Engineering Tripos

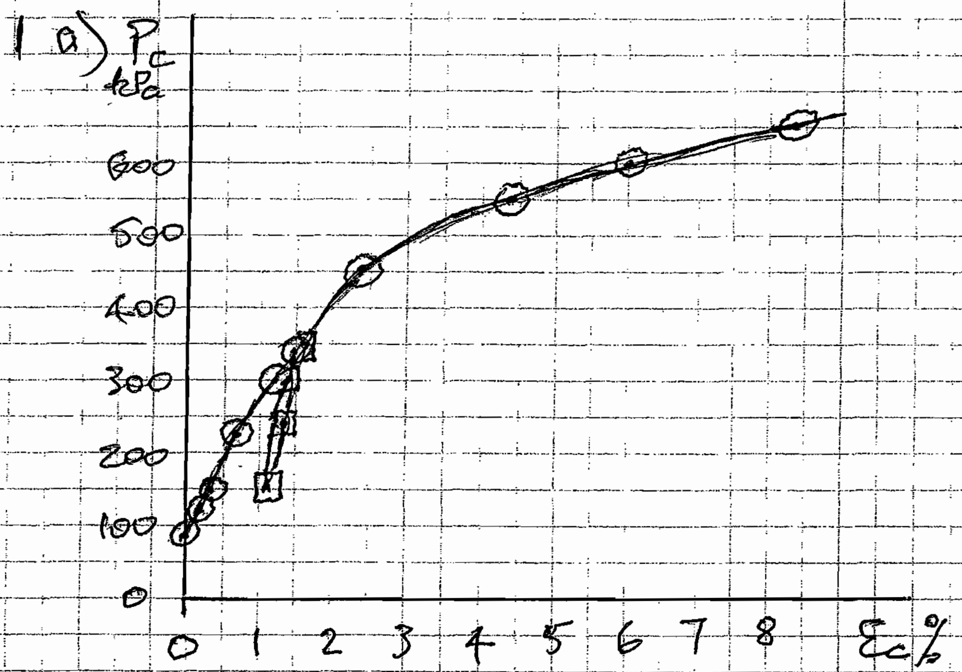
Part IIA Module 3D2

Geotechnical Engineering II

2008 Solutions

M. D. Bolton

1 a)



$\sigma_{h,0} \simeq 90$ kPa

$\sigma_{v,0} \simeq 4 \times 19 = 76$ kPa

$u_0 \simeq 1.5 \times 10 = 15$ kPa

$\therefore K_0 \simeq (90-15)/(76-15) = 75/61 = 1.23$

This shows overconsolidation, perhaps to OCR ~ 4.

b) Take G_i from gradient between 150 & 225 kPa

$$\frac{dP_c}{d\varepsilon_c} = \frac{75}{0.4\%} = 18750 \text{ kPa}$$

But $\gamma_c = 2\varepsilon_c$ and $d\sigma_c = G\, 2\pi r_c\, \rho_c / \pi r_c^2 = G\gamma_c$

$\therefore G_i \simeq 9375$ kPa

G_{ur} for 0.4% cavity strain reduction

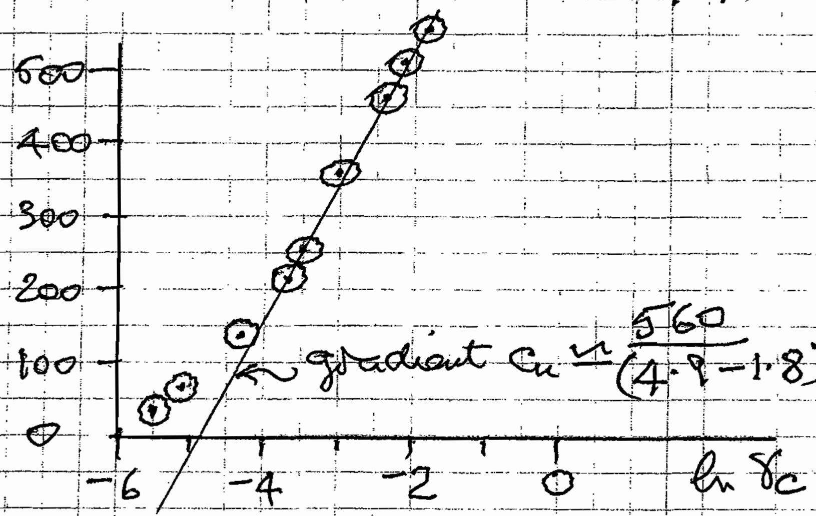
$$G_{ur} \simeq \frac{(345-150)}{0.4\%} \times 1/2 \simeq 24400 \text{ kPa}$$

