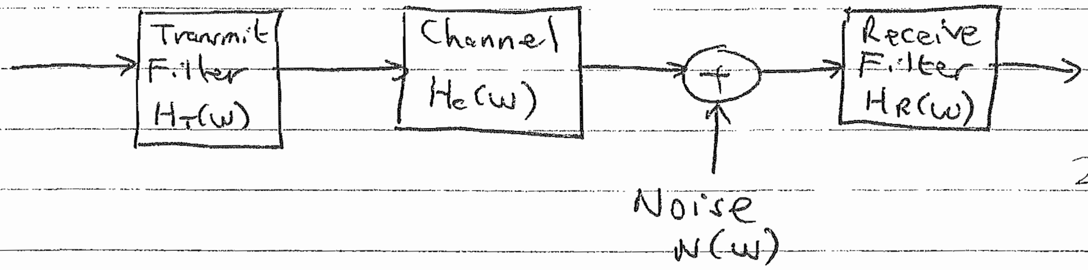
3F4 07/08 Dr I Wassell.

1. (a)



2

$H_T(\omega)$ shapes transmitted power spectrum. 1

$H_c(\omega)$ is the channel frequency response 1

$H_R(\omega)$ reduces the effect of noise. 1

$N(\omega)$ is the additive noise power spectrum

- usual assumptions:
  - Gaussian pdf with zero mean
  - Uniform spectrum (i.e., white). 1

[6]

(b) $$E_T = \int_{-\infty}^{\infty} (h(t))^2 dt.$$ 1

From Parsevals theorem

$$E_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H_T(\omega)|^2 d\omega.$$ 1

We are told that

$$H_T(\omega)\, H_c(\omega)\, H_R(\omega) = k\, P_R(\omega)$$

so,

$$H_T(\omega) = \frac{k\, P_R(\omega)}{H_c(\omega)\, H_R(\omega)}$$ 1

giving,

$$E_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} k^2 \frac{|P_R(\omega)|^2}{|H_c(\omega)|^2 |H_R(\omega)|^2} d\omega$$

rearranging yields

$$k^2 = \frac{E_T}{\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|P_R(\omega)|^2}{|H_c(\omega)|^2 |H_R(\omega)|^2} d\omega}$$ 1

Now considering noise, the psd of the receive noise at the receive filter output is,

$$S_v(\omega) = N(\omega)\,|H_R(\omega)|^2$$

Hence the noise power at the filter output is,

$$\sigma_v^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_v(\omega)\,d\omega = \frac{1}{2\pi}\int N(\omega)\,|H_R(\omega)|^2\,d\omega.$$

We wish to minimise

$$\frac{\sigma_v^2}{k^2} = \frac{\frac{1}{2\pi}\int_{-\infty}^{\infty} N(\omega)\,|H_R(\omega)|^2\,d\omega}{\dfrac{E_T}{\frac{1}{2\pi}\int_{-\infty}^{\infty}\dfrac{|P_R(\omega)|^2}{|H_C(\omega)|^2\,|H_R(\omega)|^2}\,d\omega}}$$

$$\frac{\sigma_v^2}{k^2} = \frac{1}{(2\pi)^2 E_T}\int_{-\infty}^{\infty} N(\omega)\,|H_R(\omega)|^2\,d\omega \int_{-\infty}^{\infty}\frac{|P_R(\omega)|^2}{|H_C(\omega)|^2\,|H_R(\omega)|^2}\,d\omega$$

Schwartz inequality states that

$$\int_{-\infty}^{\infty}|F(\omega)|^2\,d\omega\int_{-\infty}^{\infty}|G(\omega)|^2\,d\omega \geq \left|\int_{-\infty}^{\infty}F(\omega)\,G(\omega)\,d\omega\right|^2$$

with equality when $F(\omega) = \lambda\, G^*(\omega)$ where λ is an arbitrary constant.

