ENGINEERING TRIPOS   PART IIA

Friday 25th April 2008   9 to 12

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed Compressible Flow Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A supersonic wind tunnel has a working section of 625 cm^2 and is designed to run continuously at a Mach number of 3.0. It is supplied at entry to the nozzle with air at a stagnation pressure of 101 kPa and a stagnation temperature of 300 K.

(a) Draw a diagram and describe the operation of the tunnel. Explain why it is not possible in practice to run the tunnel under fully isentropic conditions. [25%]

(b) Calculate the area of the nozzle throat. [20%]

(c) Calculate the minimum area of the diffuser throat under starting conditions. [25%]

(d) Under starting conditions, the flow at the exit of the diffuser is found to have a stagnation pressure of 33 kPa. Assuming that the compressor is isentropic, how much power is required to start the tunnel? [30%]

2 (a) A small change in the impulse function F in a duct of cross-sectional area A may be expressed as

$$\delta\left(\frac{F}{A}\right)=P\left(M^2-1\right)\frac{\delta V}{V}$$

where P is the static pressure, M is the Mach number and V is the velocity. Using this expression, or otherwise, show that in a duct with friction a subsonic flow will accelerate and that a supersonic flow will decelerate. Illustrate each of the two situations on a *T-s* diagram. [30%]

(b) A discharge pipe is fed from a large vessel containing air at a pressure of 15 bar and a temperature of 288 K. The valve at the entrance to the pipe forms a convergent-divergent nozzle which may be assumed to be frictionless up to the end of the divergent section. The pipe is designed for a flow rate of 25 $\mathrm{kgs^{-1}}$, and has a skin friction coefficient $c_f = 0.0025$. The Mach number at the start of the pipe is $M = 1.4$.

(i) Calculate the diameter of the pipe. [15%]
(ii) Calculate the length of pipe required to give $M = 1$ at the exit. [15%]

(c) The length of the pipe is increased with the exit still at $M = 1$. A shock wave is found to occur at 3.5 cm downstream of the pipe entrance. Draw a *T-s* diagram and determine the strength of the shock and the new length of the pipe. [40%]

3 (a) Show that the velocity of the flow V_1 ahead of a stationary hydraulic jump is related to the velocity of the flow V_2 behind the jump according to

$$V_1 V_2 = \frac{1}{2}(h_1 + h_2)g$$

where h_1 and h_2 are the corresponding depths of water, and g is the acceleration due to gravity. [25%]

(b) A large reservoir of water is connected to a long horizontal channel of constant rectangular cross-section. A sluice gate is located at the entrance to the channel. Initially the sluice gate is kept closed, there is no flow, and the water level in the reservoir is higher than that in the channel. At a particular instant the sluice gate is opened and the water level in the channel just downstream of the sluice gate rises by 20%, generating a hydraulic jump which moves off to the right along the channel.

(i) Express the velocity of the hydraulic jump in terms of the speed of water waves in the undisturbed channel. Hence find an expression for the velocity of the flow into the channel at a location a short distance downstream of the sluice gate. [35%]

(ii) After a delay of one second the sluice gate is opened slightly further. Draw a space-time diagram and describe the development of the flow. Calculate the time taken for the new disturbance to catch up with the original hydraulic jump. [40%]

4 A cross-section through a two-dimensional aircraft jet engine ramp intake is sketched in Fig. 1. The intake is designed to produce a focused two-shock system for external compression. Conditions may be assumed to be uniform in the direction into the page.

(a) Draw a carefully labelled sketch of the shock system for a flight Mach number of 2.0. Assume that the flow upstream of the intake is not affected by the aircraft fuselage and is parallel to the intake axis. Calculate the static pressure recovery through the shock system. [30%]

(b) The aircraft now accelerates to a flight Mach number of 2.5. A computer-controlled mechanism causes the central ramp of the intake to translate along its axis in order to preserve a focused shock system. Explain in which direction the ramp must translate and calculate the distance it must move, given that the distance to the cowl lip is 0.5 m. Calculate the static pressure recovery through the new shock system, and comment on its magnitude compared with that in section (a). [40%]

(c) As the aircraft accelerates, the ramp jams when it has only moved *half* of the distance calculated in section (b). At what flight Mach number will a focused two shock system be achieved with the intake jammed in this fault configuration? [10%]

(d) The pilot is alerted to the mechanism failure and slows the aircraft back down to a flight Mach number of 2.0 with the intake ramp still jammed in the fault configuration. Draw a carefully labelled sketch of the resulting shock system and briefly discuss its implications for both the engine and the airframe. [20%]

