ENGINEERING TRIPOS PART IIA

Thursday 8 May 2008 2:30–4:00

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A fuel rod in a nuclear reactor consists of an inner core of radius r_a, generating heat uniformly at a rate of α $\mathrm{W m^{-3}}$. This core is contained within a case of outer radius r_b, with radial fins of thickness t. The case and the fins are made of the same material. The fuel rod assembly is sufficiently long so that the heat conduction occurs only in the radial direction. A cross–sectional view of this arrangement is shown in Fig. 1. The thermal conductivities of the core and the case are k_1 and k_2 respectively. The fuel rod assembly is in steady state by exchanging heat with the surrounding at temperature T_∞ through the fins.

(a) By considering the energy balance across suitable control volumes in the fuel core and in the case, show that the temperatures at $r = r_b$ and $r = 0$ are related by [40%]

$$T(0) - T(r_b) = \frac{\alpha r_a^2}{2}\left[\frac{1}{2k_1} + \frac{1}{k_2}\ln\left(\frac{r_b}{r_a}\right)\right].$$

(b) The fins extend far enough to have their tip temperatures as T_∞ and the convective heat transfer coefficient is h. The heat loss from regions between the fins in the outer case is negligible. Derive an expression relating $T(0)$ and T_∞ in terms of t, h, k_1, k_2, r_a, r_b and α. [45%]

(c) Sketch the temperature distribution in the fuel rod, between the centre and the tip of a fin for $k_1 >> k_2$ and $k_1 << k_2$. [15%]

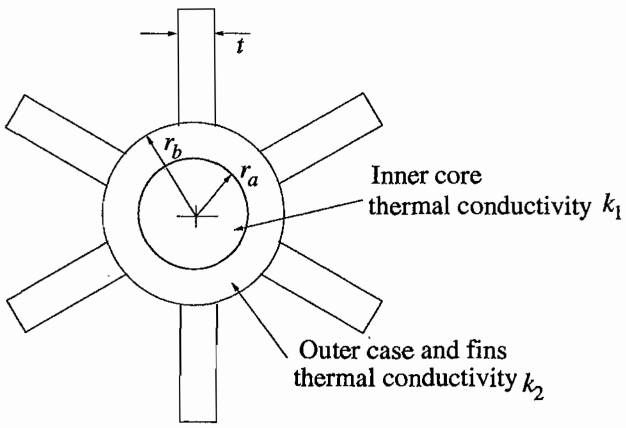

Fig. 1

2 A cylindrical radiant heating element is located at the centroid of an enclosure whose cross–section is an equilateral triangle as shown in Fig. 2. This arrangement can be considered to be a 2-D system. Two walls of the enclosure are at the same temperature and thus they can be considered to be part of a single surface.

(a) Define the term "view factor". State two key properties of view factors.

(i) Show that the view factor, F_{1e}, from wall 1 to the element is $\frac{\pi}{30}$;

(ii) What is the view factor, F_{2e}, from wall 2 to the heating element?

(iii) What is the view factor, F_{12}, from wall 1 to wall 2? [50%]

(b) Describe how a grey body differs from a black body. [10%]

(c) Consider the walls of the enclosure as a black body. The heating element is a grey body with an emissivity of 0.8 and it is at temperature 1200 K. Wall 1 and wall 2 are at 900 K and 700 K respectively.

(i) Draw the equivalent resistance circuit;

(ii) Calculate the power per unit length radiated by the heating element. [40%]

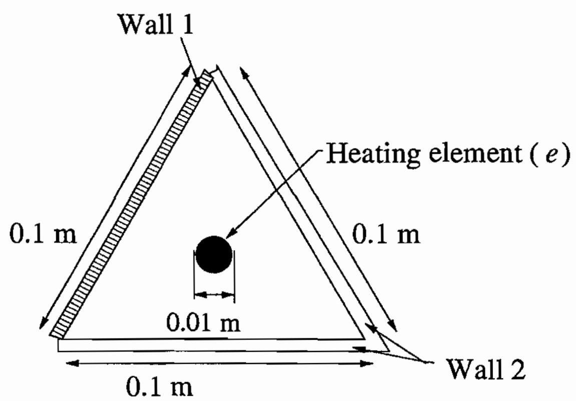

Fig. 2

(TURN OVER

3 The arrangement shown in Fig. 3 is used to produce a hot liquid at temperature 353 K and mass flow rate of $\dot{m}$. The inner tube is of length L_i and diameter D_i and it is surrounded by an electric heater having a uniform volumetric heat generation rate, $\dot{Q}$. This arrangement gives a constant surface heat flux $\dot{q}_s$ to heat the liquid. The flow is fully developed from x_o.