Let $F(\omega) = \sqrt{N(\omega)}\,|H_R(\omega)|$ and

$$G(\omega) = \frac{|P_R(\omega)|}{|H_C(\omega)|\,|H_R(\omega)|}$$

and so we obtain

$$\frac{\sigma_v^2}{k^2} = \frac{1}{(2\pi)^2 E_T}\int_{-\infty}^{\infty} N(\omega)\,|H_R(\omega)|^2\,d\omega\int_{-\infty}^{\infty}\frac{|P_R(\omega)|^2}{|H_C(\omega)|^2\,|H_R(\omega)|^2}\,d\omega$$

$$\geq \frac{1}{(2\pi)^2 E_T}\left|\int_{-\infty}^{\infty}\sqrt{N(\omega)}\,\frac{|P_R(\omega)|}{|H_C(\omega)|}\,d\omega\right|^2$$

[8]

c) All the terms in the right hand side of the expression in (b) are fixed, hence $\frac{\delta_v^2}{k^2}$ is minimised when the 2 sides of the expression are equal, i.e.,

$$\frac{\delta_v^2}{k^2} = \frac{1}{(2\pi)^2 E_T} \left| \int_{-\infty}^{\infty} \sqrt{N(\omega)} \frac{|P_R(\omega)|}{|H_c(\omega)|} d\omega \right|^2 .$$

Now substitute for $N(\omega) = N_0$, $H_c(\omega) = 1$ and $P_R(\omega) = T\cos^2\left(\frac{\omega T}{4}\right) \quad \frac{-2\pi}{T} \leq \omega \leq \frac{2\pi}{T}$

gives,

$$\frac{\delta_v^2}{k^2} = \frac{1}{(2\pi)^2 E_T} \left| \int_{-\frac{2\pi}{T}}^{\frac{2\pi}{T}} \sqrt{N_0}\, T\cos^2\left(\frac{\omega T}{4}\right) d\omega \right|^2$$

$$= \frac{N_0 T^2}{4\pi^2 E_T} \left| \int_{-2\pi/T}^{2\pi/T} \cos^2\left(\frac{\omega T}{4}\right) d\omega \right|^2$$

$$= \frac{N_0 T^2}{4\pi^2 E_T} \left| \frac{1}{2} \left[\omega + \frac{2}{T} \sin\left(\frac{\omega T}{2}\right) \right]_{-2\pi/T}^{2\pi/T} \right|^2$$

$$= \frac{N_0 T^2}{4\pi^2 E_T} \left(\frac{2\pi}{T}\right)^2$$

$$= \frac{N_0}{E_T} . \qquad 3$$

Now,

$$P_e = Q\left(\frac{V_1 - V_0}{2\delta_v}\right) = Q\left(\frac{(A_1 - A_0)\, k\, P_R(0)}{2\delta_v}\right)$$

$$P_e = Q\left(\frac{(1-0)\times 1}{2} \cdot \sqrt{\frac{E_T}{N_0}}\right)$$

$$= Q\left(\frac{1}{2}\sqrt{\frac{E_T}{N_0}}\right) = Q\left(\sqrt{\frac{E_T}{4N_0}}\right) \qquad 2$$

[6]

2)

(a) To match the PSD of the transmitted signal to suit the frequency response of the transmission channel. For example, 1
many baseband channels have a poor response at d.c. and low frequencies. Also a 1
low-pass channel limits the ability of the channel to carry high frequency components!

To permit self-synchronisation i.e. there should be sufficient information in the transition 1
of the transmitted signal to allow timing 1
regeneration (symbol) to be performed at the receiver.

[5]

(b) Now,

$$S_{xx}(\omega) = \frac{1}{T} \sum_{m=-\infty}^{\infty} R(m)\, e^{jm\omega T}$$

where $R(m) = E[a_k \, a_{k+m}]$ is the discrete autocorrelation function. 1

Now need to calculate $R(m)$ for the bipolar line coding scheme.

now b_k $\{0, 1)$

a_k $\{1.2, 0, -0.8\}$

Now construct table,

m	b_k	b_{k+m}	a_k	a_{k+m}	R_i	p_i	R_m	
0	0	0	0	0	0	0.5		
	1	1	−0.8	−0.8	0.64	0.25	0.52	1
			1.2	1.2	1.44	0.25		
1	0	0	0	0	0	0.25		
	0	1	0	−0.8	0	0.125		
			0	1.2	0	0.125	−0.24	1
	1	0	−0.8	0	0	0.125		
			1.2	0	0	0.125		
	1	1	−0.8	1.2	−0.96	0.25		
			1.2	−0.8				
2	0	0	0	0	0	0.25		
	0	1	0	−0.8	0	0.125		
			0	1.2	0	0.125		
	1	0	−0.8	0	0	0.125		
			1.2	0	0	0.125	0.01	1
	1	1	1.2 (0)	−0.8	−0.96	0.0625		
			−0.8 (0)	1.2	−0.96	0.0625		
			1.2 (−0.8)	1.2	1.44	0.0625		
			−0.8 (1.2)	−0.8	0.64	0.0625		
≥3								

