quad T_A\omega_A^* + T_c\omega_c^* + T_s\omega_s^* = 0$

At wheels $\quad T_W = F.r$ and $T_W = -5T_A$

(a) choose $\omega_s^* = 0$ then $55\omega_c^* = 45\omega_A^*$

$11\omega_c^* = 9\omega_A^*$

$$\therefore \quad T_A \cdot \frac{11}{9}\omega_c^* + T_c\omega_c^* = 0$$

$$T_c = -\frac{11}{9}T_A$$

But $T_A + T_c + T_s = 0 \quad \therefore \quad T_s = -T_A + \frac{11}{9}T_A = \frac{2}{9}T_A$

$$\therefore \quad T_A : T_c : T_s = 1 : -\frac{11}{9} : \frac{2}{9}$$

(b) (i) If $V = 25\,ms^{-1}$, $F = 150 + 0.2 \times 25^2 = 275\,N$

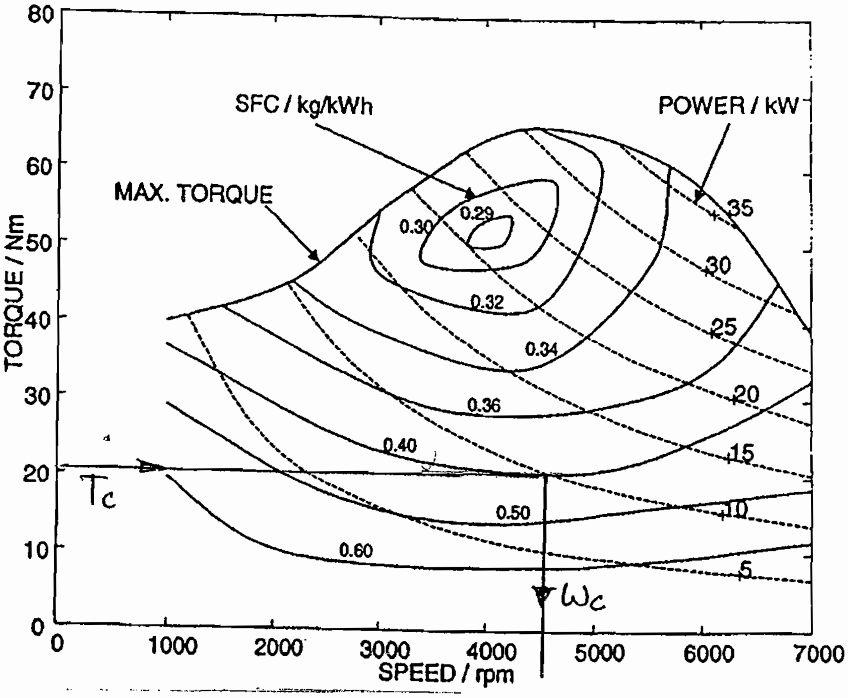
$$\therefore \quad T_W = 275 \times .3 \text{ Nm}$$

and $T_A = -\frac{1}{5} \times \frac{3}{10} \times 275 = -16.5 \text{ Nm}$

$$T_c = -\frac{11}{9} \times -16.5 = 20.17 \text{ Nm}$$

$$T_s = \frac{2}{9} \times -16.5 = -3.67 \text{ Nm}$$

(ii)



If $|T_c| = 20.17$ Nm for min SFC $\omega_c \simeq 4500$ rpm

0.40 kg/kWh NOT 0.29!

$$\Rightarrow \frac{4500 \times 2\pi}{60} \simeq 471\ s^{-1}$$

Since power is being delivered to wheels then ω_W must be in same sense as T_W, i.e. ω_W is -ve.

