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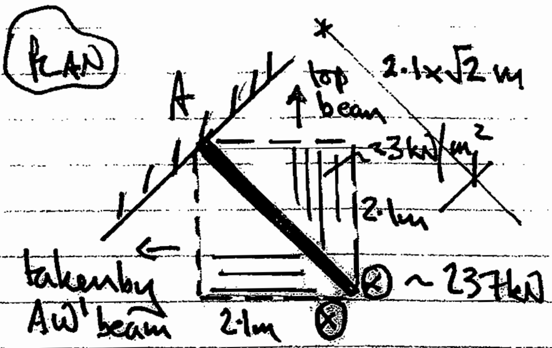
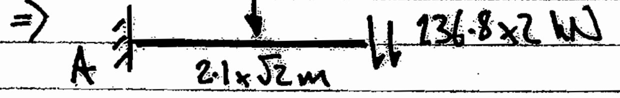
For bending moment at A, bending in the direction of AW, then we may regard A as being clamped (like cantilever – no rotation at root), loaded by 1/4 total slab + 2 × corner S.F.



Slab quarter wt = $\underbrace{23\,\text{kN/m}^2 \times 2.1^2}_{101.4\text{kN}}$, acts at $\frac{2.1\times\sqrt{2}}{2}$ from A on AW

⇒



$\Rightarrow M_A = 101.4 \times \frac{2.1\sqrt{2}}{2} + 2\times 236.8 \times (2.1\times\sqrt{2})$

$\Rightarrow \underline{M_A = 1557.1\,\text{kNm}}$

1b ii) Max B.M. in strips inside $\boxed{\;}$ 5.8m $= \left(\frac{23\,\text{kN/m}^2}{2}\right) \times \frac{5.8^2}{8}$ $\left[\frac{wL^2}{8}\right]$

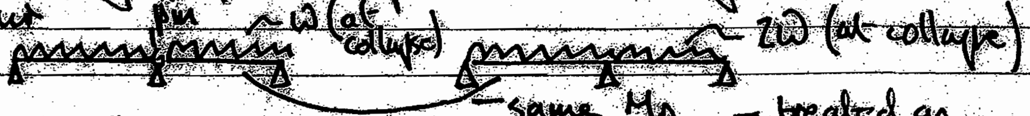
$= \underline{48.4\,\text{kNm}}$ (A) — due to double strips ↕ ↔

Max B.M. in strips inside ⬡ $= \frac{23\,\text{kN/m}^2 \times 4.2^2}{8} = \underline{50.7\,\text{kNm}}$ (B)

single load carrying dirn

If WW' = WX/√2 (ie. take exact value), then (A) & (B) would be equal ⇒ desirable, for it implies same moment capacity in both ⇒ SAME REINFORCEMENT DETAIL IN FLOOR REGIONS

If beam strips considered to continuous over main beams then maximum moment in floor can be reduced by a factor up to two: analogous to continuous beam behaviour



∴ beams strips in floor can be more efficient if treated as continuous over beams [but hogging reinforcement now needed]

1c) This is an exposition of LOWER BOUND THEOREM: any equil set is safe / acceptable provided no yield; material has to be ductile.

Common failings: forgetting about double strips inside □; making moment at A hard to calculate by not considering symmetry; forgetting about 2 shear forces at W; not performing overall statical check at A for column force; Realising max moment capacity is enhanced by treating ~~beams as~~ floor strips as continuous over beams.