$$S_x(\omega) = \frac{1}{T}\left\{ \cdots + 0.01 e^{-j2\omega T} - 0.24 e^{-j\omega T} + 0.52 - 0.24 e^{j\omega T} + 0.01 e^{j2\omega T} + \cdots \right\}$$

$$= \frac{1}{T}\left\{ \cdots + 0.01 e^{-j2\omega T} + 0.01 e^{-j\omega T} + 0.01 + 0.01 e^{j\omega T} + 0.01 e^{j2\omega T} + (-0.25) e^{-j\omega T} + 0.51 + (-0.25) e^{j\omega T} + \cdots \right\}$$

$$S_x(\omega) = \frac{1}{T}\left\{ \sum_{m=-\infty}^{\infty} 0.01 e^{jm\omega T} + (-0.25) e^{-j\omega T} + 0.51 + (-0.25) e^{j\omega T} \right\}$$

$$= \frac{1}{T}\left\{ \sum_{m=-\infty}^{\infty} 0.01 e^{jm\omega T} + 0.51 - 0.25 \left(e^{j\omega T} + e^{-j\omega T} \right) \right\}$$

$$S_x(\omega) = \frac{1}{T}\left\{\sum_{m=-\infty}^{\infty} 0.01\, e^{jm\omega T} + 0.51 - 0.5\cos\omega T\right\}$$

$$= \frac{1}{T}\left\{\sum_{m=-\infty}^{\infty} 0.01\, e^{jm\omega T} + 0.01 + 0.5\left(1-\cos\omega T\right)\right\}$$

$$= \frac{1}{T}\left\{\sum_{m=-\infty}^{\infty} 0.01\, e^{jm\omega T} + 0.01 + 0.5\left(2\sin^2\left(\frac{\omega T}{2}\right)\right)\right\}$$

$$= \frac{1}{T}\left\{\sum_{m=-\infty}^{\infty} 0.01\, e^{jm\omega T} + 0.01 + \sin^2\frac{\omega T}{2}\right\}$$ 2

The sum of exponentials is equivalent to a series of impulses in the frequency domain,

$$S_x(\omega) = \frac{1}{T}\left\{0.01 \times \frac{2\pi}{T}\sum_{m=-\infty}^{\infty}\delta\left(\omega - m\frac{2\pi}{T}\right) + 0.01 + \sin^2\frac{\omega T}{2}\right\}$$

$$S_x(\omega) = \underbrace{\frac{0.01\times 2\pi}{T^2}\sum_{m=-\infty}^{\infty}\delta\left(\omega - m\frac{2\pi}{T}\right)}_{\text{set of impulses spaced at bit rate owing to d.c. offset.}} + \underbrace{\frac{0.01}{T} + \frac{1}{T}\sin^2\frac{\omega T}{2}}_{\text{continuous psd.}}$$