for a shear strain excursion $\Delta\gamma_c = 0.8\%$

The latter is free from disturbance errors, so more useful.

1 c) Plot $(\sigma_c - \sigma_o)$ versus $\ln \gamma_c$

$\sigma_c - \sigma_o$ kPa	γ_c	$\ln \gamma_c$
30	4×10^{-3}	-5.52
60	6×10^{-3}	-5.12
135	14×10^{-3}	-4.27
210	24×10^{-3}	-3.73
255	30×10^{-3}	-3.51
360	48×10^{-3}	-3.04
460	88×10^{-3}	-2.43
510	0.120	-2.12
560	0.168	-1.78



gradient $c_u \simeq \dfrac{560}{(4.9-1.8)} \simeq 181$ kPa

d) At $\sigma_c = 650$ kPa, $\sigma_\theta = 650 - 2\times181$

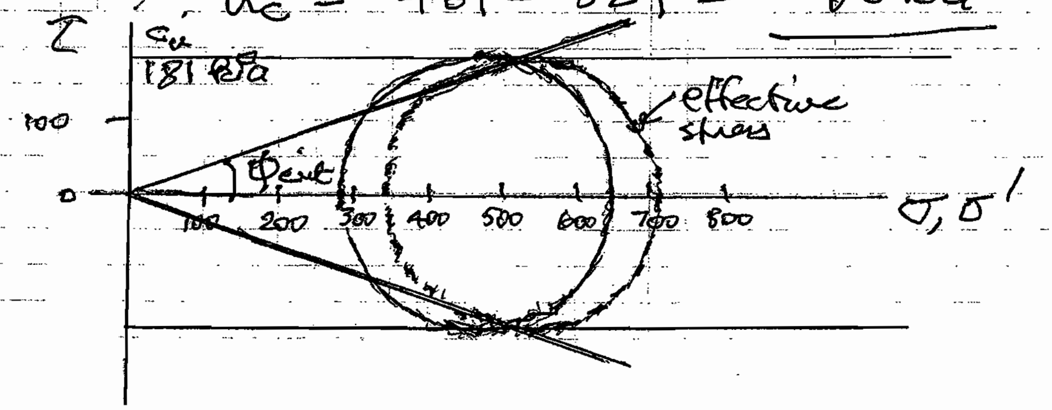
So $\sigma_\theta = 288$ kPa.

Assume $\delta\sigma_v \simeq 0.5(\delta\sigma_r + \delta\sigma_\theta)$

Then $p \simeq 0.5(\sigma_r + \sigma_\theta) \simeq 469$ kPa

But $p' \simeq c_u / \sin\phi_{crit} \simeq 181/0.342 \simeq 529$ kPa

$\therefore u_c \simeq 469 - 529 \simeq \underline{-60 \text{ kPa}}$



Q1 Examiners comments:

