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Module 3D5, Water Engineering, 28 May 2008

Question 1

(a) The time of concentration is the time required for rain drops falling at the most remote point of the catchment to flow to the outlet. Thus after the time of concentration, the whole catchment begins to contribute to the outflow. The rational method states that for mean rainfall of i over a duration equal to the time of concentration, the outflow is

$$Q_{\max} = CiA$$

where C is the runoff coefficient and A is the area of the catchment. i is the overall rain intensity, so C depends on the soil condition.
For large catchments, the rainfall intensity cannot be assumed to be uniform across the catchments.

(b)
(i)The infiltration rate f is

$$f(t) = f_c + (f_0 - f_c)e^{-K_f t} = 10 + 20e^{-0.5t}$$

$f(0) = 30$ mm/hr
$f(2) = 17.36$ mm/hr
$f(4) = 12.71$ mm/hr

Not all the rain falling in the second two hours infiltrates into the ground, so the rain falling in the second two hours also contributes to the outflow.
The amount of water infiltrated is

$$\int_{t_0}^{t_1}\left[f_c + (f_0 - f_c)e^{-K_f t}\right]dt = f_c(t_1 - t_0) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_1} - e^{-K_f t_0}\right)$$

1st hour: $$\int_0^2 \left(10 + 20e^{-0.5t}\right)dt = 10\times 2 - \frac{1}{0.5}\times 20\times\left(e^{-1} - 1\right) = 45.28 \text{ mm}$$

2nd hour: $$\int_2^4 \left(10 + 20e^{-0.5t}\right)dt = 10\times 2 - \frac{1}{0.5}\times 20\times\left(e^{-2} - e^{-1}\right) = 29.30 \text{ mm}$$

Total volume of runoff is
1st hour: (2×40-45.28)×0.001 × 20×1000000 = 694400 m^3
2nd hour: (2×20-29.30)×0.001 × 20×1000000 = 214000 m^3

Sum the corresponding volumes in each two hours

	time (hour)		1	3	5	7	9	11	13
	1st two hours:	694400 m^3×	3%	16%	35%	27%	15%	4%	
+	2nd two hours:	214000 m^3×	0	3%	16%	35%	27%	15%	4%
=	total volume	m^3	20832	117524	277280	262388	161940	59876	8560

The peak discharge occurs at the third two-hour period. The discharge is

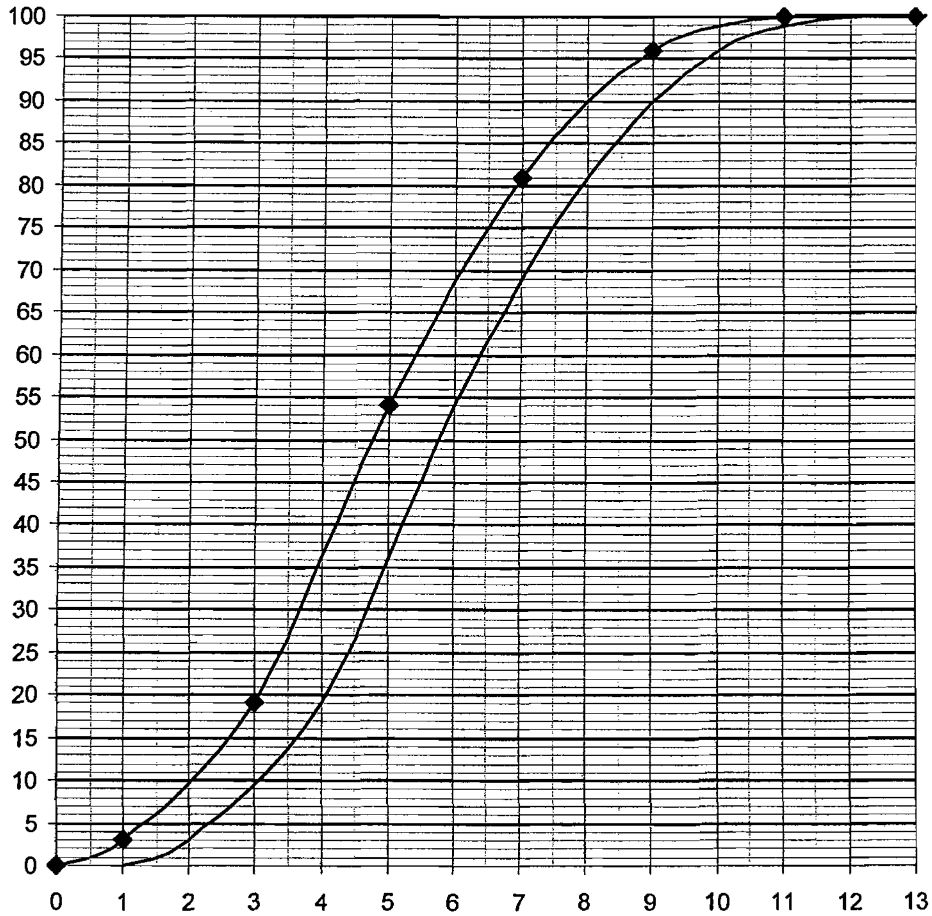
$$\frac{277280}{7200} = 38.51 \text{ m}^3/\text{s}$$