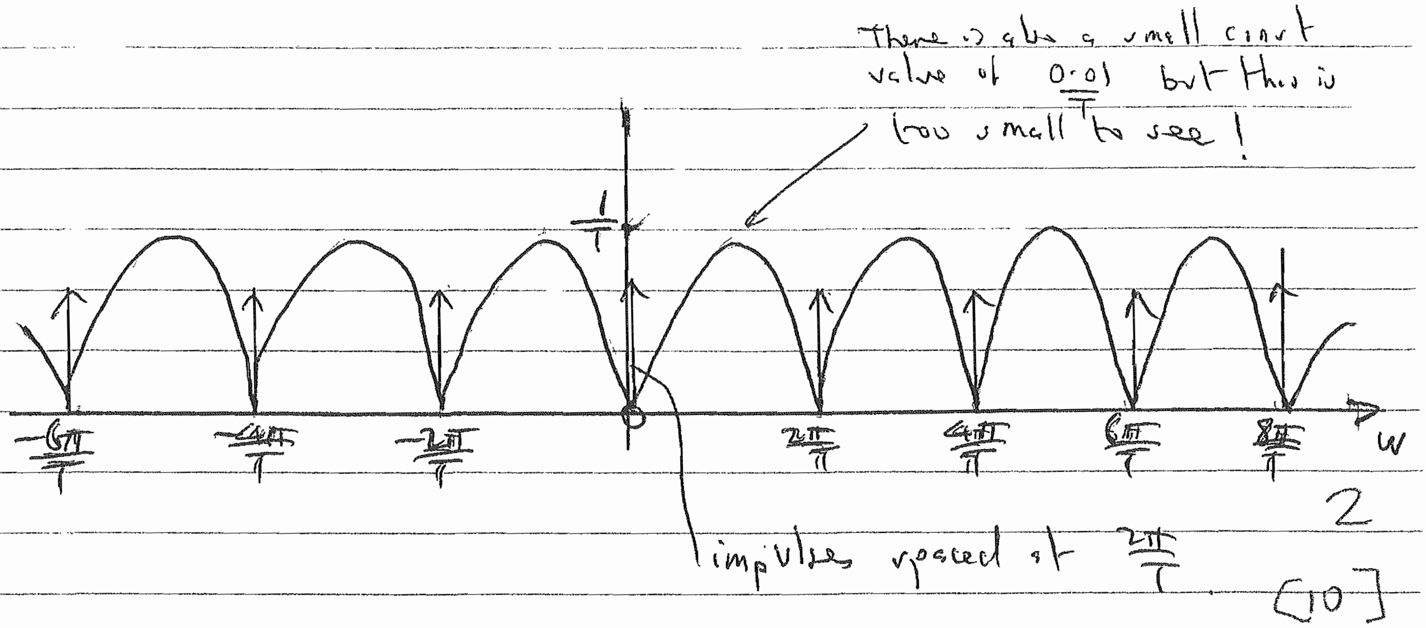
2

Alternative solution:

(i) Calculate continuous spectrum owing to bipolar coding using nominal voltage levels

(ii) Calculate spectrum due to unipolar code with amplitude 0.2V – This will offset the 'marks' by the required amount of +0.2V to create the required signal.

Add spectrums due to (i) and (ii).



2

[10]

c) Using the E and I Data Book, the rect function → sinc function

$b = T$ and $a = 1$

$$\therefore \quad H(w) = T \operatorname{sinc}\left(\frac{wT}{2}\right) \qquad 1$$

which also has zeros at multiples of $\frac{2\pi}{T}$ (or $\frac{1}{T}$) which is the symbol (bit) rate.

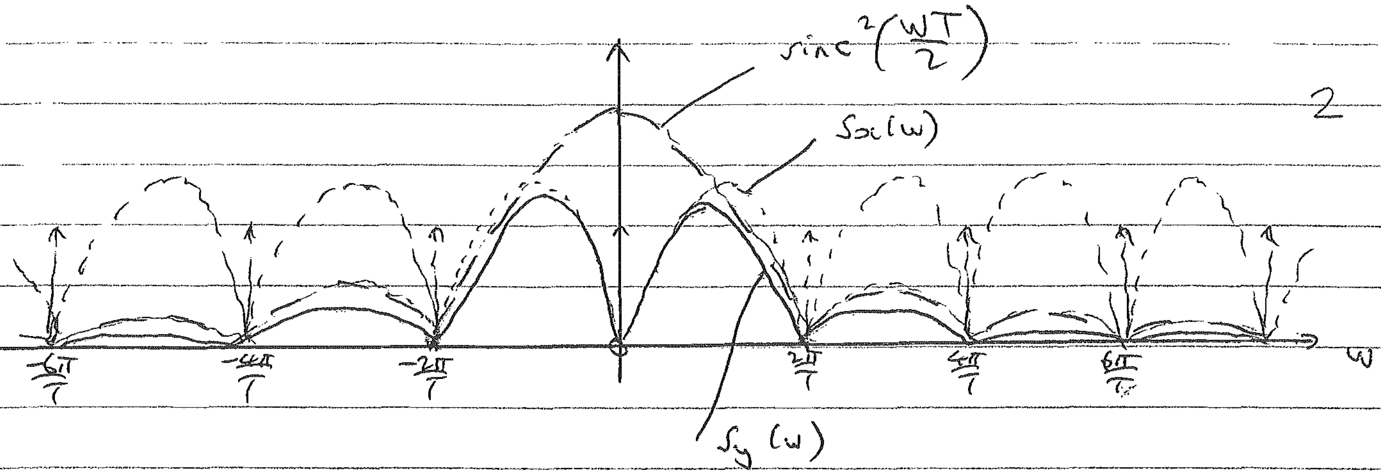
$$\text{now,} \quad S_y(w) = S_x(w)\,|H(w)|^2 \qquad 1$$