Dr K Seffen

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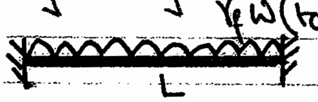
1a) Using the yield stress to the full $\Rightarrow$ the beam is on the point of collapse



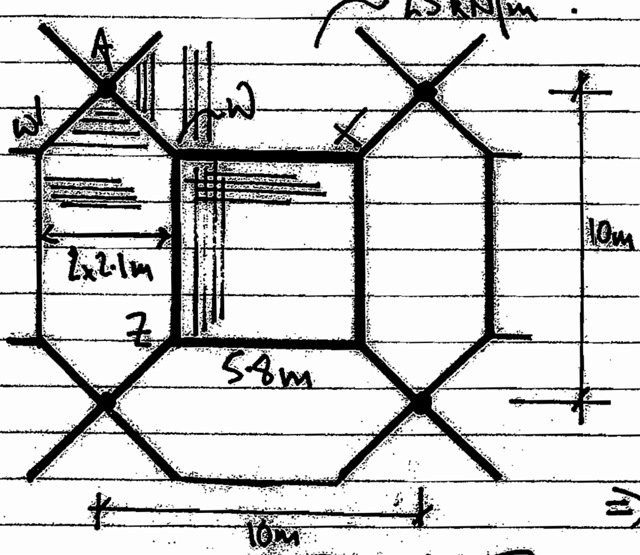
$\Rightarrow \gamma_f W = \frac{16 M_p}{L}$ (3 plastic hinges), $M_p = \sigma_y \cdot Z_p$ — ①

Under load cW, midspan $\delta \le \frac{L}{F}$, where $\delta = cWL^3/384EI$

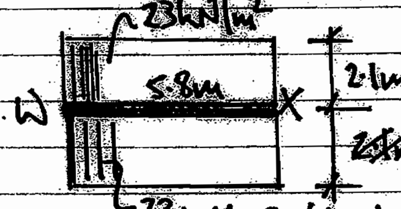
$\Rightarrow \frac{cWL^3}{384EI} \le \frac{L}{F}$ — ② : ② ÷ ① ÷ d $\Rightarrow$ $\boxed{\frac{L}{d} \le \underbrace{\frac{24\gamma_f}{cF}}_{\text{constants}} \cdot \underbrace{\frac{E}{\sigma_y}}_{\text{material}} \cdot \underbrace{\frac{I}{dZ_p}}_{\text{geom.}}}$

With greater L/d ratios (span relatively longer) then behaviour governed by stiffness rather than strength. L is usually known at start: choose d so that inequality not violated.

1bi)



═ direction in which floor resists bending.



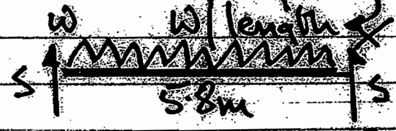
(PLAN)

$\frac{23}{2}$ kN/m² (due to ↕ ⟺ capability)

force/length $= 23 \times 2.1 + \frac{23}{2} \times \frac{5.8}{2}$ kN/m $= w$

$\Rightarrow w = 81.65$ kN/m

No moment here as suggested in question

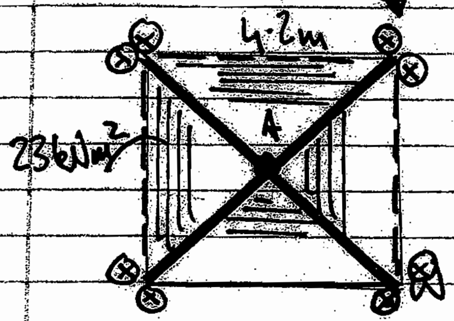


(SIDE)

$\Rightarrow$ B.M.(max) in WX (= WZ, due to symmetry) $= \frac{wL^2}{8} = \frac{81.65 \times 10^3 \times 5.8^2}{8}$ Nm $= \underline{\underline{343.3 \text{ kNm}}}$

S.F.: $2S = wL \Rightarrow S = 81.65 \times 10^3 \times \frac{5.8}{2}$

$= \underline{\underline{236.8 \text{ kN}}}$



The S.F. at W consists of two components either side, due to shear force in WX and WZ: it acts downwards, hence ⊗ notation.