(ii) Answering this part needs to construct the hydrograph on the basis of a unit time of 1 hour.
Plot S curve:

t:	0	1	3	5	7	9	11
%:	0	3	19	54	81	96	100

Shift by one hour and subtract:

t	0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
%:	0	1	5	7.5	13	18.5	16.5	13.5	11	7	5	1.5	0.5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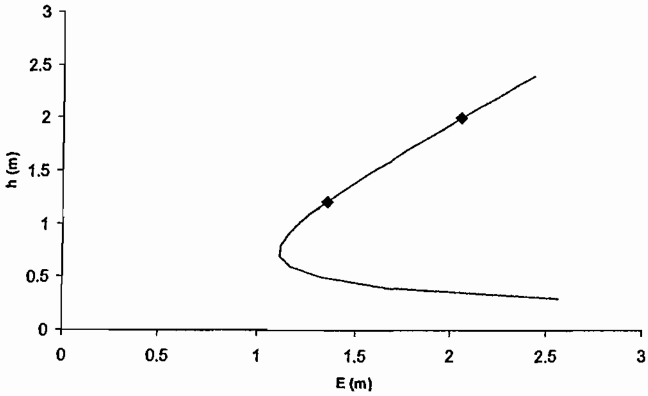
Question 2

(a) The easiest way is to draw the specific energy graph for $q = U_1 h_1 = 2$ m²/s

$$E_1 = h_1 + \frac{U_1^2}{2g} = 2 + 0.051 = 2.051 \text{ m}$$

$$E_2 = E_1 - 0.7 = 1.351 \text{ m}$$

Find from following graph $h_2 = 1.213$ m



The critical depth for $q = 2$ m²/s is $h_c = \left(\frac{2}{\sqrt{g}}\right)^{2/3} = 0.74$ m and $E_c = 1.5\,h_c = 1.11$ m, so road height should be E_1 - E_c = 0.941 m

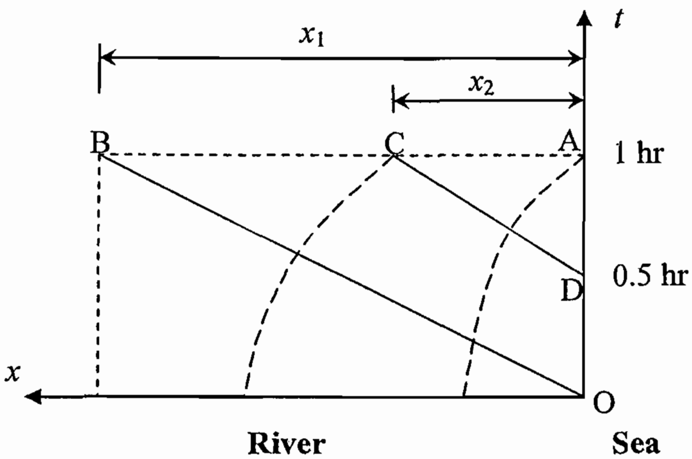
(b) It only needs to prove that $\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = \left(1 - Fr^2\right)\frac{dh}{dx}$.

For steady flow, q must be constant along the channel, so it is not a function of x.

$$\frac{d}{dx}\left(h+\frac{U^2}{2g}\right) = \frac{dh}{dx} + \frac{d}{dx}\left(\frac{q^2}{2gh^2}\right) = \frac{dh}{dx} + \frac{q^2}{2g}\frac{d}{dx}\left(\frac{1}{h^2}\right) = \frac{dh}{dx} + \frac{q^2}{2g}\frac{-2}{h^3}\frac{dh}{dx} = \left(1 - \frac{q^2}{g}\frac{1}{h^3}\right)\frac{dh}{dx} = \left(1 - Fr^2\right)\frac{dh}{dx}$$

The hydraulic jumps are rapidly varied flows, so cannot be analysed using these equations.

(c)



Suppose it is point C, so $h_C = 3.5$ m.