$$S_y(w) = \left\{\frac{0.01}{T} + \frac{1}{T}\sin^2\left(\frac{wT}{2}\right) + \frac{0.01\,2\pi}{T^2}\sum_{n=-\infty}^{\infty}\delta\left(w - \frac{n2\pi}{T}\right)\right\} T^2 \operatorname{sinc}^2\left(\frac{wT}{2}\right)$$

$$= \left[0.01\,T + T\sin^2\left(\frac{wT}{2}\right) + 0.01\,2\pi\sum_{n=-\infty}^{\infty}\delta\left(w - \frac{n2\pi}{T}\right)\right] \operatorname{sinc}^2\left(\frac{wT}{2}\right)$$

1

The spectrum is now that of (b) multiplied by the $\operatorname{sinc}^2\left(\frac{wT}{2}\right)$ function, consequently the high frequency components will be reduced in amplitude.



$\mathrm{sinc}^2\left(\frac{WT}{2}\right)$
$S_x(w)$
$S_y(w)$
$-\frac{6\pi}{T}$
$-\frac{4\pi}{T}$
$-\frac{2\pi}{T}$
$\frac{2\pi}{T}$
$\frac{4\pi}{T}$
$\frac{6\pi}{T}$
w

2

[5]

3 (a) $$s(t) = a(t)\cos(\omega_c t + \phi(t))$$

$a(t)$ is the amplitude of the wave & thus represents amplitude modulation if it varies with time. 1

ω_c is the carrier frequency of the wave, & is usually constant. It is in rad/sec. 1

$\phi(t)$ is the phase of the wave (in radians) & thus represents phase modulation if it varies with time. Alternatively $\phi(t)$ can represent frequency modulation, since $\frac{d\phi}{dt}$ = instantaneous frequency offset from ω_c. 1

We can write

$$s(t) = Re\{a(t)\, e^{j(\omega_c t + \phi(t))}\} \text{ if } a(t) \text{ is real}$$

$$= Re\{a(t).e^{j\phi(t)}.e^{j\omega_c t}\}$$

$$= Re\{p(t)\, e^{j\omega_c t}\} \quad \text{if } p(t) = a(t)\, e^{j\phi(t)}$$

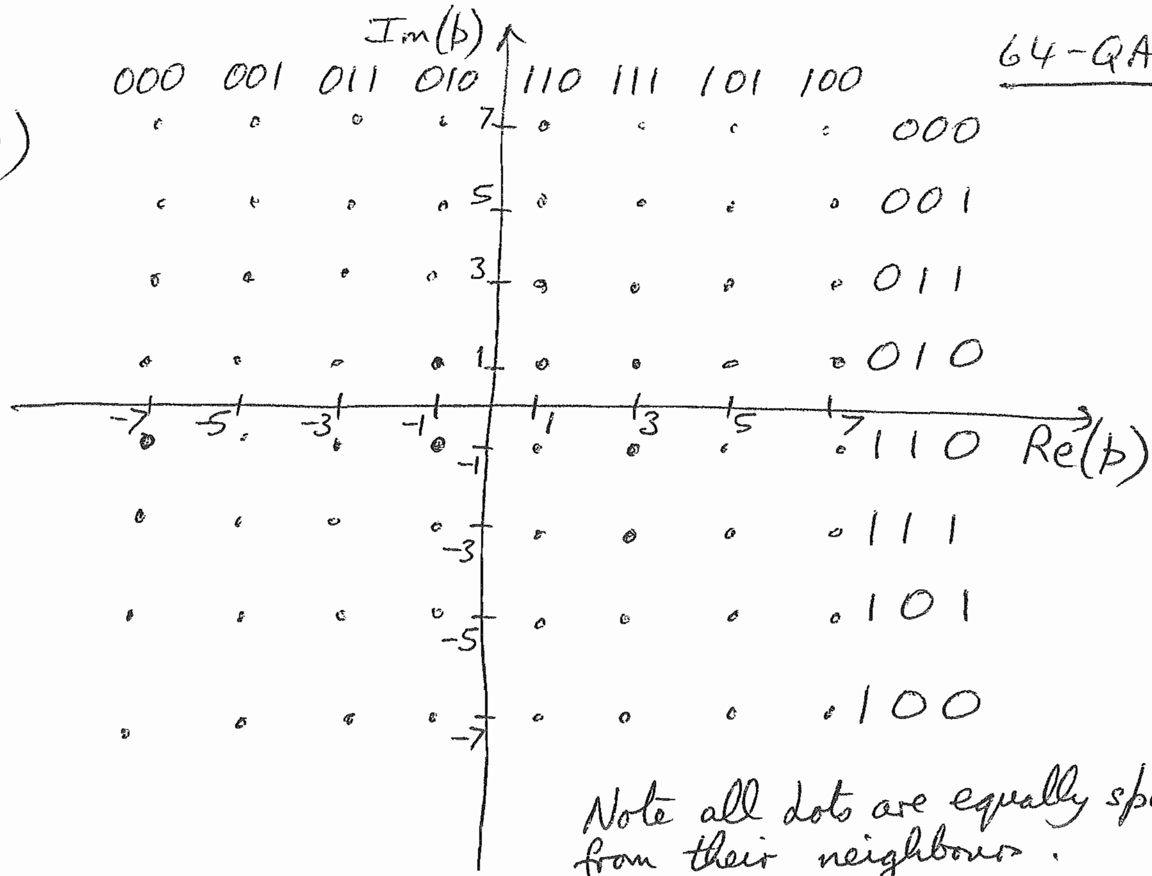
$p(t)$ is complex with amplitude $a(t)$ & phase $\phi(t)$.