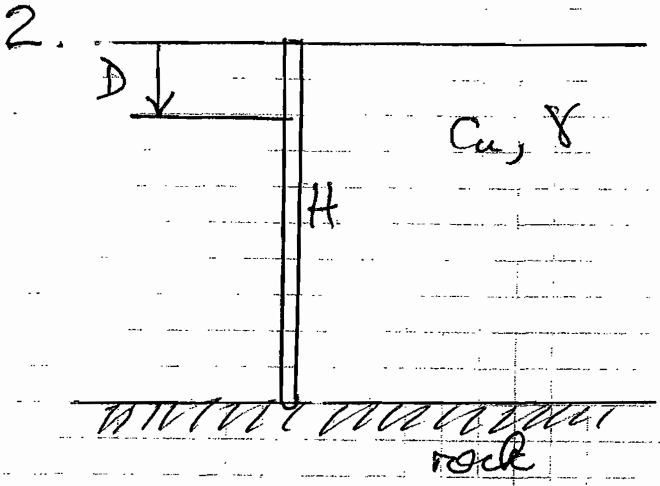
a) Some candidates failed to recall that K_o is an effective stress ratio.

b) Too many candidates wrongly took cavity strain ε_c as the shear strain γ_c, and so created factor 2 errors in stiffness.

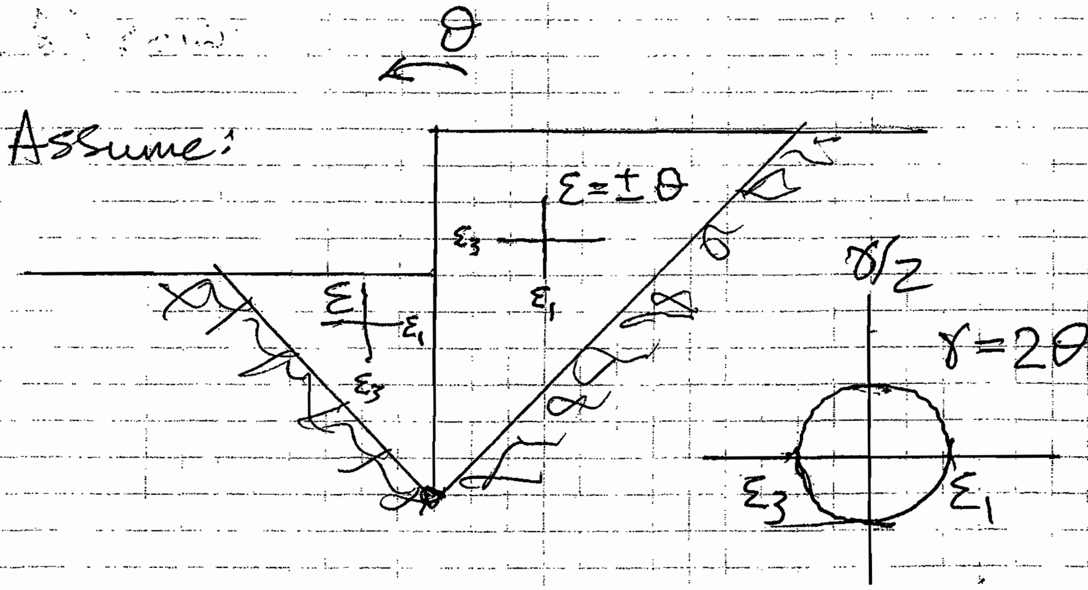
c) Many candidates tried to find yield stress from a natural stress - strain plot, and wrongly associated this with the ultimate undrained shear strength c_u. Real soil is non-linear, so "yield point" can not reliably be discerned.

d) Only 2 candidates drew Mohr circles of total and effective stress. Most messed about with total stresses alone, and overlooked the provision of a ϕ value, so they got 0 marks for this section.

2.