Along –ve line from C: $U_C - 2\sqrt{3.5g} = 0 - 2\sqrt{4g}$. So $U_C = -0.81$ m/s

Along +ve line CD, $U_D = U_C = -0.81$ m/s and $h_D = h_C = 3.5$ m.
This corresponds to a time of 0.5 hour at D. Consider the slope of CD:

$$\frac{x_2}{1800} = -0.81 + \sqrt{3.5g}$$

So $x_2 = 9091$ m

Question 3

(a) $$d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.2\times10^{-3}\times\left(\frac{9.81\times(2.65-1)}{10^{-12}}\right)^{1/3} = 5.06$$

The fall velocity:

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2+1.049d_*^{\,3}}-10.36\right] = \frac{10^{-6}}{0.2\times10^{-3}}\left(\sqrt{10.36^2+1.049\times5.06^3}-10.36\right) = 0.0262 \text{ m/s}$$

Total shear stress: $$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{ghS_b} = \sqrt{9.81\times10\times4\times10^{-5}} = 0.0626 \text{ m/s}$$

$$\frac{u_*}{w_s} = \frac{0.0626}{0.0262} = 2.39,\quad \mathrm{Re}_* = \frac{u_* d}{\nu} = \frac{0.0626\times0.2\times0.001}{10^{-6}} = 12.52$$

So from Liu's graph, the predominant bedform is dunes or transition between dunes and antidunes depending on how the graph is read.

(b) According to Chezy formula: $C = \frac{U}{\sqrt{hS_b}} = \frac{1}{\sqrt{10\times4\times10^{-5}}} = 50$

According to $C = 7.8\ln\left(\frac{12h}{k_s}\right)$: $\quad k_s = \frac{12h}{e^{\frac{C}{7.8}}} = \frac{12\times10}{e^{\frac{50}{7.8}}} = 0.197 \text{ m}$

(c) $$\frac{w_s}{\kappa u_*} = \frac{0.0262}{0.4\times0.0626} = 1.046$$

Concentration at 0.2 m: $\quad \bar{c}(0.2) = \bar{c}(1)\cdot\left(\frac{10-0.2}{0.2}\cdot\frac{1}{10-1}\right)^{1.046} = 0.359 \text{ kg/m}^3$

$$q_s = \int_a^h \bar{c}(z)u(z)dz = \int_a^h \bar{c}(z)u(z)dz = 11.6\cdot u_*\cdot\bar{c}(a)\cdot a\cdot\left[I_1\ln\left(\frac{30h}{k_s}\right)+I_2\right]$$

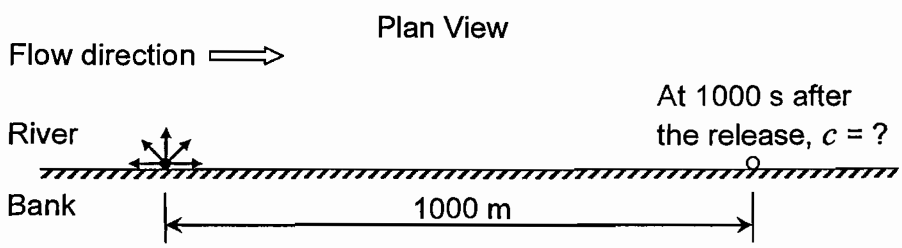
$$= 11.6\cdot0.0626\cdot0.359\cdot0.2\cdot\left[I_1\ln\left(\frac{30\times10}{0.197}\right)+I_2\right] = 0.0521\cdot[7.33I_1+I_2]$$

Check table in the data sheet. Corresponding to a/h = 0.2/10 = 0.02 and $w_s/\kappa u_*$ = 1.046,

$I_1 = 0.646$, $I_2 = -1.448$

So $q_s = 0.0521\cdot[7.33\times0.646-1.448] = 0.17$ kg/(m·s)

(d)



$D_y = 0.15hu_* = 0.15\times10\times0.0626 = 0.0939 \text{ m}^2/\text{s}$

$D_x = (0.15+5.93)hu_* = (0.15+5.93)\times10\times0.0626 = 3.8 \text{ m}^2/\text{s}$

$$c(x,y,t)=\frac{M/h}{4\pi t\sqrt{D_xD_y}}\exp\left(-\frac{(x-u_0t)^2}{4D_xt}-\frac{y^2}{4D_yt}\right)+\frac{M/h}{4\pi t\sqrt{D_xD_y}}\exp\left(-\frac{(x-u_0t)^2}{4D_xt}-\frac{y^2}{4D_yt}\right)$$

$$=\frac{2M/h}{4\pi t\sqrt{D_xD_y}}\exp\left(-\frac{(x-u_0t)^2}{4D_xt}-\frac{y^2}{4D_yt}\right)$$

$$c(1000,0,1000)=\frac{2\times 10/10}{4\pi\times 1000\times\sqrt{0.0939\times 3.8}}\exp\left(-\frac{(1000-1000)^2}{4\times 3.8\times 1000}\right)$$

$$=2.666\times 10^{-4}\ \text{kg m}^{-3}$$

Question 4

(a) The specific speed increases from centrifugal, mixed to axial pumps, meaning that the centrifugal pump is suitable for delivering large head with a small discharge while the axial pump is suitable for delivering small head with a large discharge.