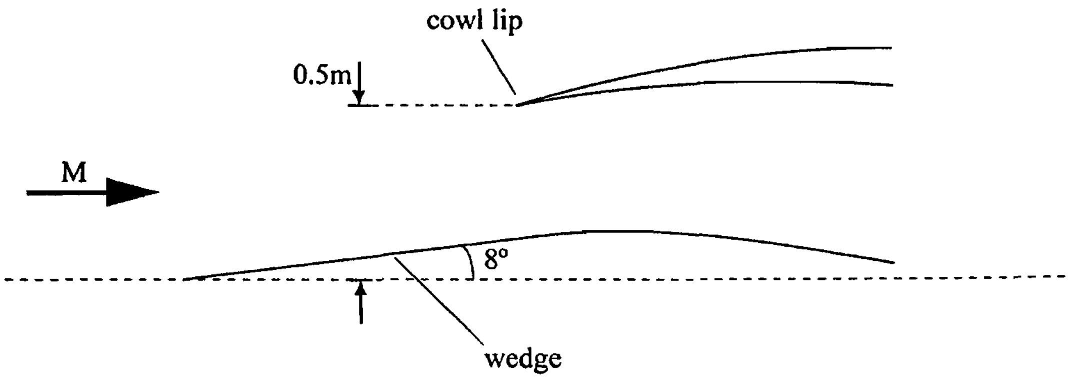

Fig. 1.

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5 A supersonic aircraft is flying at high altitude at a flight Mach number of 1.80. Its wing is of symmetrical diamond 8° double-wedge section, as sketched in Fig. 2. The wing is at 6° incidence to the oncoming flow.

(a) Draw a carefully labelled sketch of the shock / expansion system around the aerofoil including a short length of its wake. [30%]

(b) If the ambient static air pressure is 2.0 Nm^{-2} and each facet of the aerofoil is 1 m in length, calculate:

(i) the section lift coefficient; [20%]

(ii) the section pressure drag coefficient. [20%]

Briefly comment on the section lift / pressure drag ratio. [5%]

(c) Estimate the centre of lift for the section at the conditions shown. [15%]

(d) If the aircraft slows to a subsonic flight speed, and assuming that the flow remains attached, do you expect the centre of lift to move forwards or aft? Briefly explain your answer. [10%]

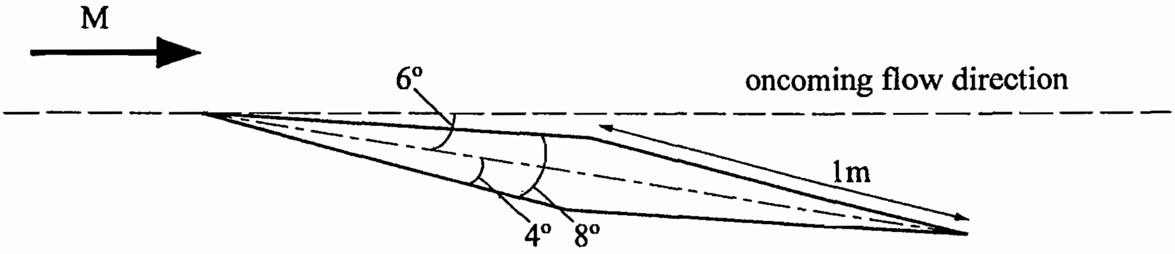

Fig. 2.

6 (a) Give a simple example of each of the following classes of partial differential equation:

- (i) Hyperbolic;
- (ii) Elliptic;
- (iii) Parabolic. [30%]

(b) For each class of equation listed above, explain its properties in the context of its numerical solution, describe the required boundary conditions and provide an example of an application to a fluid flow problem. [70%]

7 *Note that parts (a) and (b) of this question are not related to each other.*

(a) A model one-dimensional scalar convection equation is

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0,$$

where A is a positive constant.