Total downwards load $= \underbrace{8 \times 236.8 \text{ kN}}_{\text{S.F.}} + \underbrace{4.2^2 \times 23 \text{ kN}}_{\text{slab wt}}$

$= \underline{\underline{2300.0 \text{ kN}}}$ = column force at A [also $= 23 \times 10^2$ ↑ unit cell in overall plan]

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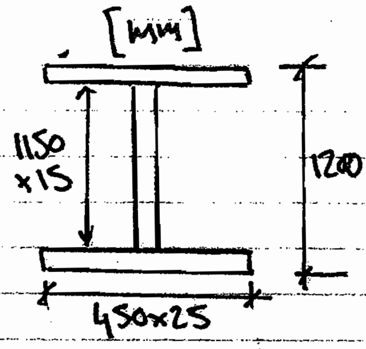
2a)



$$Z_p = \sum A_r d_r = 2 \times 450 \times 25 \times \left[\frac{1150+25/2}{2}\right] + 2\times\left(\frac{1150}{2}\times 15\right)\times\frac{1150}{4} \quad mm^3$$

$$\Rightarrow \quad Z_p = \underline{18.04\times10^6\ mm^3}$$

$$M_p = \frac{\sigma_y}{\gamma_{2\ 1.05}} Z_p = \frac{355\times10^6\ (Pa)}{1.05} \times \underbrace{18.04\times10^{-3}}_{m^3} = \underline{\underline{6100\,kNm}}$$

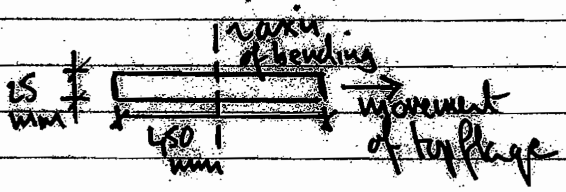
Load $\underbrace{1970\,kN}_{W\ (\text{total})}$ distributed over 25m span $\Rightarrow$ max b.M $= \underbrace{\frac{WL}{8}}_{\text{simp.sup}} = \underline{\underline{6156\,kNm}}$

$\Rightarrow$ Bending resistance exhausted at midspan (just).

2b) At ends, there is maximum shear force $\Rightarrow$ max shear stress in web = S.F / A_{web} $\approx$ 60MPa, quite low c.f. to shear yield stress. For bearing at ends, need to check there is sufficient material over bearing length to avoid local yield at point of support: if not, add vertical stiffeners to increase bearing area; this might also help reinforce against web crippling in region just above lower flange.

At mid-span need to check for local compressive (buckling) behaviour in unsupported top flange and for lateral torsional buckling overall. For welds, the shear force / length of the web/flange junction determines the weld geometry.

c)



$$r = \sqrt{I/A} = \sqrt{\frac{bd^3}{12}\cdot\frac{1}{bd}} = d/\sqrt{12} = \frac{450}{\sqrt{12}} = \underline{130\,mm}$$

$$r/\underset{\substack{\uparrow\\ \text{max fibre dist}}}{y} = \frac{130}{225} = 0.57 \Rightarrow \underline{\text{curve (b) on chart}}$$

If $\bar{\lambda} = 0.9$ (given), curve b $\Rightarrow$ $\underline{\bar{\lambda} = 0.5}$

$$\lambda_0 = \pi\sqrt{\frac{E}{\sigma_y}} = \pi\sqrt{\frac{210\times10^9}{355\times10^6}} = 76.4 \quad : \quad \lambda = \bar{\lambda}\lambda_0$$

$$\Rightarrow \lambda = 0.5\times76.4 = 38.4 = \frac{\overset{\text{intended buckling length}}{L}}{r} \Rightarrow L = 38.4\times0.13\,m = 4.99\,m$$

$\Rightarrow$ If top flange restrained less than every 5m, then buckling prevented (just)