$$\therefore \omega_W = \frac{-25}{0.3}\ s^{-1} \quad ; \quad \omega_A = \frac{5 \times 25}{0.3} = +417\ s^{-1}$$

then $55\,\omega_c = \frac{1}{2}(20\,\omega_S + 90\,\omega_A)$

provides $55 \times 471 = \frac{1}{2}(20\,\omega_S + 90 \times 417)$

$$\omega_S = \frac{2 \times 55 \times 471 - 90 \times 417}{20} = 714\ s^{-1}$$

$\omega_A = 417\ s^{-1}$; $\omega_c = 471\ s^{-1}$; $\omega_S = 714\ s^{-1}$

check power eqn. $T_A\omega_A + T_c\omega_c + T_S\omega_S$

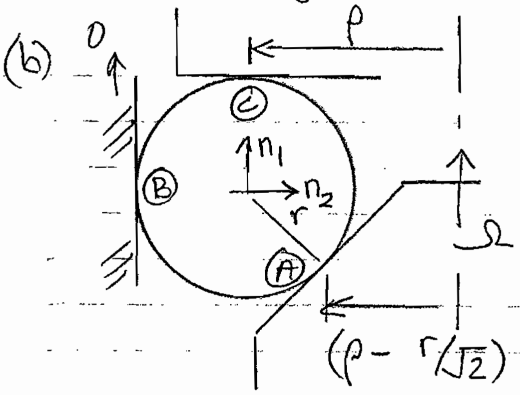
$$\Rightarrow (-16.5 \times 417) + (20.17 \times 471) + (-3.67 \times 714)$$

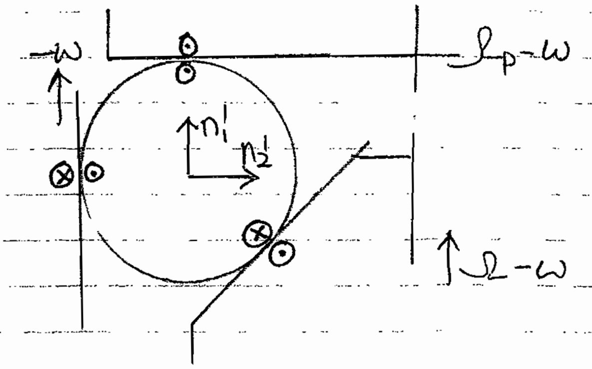
$= -0.81$ sufficiently near 0!

Popular and generally well done. The most common error in part b(ii) was to assume that the ICE can operate at its absolute minimum SFC which is not compatible with the required engine torque.

3/ (a) Bookwork from notes.

(b)





actual

bring centre of ball to rest

Suppose centres of balls rotating at ω.

Bring centres of balls to rest, then

(i) pure rolling at A: $\left(\rho - \frac{r}{\sqrt{2}}\right)(\Omega - \omega) = -\frac{n_1'}{\sqrt{2}}.r - \frac{n_2'}{\sqrt{2}}r$

where n_1' and n_2' are components of angular velocity of ball as shown.

Note that $n_1' = n_1 - \omega$; $n_2' = n_2$

$$\therefore -(\sqrt{2}\rho - r)(\Omega - \omega) = n_1' r + n_2' r$$

or $-(\sqrt{2}\,\rho/r - 1)(\Omega - \omega) = n_1' + n_2'$

Pure rolling at B $\quad r n_1' = -(\rho + r)\omega$

$$n_1' = -(\rho/r + 1)\omega$$

So $n_2' = (\rho/r + 1)\omega - (\sqrt{2}\rho/r - 1)(\Omega - \omega)$

$$= \rho/r\,\omega + \cancel{\omega} - \sqrt{2}\rho/r\,\Omega + \sqrt{2}\rho/r\,\omega + \Omega - \cancel{\omega}$$

$$n_2' = (\sqrt{2} + 1)\rho/r\,\omega - (\sqrt{2}\rho/r - 1)\Omega$$

So spin velocity at A is $\frac{n_1'}{\sqrt{2}} - \frac{n_2'}{\sqrt{2}} - \frac{(\Omega - \omega)}{\sqrt{2}}$