(b) If the pressure reaches the vapour pressure of water, cavitations will occur, causing great damages.

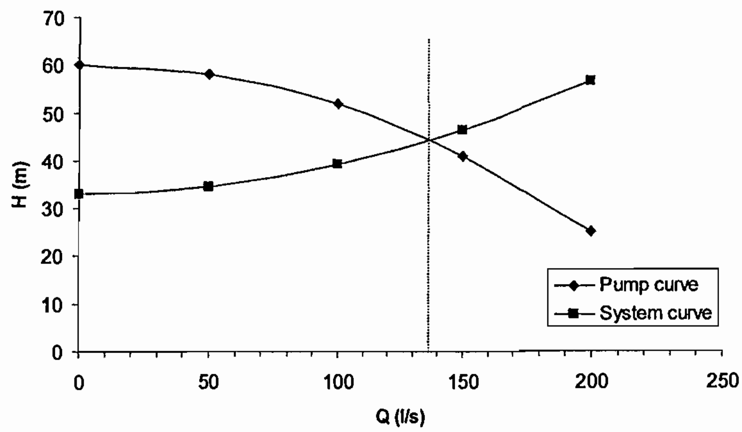
(c) The pipeline curve is: $H = 85 - 52 + \left(f\frac{2000}{0.35}+10\right)\frac{Q^2}{2g\left(\frac{\pi\cdot 0.35^2}{4}\right)^2} = 33 + (31495.7f + 55.1)Q^2$

f is obtained from Moody diagram based on: $\mathrm{Re} = \frac{UD}{\nu} = \frac{Q\cdot 0.35}{\frac{\pi\cdot 0.35^2}{4}\cdot 10^{-6}} = 3.64\times 10^6 Q$

$$\frac{k_s}{D} = \frac{0.15}{350} = 4.3\times 10^{-4}$$

Q (l/s)	0	50	100	150	200
Re ($\times 10^5$)	0	1.82	3.64	5.475	7.28
k_s/D	0.00043	0.00043	0.00043	0.00043	0.00043
f		0.0185	0.0175	0.0172	0.017
H (m)	33	34.6	39.1	46.4	56.6

From the following figure, $Q = 137$ l/s



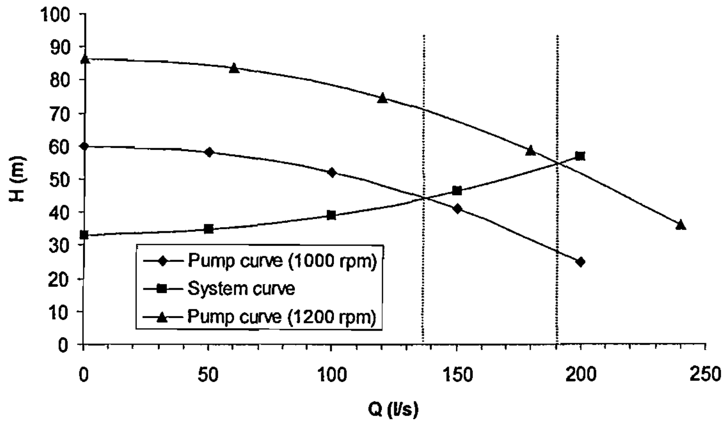
(d) When the pump runs at 1200 rpm, suppose it generates a discharge Q_{p2} against head H_{p2} with the corresponding operation point at the speed of 1000 rpm being Q_{p1} and H_{p1}. Then,

$$\frac{Q_{p2}}{N_{p2}} = \frac{Q_{p1}}{N_{p1}},\quad \frac{H_{p2}}{N_{p2}^2} = \frac{H_{p1}}{N_{p1}^2}.$$

So

$$Q_{p2} = \frac{N_{p2}Q_{p1}}{N_{p1}} = 1.2Q_{p1},\quad H_{p2} = \frac{N_{p2}^2 H_{p1}}{N_{p1}^2} = 1.44H_{p1}.$$

Q_{p1} (litre/s)	0	50	100	150	200
H_{p1} (m)	60	58	52	41	25
Q_{p2} (litre/s)	0	60	120	180	240
H_{p2} (m)	86.4	83.52	74.88	59.04	36



From the above figure: $Q = 191$ l/s