(i) Discretise this equation using first-order forward differencing in time and first-order upwind differencing in space. Explain briefly what are the advantages of upwind differencing. [25%]

(ii) Show that the stability of the resulting difference equation is governed by the criterion

$$\frac{A\Delta t}{\Delta x} \leq 1$$ [25%]

(b) A stator blade for a low-speed axial compressor has been designed to operate in a uniform inlet flow of stagnation pressure p_{01}, velocity V_1, at an angle α_1 and to deliver axial flow at the trailing edge. The thickness of the trailing edge is very small compared to the blade pitch s. The flow can be assumed to be two-dimensional and incompressible with density ρ, and it may be assumed that just downstream of the blade trailing edge the static pressure p_{te} and the velocity outside of the boundary layers V_{te} are uniform. In Fig. 3 the blade is shown operating at its design flow conditions:

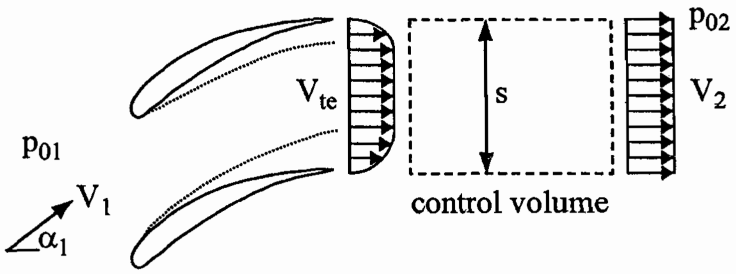

Fig. 3

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(i) Explain why, at the trailing edge, the momentum thickness θ_{ss} of the blade suction surface boundary layer would be expected to be larger than the momentum thickness θ_{ps} of the pressure surface boundary layer. [10%]

(ii) The flow at the trailing edge plane of the blade has non-uniform stagnation pressure. What mechanism is responsible for converting it into a uniform flow far downstream with stagnation pressure p_{02} ? [5%]

(iii) By applying an appropriate conservation law to the flow through the control volume identified in Fig. 3, show the following:

$$\dot{m}V_{\text{te}} - \rho V_{\text{te}}^2(\theta_{\text{ss}} + \theta_{\text{ps}}) + s\,p_{\text{te}} = \dot{m}V_2 + s\,p_2$$

where the mass flow rate through the stator blade per unit height is $\dot{m}$ and the velocity and pressure at the far downstream location are V_2 and p_2 respectively. [15%]

(iv) Assuming that $V_2 \approx V_{te}$ show that the stagnation pressure loss coefficient Y_P for the compressor stator can be approximated by:

$$Y_{\text{P}} \approx \frac{2(\theta_{\text{ss}} + \theta_{\text{ps}})}{s}\cos^2\alpha_1$$

Under what conditions would it be reasonable to assume that $V_2 \approx V_{te}$? [20%]

8 The cross-section of a radial inflow turbine stage is shown in Fig. 4. The mass flow rate through the turbine is 1.3 kgs^{-1}. At inlet to the nozzle-ring there is no swirl and the stagnation temperature and stagnation pressure are $T_{01} = 1400$ K and $p_{01} = 12.4$ bar respectively. The tip radius of the rotor is $r_2 = 80$ mm and, at that location, in the absolute frame the flow is at 70° to the radial direction, the Mach number is unity and the absolute stagnation pressure is $p_{02} = 12.0$ bar. The working fluid has the same properties as air, with $\gamma=1.4$, $R = 287$ Jkg$^{-1}K^{-1}$ and $c_p = 1005$ Jkg$^{-1}K^{-1}$

(a) Calculate the stagnation pressure loss coefficient for the nozzle-ring, and the torque on the nozzle-ring. [20%]

(b) Explain why it is usual for the rotor blade at the inlet to be radial. Calculate the required operating speed of the radial inflow turbine so that there is no incidence on to the rotor. Sketch and clearly label the rotor inlet velocity triangle. [20%]

(c) At the exit plane of the rotor there is no swirl in the absolute frame.

(i) Explain why this is a good design choice for the last turbine in a turbomachine. Calculate the power output. [20%]

(ii) If the overall total-to-total isentropic efficiency of the radial inflow turbine stage is 90% calculate the absolute stagnation pressure at rotor exit. [20%]

(d) At the exit of the rotor the hub radius is 12 mm and the tip radius is 40 mm. Calculate the axial velocity of the exit flow. Sketch and clearly label the rotor outlet velocity triangle. [20%]

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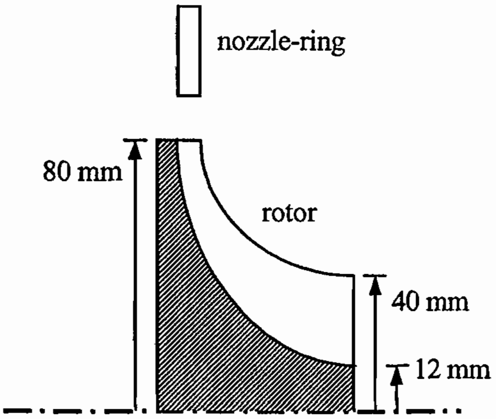

Fig. 4

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