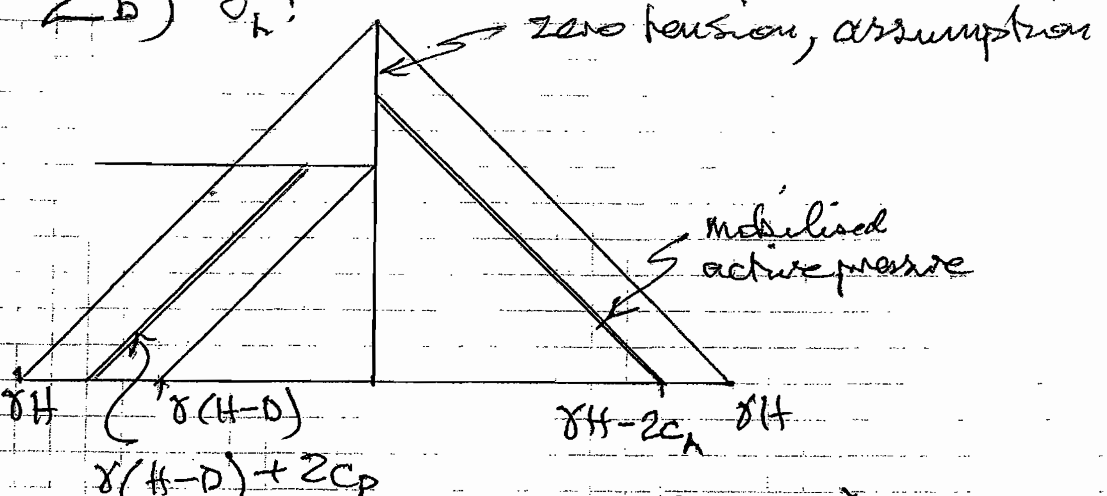


a)



Principal stresses are taken to be vertical and horizontal within the triangles of assumed deformation.

2b) σ_h:



mobilised passive pressure for $z \geq D$

The slopes of all pressure lines is γ.

The strains are of the same magnitude either side of the wall. So if the soil is isotropic, the mobilised shear strength will be the same: $c_A = c_p$.

It is evident that the wall will be held approximately in equilibrium when the lateral pressures balance either side of the embedded portion.

$$\therefore\ \gamma(H-D) + 2c_{mob} \simeq \gamma H - 2c_{mob}$$

$$\therefore\ \underline{\frac{c_{mob}}{\gamma D} \simeq 0.25}$$

For better accuracy, take moments about the base socket:

$$\frac{1}{6}\gamma(H-D)^3 + 2c_{mob}\frac{(H-D)^2}{2} = \frac{1}{6}\gamma\left[H - \frac{2c_{mob}}{\gamma}\right]^3$$

Let $c_{mob}/\gamma H$ = normalised strength x

D/H = normalised depth y

$$\frac{1}{6}(1-y)^3 + x(1-y)^2 = \frac{1}{6}(1-2x)^3$$

2 b) cont.

As $y \to 0$, ignore highest order terms

$$\frac{1}{6}(1 - 3y + 3y^2) + x(1 - 2y + y^2) \simeq \frac{1}{6}(1 - 6x + 12x^2)$$

So $$-\frac{y}{2} + \frac{y^2}{2} + x - 2xy + xy^2 = -x + 2x^2$$

Ignoring 2nd order terms: $x = y/4$

So $C_{mob}/\gamma D \to 1/4$ as $D/H \to 0$

c) Accept the estimate $C_{mob} \equiv \gamma D/4$

$\therefore C_{mob} = 18 \times 2/4 = 9$ kPa

Assuming Tresca's condition holds in a triaxial test:

$q_{mob} \equiv 2C_{mob} = 18$ kPa

and $\varepsilon_a \equiv 0.1\%$

In a triaxial test, undrained, $\varepsilon_r = -0.5\,\varepsilon_a$

So shear strain $\gamma = \varepsilon_1 - \varepsilon_3 \simeq 0.15\%$

In plane strain, Mohr's circle gives $\varepsilon_h = -\gamma/2$

$\therefore \varepsilon_h \simeq -0.075\%$

The width of the active zone is $\sim H = 10$ m

$\therefore \delta_h \simeq 7.5$ mm

Could: — Solve the quadratic, giving $C_{mob}/\gamma D = 0.265$

— could demand plane strain test data

— could perform both compression & extension and therefore have $C_A \neq C_B$

— could allow for wall friction in the equilibrium expression, reducing $C_{mob}/\gamma D$ by factor $4/5.14 \simeq 0.78$

Q2 Examiner's comments:

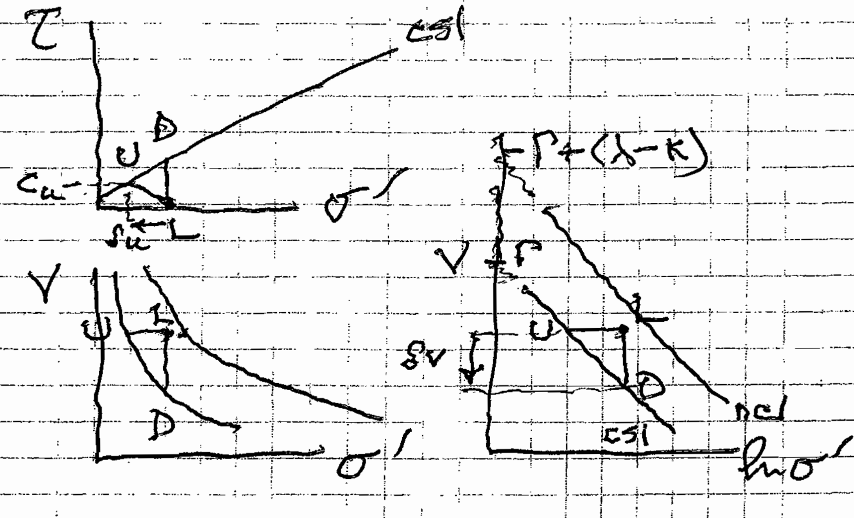
Quite unpopular, but this question represents the new material on serviceability and limiting displacements that was introduced into 3D2 in 2006-7. It rewarded the 18/43 attempts with the highest average marks of 63% recorded for these questions.

a) A few candidates failed to show sufficient information to define the deformations.

b) Almost all candidates set off in the right direction with moment equilibrium. Many failed to create the general expression for C_{mob}, but most produced the limit for $D/H \to 0$.

c) A lack of clarity relating C_{mob} to q and θ to ε_a led to many mistakes, and therefore to the loss of some marks. Some candidates also ignored the last part, failing to suggest improvements to the calculation method.

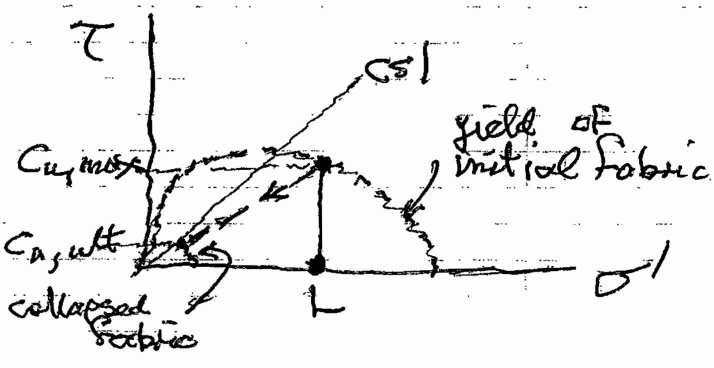
3. a)



If pore pressures, created by the collapse of the microstructure following crushing of grains/agglomerates, do not have time to dissipate, the "undrained" path LU results in low shear strength c_u. The ancillary "drained" problem is the significant loss of specific volume δv along path LD.

Typical field problem is embankment on soft mud. Typical control measure is to build in stages, monitoring pore pressures with piezometers to ensure that $c_{mob} < c_u$ at all stages as both c_{mob} and c_u increase.

For very sensitive soils, the initial strength $c_{u,max}$ following the earlier "Cam-Clay" model, suddenly reduces to $c_{u,ult} \approx c_{u,max} / S_t$ where S_t may be 10 or more. It is then essential to keep the soil from yielding. Strains should be monitored and kept within "elastic" limits, e.g through use of inclinometers.