If this is to be $\Rightarrow 0$ then

$$n_1' - n_2' - \Omega + \omega = 0$$

$$-(\rho/r + 1)\omega - (\sqrt{2}+1)\rho/r\,\omega + (\sqrt{2}\rho/r - 1)\Omega - \Omega + \omega = 0$$

$$\therefore \left[-\rho/r \cancel{-1} - \sqrt{2}\rho/r - \rho/r \cancel{+1}\right]\omega = \left[1 + 1 - \sqrt{2}\rho/r\right]\Omega$$

$$-(2+\sqrt{2})\rho/r\,\omega = (2 - \sqrt{2}\rho/r)\Omega$$

$$\omega = \frac{(\sqrt{2}\rho/r - 2)\Omega}{(2+\sqrt{2})\rho/r} \quad \text{QED}$$

or $\dfrac{(\rho/r - \sqrt{2})\Omega}{(1+\sqrt{2})\rho/r}$

(ii) If no slip at C

$$\rho(\Omega_p - \omega) = r n_2'$$

i.e. $\Omega_p = \frac{r}{\rho} n_2' + \omega$

$$= (\sqrt{2}+1)\omega - (\sqrt{2} - r/\rho)\Omega + \omega \checkmark$$

$$\Omega_p = (\sqrt{2}+2)\omega - (\sqrt{2} - r/\rho)\Omega \checkmark$$

$$= \frac{r}{\rho}(\sqrt{2}\rho/r - 2)\Omega - (\sqrt{2} - r/\rho)\Omega$$

$$\frac{\Omega_p}{\Omega} = \cancel{\sqrt{2}} - 2r/\rho - \cancel{\sqrt{2}} + r/\rho$$

$$\Omega_p = -\frac{r}{\rho}\Omega$$

~~(iii)~~ Rot'n vely at B is $n_2 \Rightarrow n_2'$

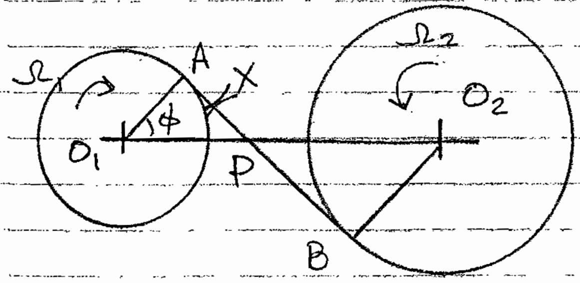
$$= (\sqrt{2}+1)\rho/r\,\omega - (\sqrt{2}\rho/r - 1)\Omega$$

$$= \frac{\cancel{(\sqrt{2}+1)}(\rho/r - \sqrt{2})\Omega}{\cancel{(1+\sqrt{2})}} - (\sqrt{2}\rho/r - 1)\Omega$$

$$= \rho/r\,\Omega - \sqrt{2}\Omega - \sqrt{2}\rho/r\,\Omega + \Omega$$

$$= -(\sqrt{2}-1)(1+\rho/r)\Omega$$

4.(a) Involute tooth profile defined by locus of end of a taut string unwound from a circular base circle.



Since both teeth in contact are involutes contact is along the 'common normal' line which forms the tangent AB to the two base circles.

This common normal intersects the line joining the centres O_1O_2 at a fixed point P, the pitch point

Equating velocities // AB $\quad O_1A.\Omega_1 = O_2B\,\Omega_2$

ie. $\dfrac{\Omega_1}{\Omega_2} = \dfrac{O_2B}{O_1A}$

which is fixed.

b(i) from Data Sheet $\quad l = [r^2\sin^2\phi + a(2r+a)]^{1/2} - r\sin\phi$

with $r = NM/2$ and $p_b = \dfrac{2\pi r\cos\phi}{N} = \pi m\cos\phi$

Since contact ratio = 2 $\quad l_1 + l_2 = 2p_b$

ie. $\left[\left(\dfrac{N_1}{2}\right)^2\sin^2\phi + k(N_1+k)\right]^{1/2} - \dfrac{N_1}{2}\sin\phi$