(a) Considering the energy balance in an appropriate control volume, deduce a differential equation for the variation of bulk mean temperature T_b of the liquid with x. Integrating this equation, show that

$$T_{b,x} = T_{b,o} + \left(\frac{\pi D_i}{\dot{m} c_p}\right) \dot{q}_s \left(x - x_o\right),$$

where c_p is the specific heat capcity of the liquid. [25%]

(b) Show that the surface heat flux is $\dot{q}_s = \frac{\dot{Q} D_i}{4}\left[\left(\frac{D_o}{D_i}\right)^2 - 1\right]$. [10%]

(c) For the given flow conditions, the Nusselt number based on D_i is 4.4. Take $\dot{m} = 0.05$ kgs^{-1}, $c_p = 4.7$ kJkg^{-1}K^{-1}, $D_i = 0.03$ m, $D_o = 0.06$ m, $T_{b,o} = 298$ K and the thermal conductivity of the liquid as 0.682 Wm^{-1}K^{-1}. If the maximum metal temperature is 400 K [65%]

(i) Sketch the variation of T_b and the surface temperature T_s with x;

(ii) Calculate the surface heat flux required to achieve $T_{b,L} = 353$ K;

(iii) Calculate the length L required to obtain $T_{b,L} = 353$K;

(iv) Determine the power of the heating element in kW and $\dot{Q}$.

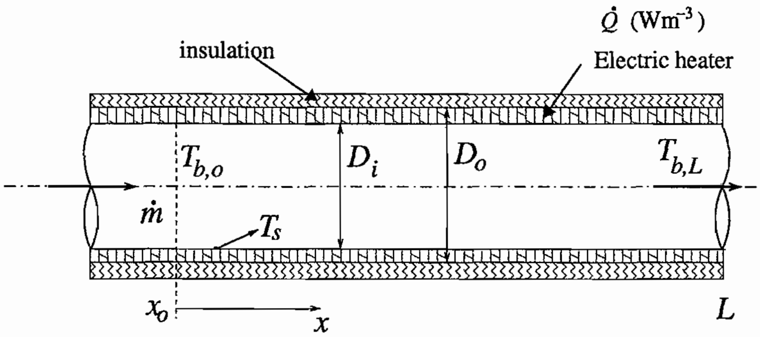

Fig. 3

4 A helium–cadmium laser lamp used in a copying machine contains helium at a constant pressure of 500 Pa supplied from a reservoir. The lamp is made of glass tube having a wall thickness of 1.5×10^{-3} m, a volume of 1.5×10^{-4} m^3 and has a surface area of 5.5×10^{-2} m^2. The glass wall reaches a temperature of 388 K and the helium attains a temperature of 473 K during the normal operating condition. The solubility and diffusion coefficients for the helium–glass system are given respectively by

$$\mathscr{S} = -1.19 \times 10^{-3} + 3 \times 10^{-5} T \quad \text{and}$$

$$\mathscr{D} = 1.4 \times 10^{-8} \exp(-3280/T) \ \ \mathrm{m^2/s},$$

where T is temperature in K. Neglect the glass tube curvature in the analysis and take the helium mass concentration in the glass wall as $\rho_{He} = \mathscr{S}\rho$, where ρ is the helium density inside the lamp. Treat the helium as an ideal gas.

(a) Draw an equivalent resistance circuit for the diffusive flux of helium through the tube wall and calculate the resistance. [10%]

(b) Calculate the helium leak rate because of its diffusion through the tube wall. [30%]

(c) Show that the change in helium mass in the lamp is linear with time. Calculate the time required to lose 2% of the initial helium mass in the lamp. [40%]

(d) How can the helium leak rate be reduced? [20%]

END OF PAPER