2

[5]

3. (b)

64-QAM



Note all dots are equally spaced from their neighbours.

2

$64 = 8^2$, so there are 8 states in each direction, conveying 3 bits each, since $2^3 = 8$ (6-bits in total).

For the ~~8 bits~~ 8 states in each direction, we label them with a 3-bit unit-distance code, so that any pair of adjacent states only differ by 1 bit in their codewords. 1

Errors are most likely to occur to an adjacent state to that which was transmitted (since they are nearest), and the unit-distance codes ensure that such errors only cause 1 of the 3 bits to be changed. Gray codes are suitable unit-distance codes & the above diagram has the states labelled with such a code. 2

[5]

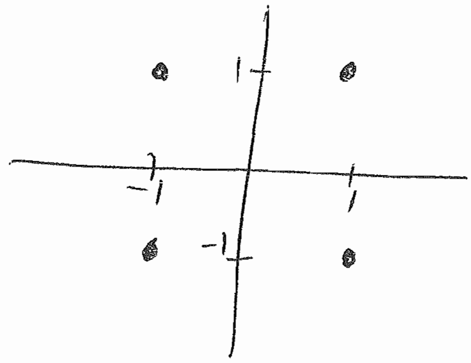
3 (c) 64-QAM fills the available 2-D space uniformly & hence achieves approximately the maximum spacing between states, for a given average mean squared amplitude (power) of the transmitted phasors. (A hexagonal lattice could do a bit better than this, but is not used in practice) 1

64-PSK has 64 states uniformly distributed around a circle of constant radius & this results in states that are much closer to their ~~nea~~ nearest neighbours for the same average signal power. This means that the probability of error is much higher for 64-PSK at a given input signal-to-noise ratio, since noise is more likely to cause errors when the ~~~~ inter-state distance gets smaller. 2

64-PSK does, however, produce phasors of constant amplitude, which means that there is no amplitude modulation & the optimum receiver does not need to estimate the amplitude of the received signal in order to detect the data correctly. 64-QAM has variable amplitude & the receiver must estimate this correctly in order to detect the data correctly. 2

[5]

3(d) QPSK uses a 4-state phasor diagram



and conveys only 2 bits per symbol, compared with 6 bits per symbol for 64-PSK & 64-QAM.

Hence, for a given bit rate, the symbol rate of QPSK is 3 times greater than the others, and so ~~it~~ QPSK requires 3 times as much bandwidth to transmit a given data rate.

However for a given ~~transmit~~ mean signal power, the 4 states of QPSK are much further apart than those of 64-QAM, which are significantly further apart than those of 64-PSK.

Thus QPSK is best against noise, & 64-PSK is worst, with 64-QAM in the middle; but QPSK needs 3 times the bandwidth of the other two schemes. QPSK is also the simplest to detect & decode.