3 b) i) $z = 15m$, $z_w = 13m$

Find σ' at 15m

At 15m, $\sigma'_c = 130$ kPa

Kaolin ncl: $v_c = \Gamma + \lambda - \kappa - \lambda \ln \sigma'_c$

$= 3.767 + 0.26 - 0.05 - 0.26 \ln 130$

$v_c = 2.711$

So buoyant unit weight $\gamma' = \frac{(G_s - 1)}{v} \gamma_w$

$\gamma' = \frac{1.61}{2.711} \times 9.8 = 5.8$ kN/m^3

and $\gamma = 15.6$ kN/m^3

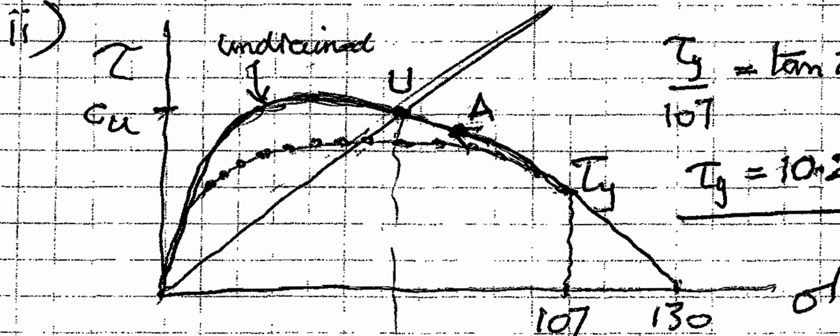
Ignoring any small reductions with depth, we get

$\sigma'_{o\,(15m)} = 13 \times 5.8 + 2 \times 15.6 = \underline{107 \text{ kPa}}$

Then $v_o \simeq 2.711 + 0.05 \ln(130/107)$

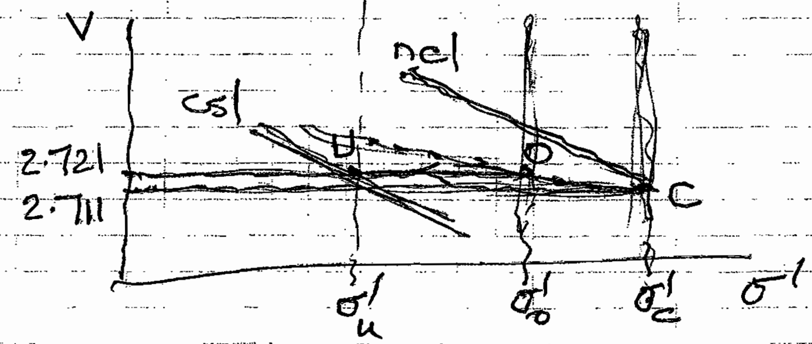
$\simeq \underline{2.721}$

ii)



$\frac{\tau_y}{107} = \tan 26° \ln \frac{130}{107}$

$\underline{\tau_y = 10.2 \text{ kPa}}$



On csl, for U: $v = \Gamma - \lambda \ln \sigma'_u = v_o$

$\therefore\ 3.767 - 0.26 \ln \sigma'_u = 2.721$

3 b ii) (cont)

$\therefore \quad \sigma'_u = 56$ kPa

and $\quad c_a = 56 \tan 26° = \underline{27 \text{ kPa}}$

iii) For FoS of 1.25, $\tau_A = 27/1.25 = 22$ kPa

This has advanced a considerable way along the undrained yield surface. It can not be considered an adequate margin for the following reasons:

- a sensitivity of only 1.25 would bring the soil to failure (ULS)

- even the quasi-elastic strains might prove to be unacceptable from the viewpoint of adjacent structures or services (SLS)

Q3 Examiner's comments:

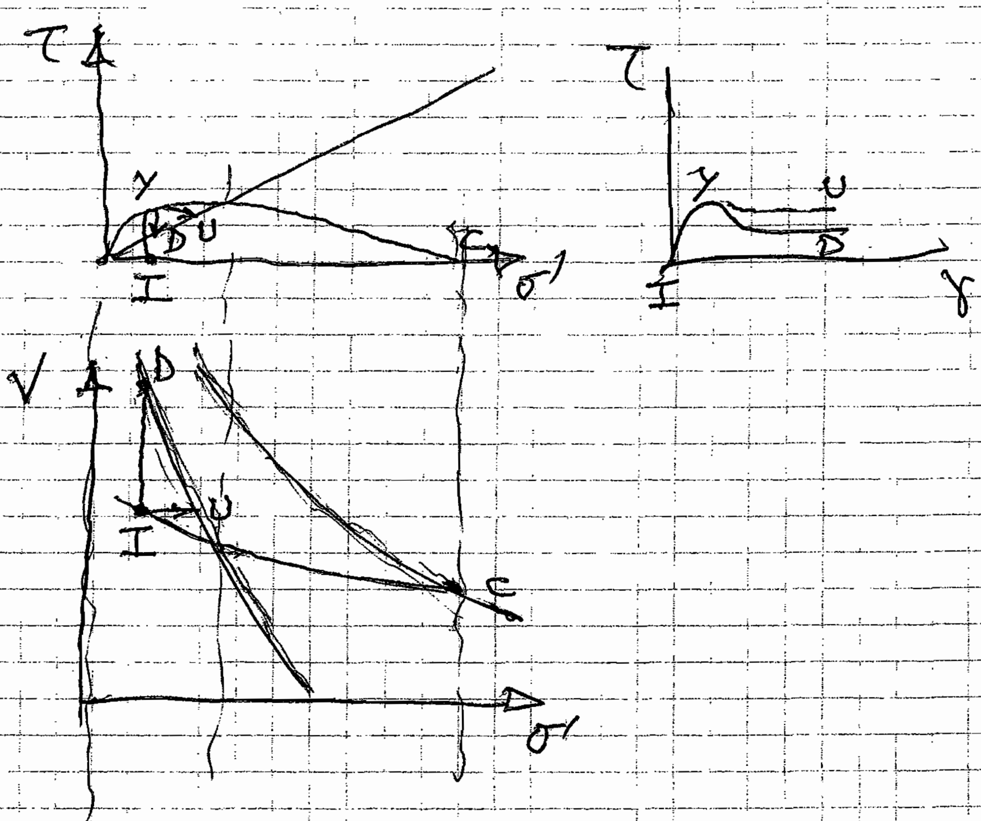
a) Many candidates failed to monitor the exact questions, and therefore failed to pick up all the available credit, having written a more general essay.

b) The main problem was the unnecessary assumption of soil unit weight γ, when it can be deduced from v and G_s.

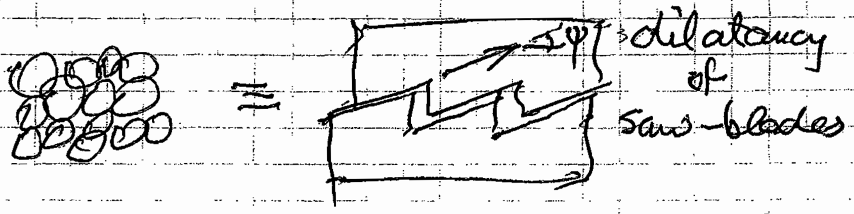
c) Quite well done, in principle.

d) Many candidates failed to get all the available marks, simply by missing important points in a hurried conclusion.

4 a)



The main challenge is brittleness, brought about by their tendency to dilate towards ultimate critical state. Undrained paths invoke $\delta u < 0$, with $\sigma'_u > \sigma'_i$, so that temporary undrained strength exceeds ultimate drained strength. Micromechanics can be visualized as:

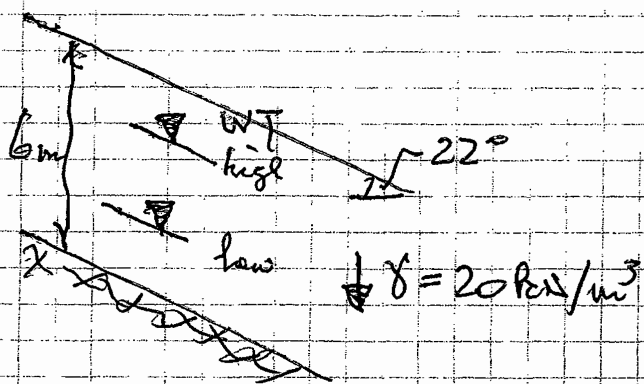


except that there is not a sudden drop from "dilatant" to "ultimate" in the case of soils. Nevertheless:

$$\phi_{max} \simeq \phi_{crit} + \psi_{max}.$$

$$\phi_{ult} \simeq \phi_{crit}$$

4 b)



		①	②	③
i)	σ_3' kPa	25	100	400
	q kPa	75	250	840
	σ_1' kPa	100	350	1240
	σ_1'/σ_3'	4.00	3.50	3.10
	ϕ_{max}	36.9°	33.7°	30.8°
	p' kPa	50	183	680
	$\ln p'$	3.91	5.21	6.52

ii)

From test ③, $\phi_{crit} = 30.8°$ consistent with the description clayey, silty sand.

From tests ① & ②, $\dfrac{\Delta\phi_{max}}{\Delta \ln p'} = \dfrac{3.2°}{1.3} = 2.5°$

which is consistent with dense sand behaviour of 3°.

Now estimate p'_{crit} from ② & ③, $\Delta\phi_{max} = 2.9°$

so $\Delta \ln p' = \dfrac{2.9°}{2.5} = 1.16$ and $p'_{crit} = 183 \times 3.19$

so $p'_{crit} \simeq 584$ kPa

Then $\phi_{max} = 30.8° + 2.5° \ln \dfrac{584}{p'}$

4 b iii) Dry season $z_w/Z = 2/6 = 1/3$

Wet season $z_w/Z = 5/6$

Since ϕ' reduces with effective stress, it also reduces with depth. So the critical slip surface is the deepest, 6 m.

In the dry season:

$$\sigma' = (6\times20 - 2\times10)\cos^2 22° = 86\,\text{kPa}$$

Assuming σ' on slip surface $\simeq p'$ we can estimate

$$\phi_{max} = 30.8° + 2.5°\ln\left(\frac{584}{86}\right)\times\frac{5}{3}$$

where the 5/3 factor mimics that found for plane strain versus triaxial strain in the data of clean sand.

$$\text{i.e. } \phi_{max} = 30.8° + 8° = 38.8°$$

But $$\tan\phi_{mob} = \frac{\tan 22°}{(1-\frac{1}{2}\cdot\frac{1}{3})} = 0.485$$

$$\underline{\phi_{mob} = 25.9°} \text{ in the dry season.}$$

In the wet season:

$$\sigma' = (6\times20 - 5\times10)\cos^2 22° = 60\,\text{kPa}$$

$$\phi_{max} = 40.3°$$

But $$\tan\phi_{mob} = \frac{\tan 22°}{(1-\frac{1}{2}\cdot\frac{5}{6})} = 0.693$$

$$\underline{\phi_{mob} = 34.7°} \text{ in the wet season}$$

This shows no 'first-time' failure if the wet season is duplicated.

iv) Nevertheless, the engineer should expect the slope to exhibit dilatant creep, since $\phi_{mob} > \phi_{crit}$ in the wet season. Also the "worst" wet season could be such that $z_w = 6$ m. Eventually, the slope will fail.

Q4 Examiner's comments:

a) Very well answered.

b) i) This was intended to be very easy, but hardly any candidates did the expected calculation of using σ_f' and q to obtain σ_a' so that ϕ_{max} could be calculated from σ_a'/σ_f'. Many "estimated" ϕ_{max} by using arbitrary ratios, such as $\tan^{-1}(q/p')$ or $\tan^{-1}(q/2p')$ which were so inaccurate that they led candidates astray in the subsequent parts.

ii) Those who had estimated ϕ_{max} tended to make poor choices on ϕ_{crit}.

iii) and iv) These sections were very well answered, showing that most candidates could assess slope stability if only someone would tell them what ϕ was!