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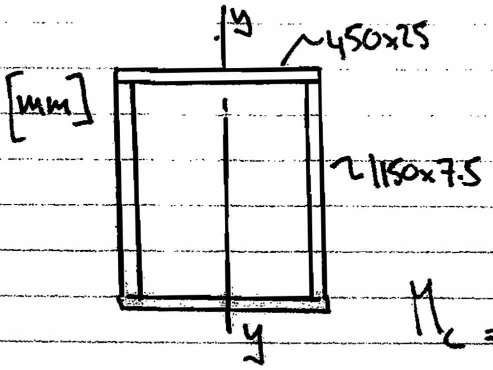

$$I_{yy} = 2 \times \left[\frac{25 \times 450^3}{12} + \left(1150 \times 7.5 \right) \times \left[\frac{450-7.5}{2} \right] \right] \; mm^4$$

$$\Rightarrow \underline{\underline{I_{yy} = 1224.1 \times 10^6 \, mm^4}}$$

$$M_c = \frac{\pi}{L} \left[E I_{yy} \left[G J + \cancel{E C_w \frac{\pi^2}{L^2}} \right] \right]^{1/2}$$

$C_w \approx 0$ from question

$$\Rightarrow M_c = \underbrace{\frac{\pi}{25}}_{span} \cdot \left[\underbrace{210 \times 10^9}_{E (Pa)} \times \underbrace{1224.1 \times 10^{-6}}_{I (m^4)} \times \underbrace{81 \times 10^9}_{G (Pa)} \times \underbrace{3 \times 10^{-3}}_{\substack{J (m^4) \\ given}} \right]^{1/2}$$

$$= \underline{31.4 \, MNm}$$

A very high moment for LTB : M_{cr} = equiv. critical moment

$= M_c / 0.8$ ← lectures

$= 39.2 \, MNm$

$$\Rightarrow \bar{\lambda} = \sqrt{\frac{M_p}{M_{cr}}} = \sqrt{\frac{6100}{39200}} = \underline{0.4}$$

From LTB datasheet $\chi_{LT} \simeq 0.95 = M_F / M_p$ ← applied

$$\Rightarrow M_F = 0.95 \times 6100 \simeq \underline{\underline{5.8 \, MNm}}$$

<u>Common failings</u>: using Z_e instead of Z_p : calculating Z_p improperly : reading wrong buckling curve in 1(c), not completing part (d), not using equivalent critical moment.)

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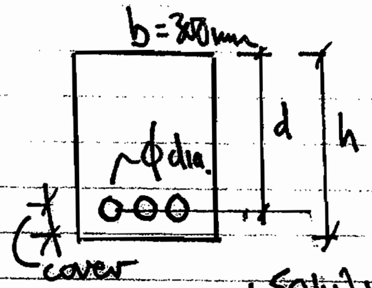
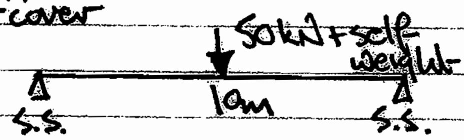
3a)



$d = h - \frac{\phi}{2} - \text{cover} = 600 - \frac{30}{2} - 50 = \underline{535\ \text{mm}}$ (50 given)

[shear links not considered].

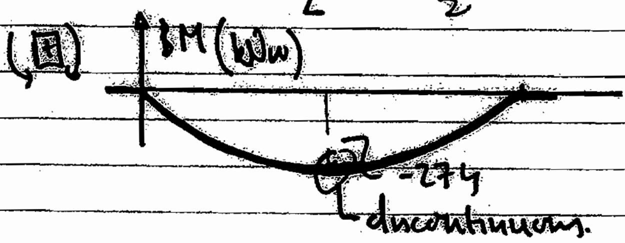


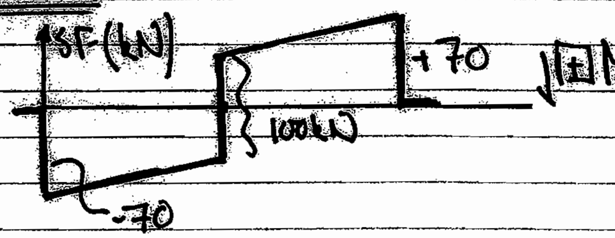
self wt $= \rho \cdot bh \cdot g = \underbrace{2400}_{\text{struct databk}}(0.3 \times 0.6) \cdot 9.81 = \underline{4.24\ \text{kN/m}}$