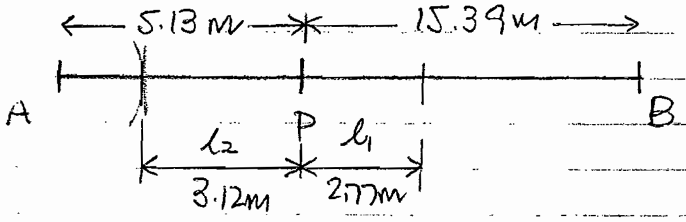
$+ \left[\left(\dfrac{N_2}{2}\right)^2\sin^2\phi + k(N_2+k)\right]^{1/2} - \dfrac{N_2}{2}\sin\phi = 2\pi\cos\phi$

(cancelling throughout by m)

put $k = 1.16$, $N_1 = 30$, $N_2 = 90$, $\phi = 20°$

LHS = $2.77 + 3.12 = 5.89$ $\quad$ RHS = 5.90

(ii)



$AP = r_1 \sin\phi$
$= \frac{N_1 m \sin\phi}{2} = 5.13\,m$

$BP = r_2 \sin\phi$
$= \frac{N_2 m \sin\phi}{2} = 15.39\,m$

Always two pairs of teeth in contact – assume load shared equally then

$$P' = T_1/2w r_{b1} = T_1/wmN_1\cos\phi = T_1/28.2wm$$

Worst case is when contact is nearest to one of base circles. With equal addenda this will be base circle of pinion
then radii of curvature are $R_1 = 5.13m - 3.12m$
$R_2 = 15.39m + 3.12m$

$$\therefore \quad R' = \frac{2.01 \times 18.5}{2.01 \quad 18.5}\,m = \underline{1.81\ m}$$

Now use Hertz line contact

$$p_0 = \sigma_c = \left\{\frac{P'E^*}{\pi R'}\right\}^{1/2} = T_1$$

$$\text{i.e.} \quad \sigma_c^2 = \frac{T_1}{28.2wm} \cdot \frac{E^*}{\pi \times 1.81\,m}$$

$$\text{i.e.} \quad T_1 = \underline{\frac{160.3\ \sigma_c^2\, w m^2}{E^*}}$$

(iii) Equivalent formula for bending $\sigma_b = \frac{P_T'}{Jm}$

$$\text{i.e.} \quad \sigma_b = \frac{P'\cos\phi}{Jm}$$

from Data Sheet $J \sim 0.40$

$$\therefore \quad \sigma_b = \frac{T_1}{wmN_1\cos\phi} \cdot \frac{\cos\phi}{0.40m}$$

ie $\underline{T_1 = \quad \sigma_b\, w m^2 .30}$

$\therefore$ for root bending before surface fatigue

$$\sigma_b\, w m^2 .30 < \frac{160\, \sigma_c^2\, w m^2}{E^*}$$

$$\text{ie.}\quad \underline{\frac{3}{16} E^* \sigma_b < \sigma_c^2}$$

Take midrange data $\begin{cases} \sigma_b \simeq 430\,\text{MPa} \\ \sigma_c \simeq 1400\,\text{MPa} \end{cases}$
$E^* = 115\,\text{GPa}$

$$\therefore\ LHS = \frac{3}{16} \times 115\times10^9 \times 430\times10^6 = 9.27\times10^{18}\ (Pa^2)$$

$$RHS = 14 \times 10^6 = 1.96\times10^{18}$$

<u>So expect surface fatigue.</u>

<u>Comments on Q3 & Q4</u>

Bringing the rotation rate of the centres of the balls to zero makes working out spin rates much easier. Most solutions ran out of steam before part b (ii).

Simply saying that an involute is the locus traced out by a taught string unwound from a circular cylinder is not enough for (a). Part b(i) was well done – some errors in b(ii) working out the effective radius of curvature when applying Hertz.

JAW
May 08