[5]

4. (a) Digital transmission uses discrete transmission states that permit the transmitted data to be recovered exactly at the receiver, as long as the channel noise is insufficient to cause data errors. Hence a signal may pass over a number of transmission ~~paths~~ links in series without incurring additional degradation. 1

In an analogue system, any channel noise causes signal degradation which cannot subsequently be removed, & the effects of many transmission links all accumulate to produce significant loss of signal quality. 1 The only loss of quality in a digital system is the quantisation noise of the A-to-D converter when the signal is originally digitised, and also any distortions due to compression algorithms. 1

The main ~~Bs~~ difficulty with digital systems is the increased bandwidth they tend to need, compared with an analogue system for the same source signal. 1

This problem is usually overcome by using

4(a) (cont)

sophisticated compression methods when the source signal is digitised (eg. JPEG, MPEG$_2$ for images + video; MP3 for audio). In the case of DAB, it has also been possible to achieve coverage of the whole of the UK without needing frequency reuse – saving a further 7:1 in bandwidth (see below) 1

[5]

4(b) Multi-path, if the differential path delays are greater than the symbol period of the modulated signal, causes severe inter-symbol interference. From a spectral viewpoint, it also causes nulls (dips) in the frequency response of the channel. 1 OFDM distributes the source data across many (typ ~ 1000) carriers so that the symbol rate on each carrier is considerably reduced compared to the source bit rate. 1

In addition, guard periods are used between consecutive symbol periods to allow for the expected maximum path delay differences. 1

~~Coding~~ Error-correction coding is used to permit successful decoding of all transmitted data even when there are nulls in the channel response which

4(b) (cont) degrade the error rate on some of the multiple carriers. 1

In the UK, a guard period of $\tilde{}$ 0·25 ms. allows all transmitters of a given block of DAB signals to operate on the same frequency, since signals from adjacent transmitters will appear at the receiver as a multi-path effect, and 0·25 ms is the path delay difference that is adequate to cope with this situation. This avoids the need for frequency reuse of 4:1 or 7:1 as are needed for current analogue FM 1 [5] transmissions.

4(c) Audio bit rate = 1·5 Mb/s

Rate 1:2 coding so transmitted bit rate = 2 . 1·5 = 3 Mb/s 1

Analysis period = 1·0 ms, Guard period = 0·25 ms

So symbol rate = $\dfrac{1}{(1{\cdot}0 + 0{\cdot}25).10^{-3}}$ = 800 sym/s. 1

Modulation is QPSK (4 levels), so this conveys 2 bit/sym.

∴ Total symbol rate needed = $\dfrac{3.10^6}{2} = 1{\cdot}5.10^6$ sym/s.

∴ No. of carriers = $\dfrac{1{\cdot}5.10^6}{800} = \dfrac{15}{8}.10^3 = \underline{\underline{1875}}$ 1

4(c) (cont)

Spacing of carriers $= \dfrac{1}{\text{analysis period}} = 1\,\text{KHz}$

(since carriers are orthogonal over the analysis period, & allow use of FFT for detection).

$\therefore$ Bandwidth of carriers $= 1875 \times 1\,\text{KHz}$

$= 1{\cdot}875\,\text{MHz}$. 1

In practice this ~~a~~ must be increased by ~10% or more to allow for the sidelobes of the $\left(\frac{\sin x}{x}\right)^2$ power spectrum of each carrier & the transition bands of typical filters. Hence the bandwidth ~ 2·1 MHz. 1 [5]

4(d) Each block of the DAB system in section (c) can convey 1·5 Mb/s of audio data, which is between 8 & 11 channels at the coded rates of 144 to 192 ~~#~~ kb/s, given. 1

Hence we would need **2** blocks for the 15 radio channels. Thus the bandwidth for the digital system ~~&~~ would be 2 × 2·1 MHz plus perhaps 0·3 MHz for additional separation, giving

4(d) about $\underline{\underline{4.5 \text{ MHz}}}$ total bandwidth. 1

For the equivalent analogue system, we would need ~~15 × 0.25 MHz~~ 250 kHz for each signal plus ~~say~~ approx 50 kHz for filter transition bands, so the 15 signals would need $15(250+50)$ $= 4500 \text{ kHz} = 4.5 \text{ MHz}$. 1 But in addition we require 7:1 frequency reuse, so the total bandwidth for the whole UK would be

$7 \times 4.5 \text{ MHz} = \underline{\underline{31.5 \text{ MHz}}}$ 1

Hence the digital system ~~is~~ requires approximately ~~7 times more bandwidth efficient~~ $\underline{\underline{\frac{1}{7}}}$ of the bandwidth of the analogue system. 1

[5]