Factored loads: $50\text{kN} \times 1.6 = \underbrace{80\text{kN}}_{\text{live}}$ + $4.24 \times 1.4 = \underbrace{5.93\ \text{kN/m}}_{\text{dead}}$

$\Rightarrow$ max B.M. $= \overset{(WL/4)}{\frac{80 \times 10}{4}} + \overset{(wL^2/8)}{\frac{5.93 \times 10^2}{8}} = \underline{274.1\ \text{kNm}}$

max S.F. $= \overset{(W/2)}{\frac{80}{2}} + \overset{(wL/2)}{\frac{5.93 \times 10}{2}} = \underline{69.7\ \text{kNm}}$





3b) Assume underreinforced $\Rightarrow$ $M_u = 0.225 f_{cu} \cdot bd^2/\gamma_c$ (datasheet)

$f_{cu} = 30$ MPa (given) $\Rightarrow M_u = 0.225 \times (30 \times 10^6) \times 0.3 \times 0.535^2 / 1.5$ (given)

$\Rightarrow \underline{M_u = 386.4\ \text{kNm}}$ (actual $= 274$ kNm $\Rightarrow$ underreinforced ok).

If singly reinforced (and underreinforced) $\Rightarrow M_u = A_s f_y \cdot \frac{d}{\gamma_s}\left(1 - \frac{x}{2d}\right)$ [datasheet]

where $x/d = \gamma_c A_s f_y \,/\, \gamma_s \cdot 0.6 f_{cu} \cdot bd$ (given)

$\Rightarrow x/d = \frac{1.5}{1.15} \cdot \frac{(460 \times 10^6)}{0.6 \times 30 \times 10^6} \cdot \frac{1}{0.3 \times 0.535} \cdot A_s \Rightarrow \underline{x/d = 207.68 A_s}$

($1.15 = \gamma_s$)

$\underbrace{\frac{274 \times 10^3 \times 1.15}{460 \times 10^6 \cdot 0.535}}_{M_u \gamma_s / f_y \cdot d \,=\, 1.2804 \times 10^{-3}} = A_s \cdot \left(1 - \frac{207.68}{2} A_s\right)$

$\underbrace{103.84 A_s^2 - A_s + 1.2804 \times 10^{-3} = 0}_{\text{solve quad, seek lowest root}}$

$A_s = \left\{1 \pm \left[1 - 4 \times 103.84 \times 1.284 \times 10^{-3}\right]^{1/2}\right\} / 2 \times 103.84$ →

3b). $\Rightarrow A_s = 1.526 \times 10^{-3} m^2 = 1525.7 mm^2$

2 bars (ϕ = 30mm) $= \pi . \frac{30^2}{4} \times 2 \simeq 1413 mm^2$; probably OK; three bars too much – select smaller ϕ ultimately.

$\Rightarrow$ 2 ϕ30 bars. [spacing OK across width]

3c). $V_{max} = 69.7 kN$. From datasheet $V_{Rd,c} = \frac{0.18}{\gamma_c} \cdot k \cdot [100 \cdot \rho_1 \cdot f_{ck}]^{1/3} \cdot bd$ (γ_c: 1.5; k: $1 + \sqrt{\frac{200}{d \text{ in mm}}}$; ρ_1: A_s/bd; f_{ck}: $0.8 f_{cu}$)

$$\frac{V_{Rd,c}}{bd} = \frac{0.18}{1.5}\left[1+\sqrt{\frac{200}{535}}\right]\left[100 \cdot \underbrace{0.799}_{\% \div 100} \times (0.8 \times \underbrace{30}_{MPa})\right]^{1/3}$$

$\Rightarrow$ $\underline{V_{Rd,c}/bd = 0.518 MPa}$ (1) : $V_{max}/bd = \frac{69.7 \times 10^3}{0.3 \times 0.535} = \underline{0.434 MPa}$ (2)

$v_{min} = 0.035 \times k^{3/2} \cdot \frac{f_{ck}}{MPa}^{1/2}$ (from datasheet) $= \underline{0.350 MPa}$ (3)

(1) must be larger than (3) to take effect w/o stirrups: and since (1) > (2), the actual shear stress, then no stirrups required.

3d). Live load increased to 100kN $\Rightarrow$ Max BM is $\underline{474 kNm} > M_u \Rightarrow$ top & btm reinforcement needed. From notes

$$\underset{(474kNm)}{M_u} = \frac{\overset{\text{top steel}}{A_s'}}{\gamma_s} \times (d - \underset{\text{as in bottom} = 65mm}{d'}) f_y + \underbrace{0.225 f_{cu} \frac{bd^2}{\gamma_c}}_{386kNm \text{ from before}} \Rightarrow \underline{A_s' = 468mm^2}$$

$\Rightarrow$ 1 $\underline{\phi 30mm \text{ bar } \& \text{ 2 } \phi 20mm \text{ bars at top}}$.

Long. eqlm (uder) $\Rightarrow$ $\overset{\text{btm steel}}{A_s}/\gamma_s \cdot f_y = A_s'/\gamma_s \cdot f_y + 0.6 f_{cu} \frac{b x}{\gamma_c}$ (x ~ d/2 (approx))

$$\Rightarrow A_s/_{1.15} \times 460 \times 10^6 = 486 \times 10^{-6}/_{1.15} \times 460 \times 10^6 + 0.6 \cdot (30 \times 10^6) \cdot 0.3 \times \frac{0.535/2}{1.5}$$

$\Rightarrow$ $\underline{A_s = 2894 mm} \rightarrow$ $\underline{\text{put over } 4\phi 30mm \text{ bars} : 5\phi 30 \text{ bars to be sure}}$

Common mistakes: making heavy work of BM/SF diagrams; not indicating a sign conventn on BM/S.F.; substituting for incorrect units into databook expression; not converting A's into an appropriate number of bars.

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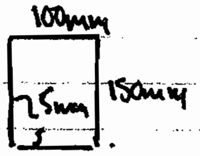
4ai)



$$I = \frac{100\times150^3}{12} - \frac{90\times140^3}{12}\ \text{mm}^4 = \underline{7.545\times10^6\ \text{mm}^4}$$

EI? $\quad E = E_f\cdot V_f + E_m(1-V_f) = 230\times10^9\cdot0.6 + 5\times10^9(1-0.6) = \underline{140\ \text{GPa}}$

$$EI = (140\times10^9)\cdot(7.545\times10^{-6}\ \text{m}^4) = \underline{\underline{1.056\times10^6\ \text{Nm}^2}}$$

4aii)

$$M_{max} = \underbrace{\sigma_{LC}}_{<\sigma_{LT},\ \text{so controls}}\cdot I/y_{max} = \underbrace{500\times10^6}_{\text{given}}\cdot\frac{7.545\times10^{-6}}{\underbrace{0.075}_{y_{max}}} = \underline{\underline{50.3\ \text{kNm}}}$$

For steel equivalent: $M_p = Z_p\cdot\sigma_y \qquad Z_p = \frac{100\times150^2}{4} - \frac{90\times140^2}{4} \quad \left(\frac{bd^2}{4}\right)$

$$M_p = \underbrace{1.215\times10^{-4}}_{Z_p,\ \text{m}^3}\cdot\underbrace{355\times10^6}_{\sigma_y,\ \text{given}} = \underline{\underline{43.1\ \text{kNm}}}$$

∴ so composite has higher moment capacity.

4bi)

$$E_{xx} = \underbrace{\frac{WL^3}{3\delta_{tip}\cdot I}}_{\text{struct. databk}} \quad L\sim0.5\text{m}$$

: assume E_{xx} in direction of cantilever tip from base

$\delta_{TIP,\ max} = 5\text{mm}$ (given); $W = 50\text{kN}$ (given)

$$\Rightarrow E_{xx} = 50\times10^3\times0.5^3 / 3\times0.005\times(7.545\times10^{-6})$$

$$\Rightarrow \underline{\underline{E_{xx} = 55.22\ \text{GPa}}}$$

$$G_{xy} = \frac{TL}{\theta_{max}J}$$ } similar to notes + examples paper : $T = 10\text{kNm}$, $L = 0.5\text{m}$, $\theta_{max} = 0.03\text{rad}$, J?

$J = 4A_e^2 / \oint\frac{ds}{t}$, part IB thin-walled + structures databk $= \frac{4(bd)^2}{(2b+2d)/t}$

$$\Rightarrow J = 2b^2d^2t/(b+d) = 2\times0.1^2\times0.15^2\cdot0.005/(0.1+0.15) = \underline{9\times10^{-6}\ \text{m}^4}$$

$$\underline{G_{xy}} = 10\times10^3\times0.5 / 0.03\times9\times10^{-6} = \underline{\underline{18.52\ \text{GPa}}}$$

4bii)

0% plies	45% plies	E_{xx} [GPa]	G_{xy} [GPa]	$\delta = WL^3/3E_{xx}I$	$\theta = TL/G_{xy}J$
0 %	80%	28	31	9.86 mm	0.0179 rad
20 "	"	50	25	5.52 "	0.0222 "
40 "	"	72	18	3.83 "	0.0309 "
60 "	"	93	13	2.97 "	0.0427 "
80 "	"	112	7.5	2.47 "	0.074

(0% and 45% plies:) total = 80% since given proportion of 90% plies = 20%

(E_{xx}, G_{xy}:) via HS carpet plot on datasheet.

(δ:) must be less than 5mm

(θ:) must be less than 0.03 rad.

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By interpolation; δ cut-off at 26% of 0% plies and more,
θ cut-off at 35% of " " less,

$\Rightarrow$ viable 0% ply range is 26 $\rightarrow$ 35%
$\Rightarrow$ [viable 45% ply range, 54 $\rightarrow$ 44%]

$\rightarrow$ Choose 50% of 45% plies ; $\underbrace{\text{20\% of 90\% plies}}_{\text{given}}$, 30% of 0%

$\Rightarrow E_{xx} = 61\,GPa$
$G_{xy} = 21.5\,GPa$

4b iii) The lay-up has to be symmetrical and balanced. If we choose two layers:

$$\underbrace{\underset{30\%}{0}\ \ \underset{25\%}{45}\ \ \underset{25\%}{-45}\ \ \underset{20\%}{90}}_{1^{st}\ layer}\ \Big|\ \underbrace{90\ \ -45\ \ 45\ \ 0}_{2^{nd}\ layer}\ \Big|$$

Max permissible strain = 0.4% [from lectures]

$\varepsilon_x = \sigma_x / E_{xx}$, where $\sigma_x = My/I_{xx}$ $[M = W_{tip}.L]$

$$\varepsilon_x = \frac{WL.y}{E_{xx} I_{xx}} = \frac{(50\times10^3 \times 0.5) \times 0.15 \sim y}{61\times10^9 \times 7.845\times10^{-6}} = 0.81\%$$

$\therefore$ too much strain: would have to revise choice of E_{xx} (at expense of lower G_{xy} and lower torsional compt. stiffness $\rightarrow$ revise box design [geometry, perhaps]

Common mistakes: mixing methods in part (a): either rule of mixtures or transform section, not both.
Forgetting that $\phi = \theta/L$ in IB torsion formula.
Not estimating J properly via IB / databook formula.
Not completing question.