ENGINEERING TRIPOS PART IIA

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9 May 2008 2.30 to 4.00

Module 3C5

DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
3C5 Dynamics and 3C6 Vibration (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Seven identical uniform solid cubes of mass m and side a are assembled to form a larger body of mass $7m$, two views of which are shown in Fig. 1(a) and Fig. 1(b). All seven cubes share a vertex at O and the vertex labelled P is opposite the "missing" cube.

(a) Explain clearly, and with sketches, why the body is an "AAC" body and why the axis OP is a principal axis. [25%]

(b) Find the moment of inertia of the body about OP. [25%]

(c) Locate the centre of mass G of the body. [25%]

(d) The body is now spun "fast" like a top on a horizontal table, as shown in Fig. 1(c). Find the rate of precession in terms of a, the spin rate ω and the acceleration due to gravity g. [25%]

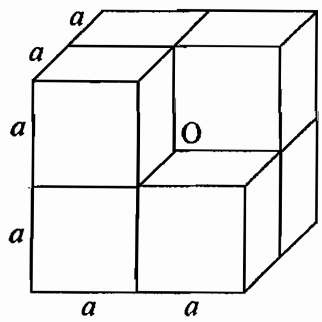

Fig. 1(a)

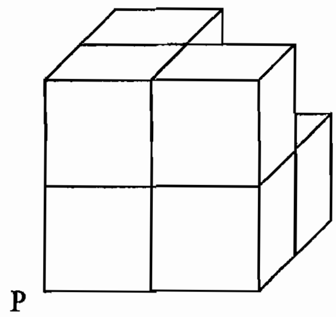

Fig. 1 (b)

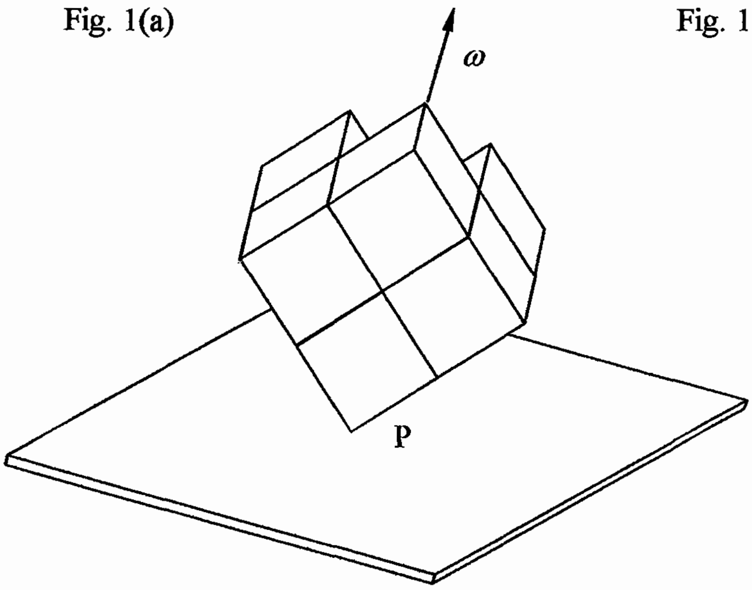

Fig. 1(c)

2 A schematic of a gyrocompass is shown in Fig. 2. The rotor spins "fast" on an axis fixed in a gimbal. The gimbal itself turns on a fixed vertical axis. A gimbal-fixed reference frame **i**, **j**, **k** is aligned with **k** along the rotor spin axis and **j** vertical. The angle between **k** and true North is θ. The angular velocity of the Earth is Ω and the gyrocompass is located at latitude λ.

(a) Show that the component of the Earth's angular velocity resolved in the **i** direction is $-\Omega \cos\lambda \sin\theta$ and find the **j** and **k** components. [30%]

(b) By using the Gyroscope Equations or otherwise show that the only stable steady-state orientation for the gyrocompass is with **k** pointing North. [30%]

(c) Find the frequency of small oscillations of the gyrocompass and explain the importance of damping in a practical device. [40%]

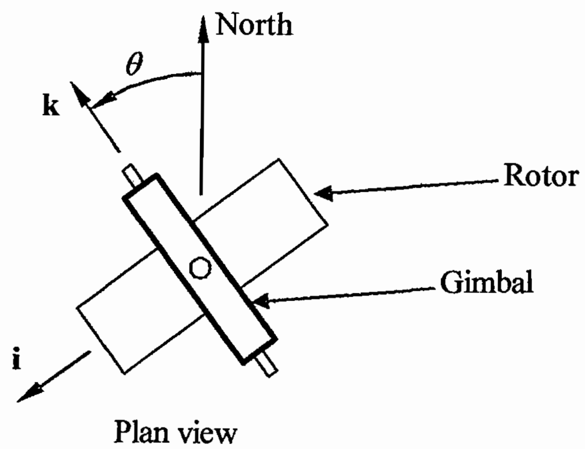

Fig. 2

3 A ball modelled as a solid uniform sphere of mass m and radius a is rolling without slip inside a rigid cylinder. The axis of the cylinder is vertical. A particular steady-state motion is observed where the centre of the ball moves on a horizontal circular path at an angular velocity Ω, as shown in Fig. 3. The non-body-fixed reference frame $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ shown in the figure rotates at $\Omega\mathbf{k}$. The angular velocity of the ball is $\omega_1\mathbf{i}+\omega_2\mathbf{j}+\omega_3\mathbf{k}$.

(a) Use a no-slip condition to show that $\omega_2 = 0$ and find an expression for ω_3 in terms of Ω, a and R. [25%]

(b) Find expressions for:

(i) the couple $\mathbf{Q}$ acting on the ball;

(ii) the moment of momentum $\mathbf{h}$ of the ball. [25%]

(c) By considering $\mathbf{Q}=\dot{\mathbf{h}}$ find the steady state value of ω_1 needed to sustain the motion. [25%]

(d) Use a free-body-diagram of the ball to show how the ball is held up against gravity. [25%]

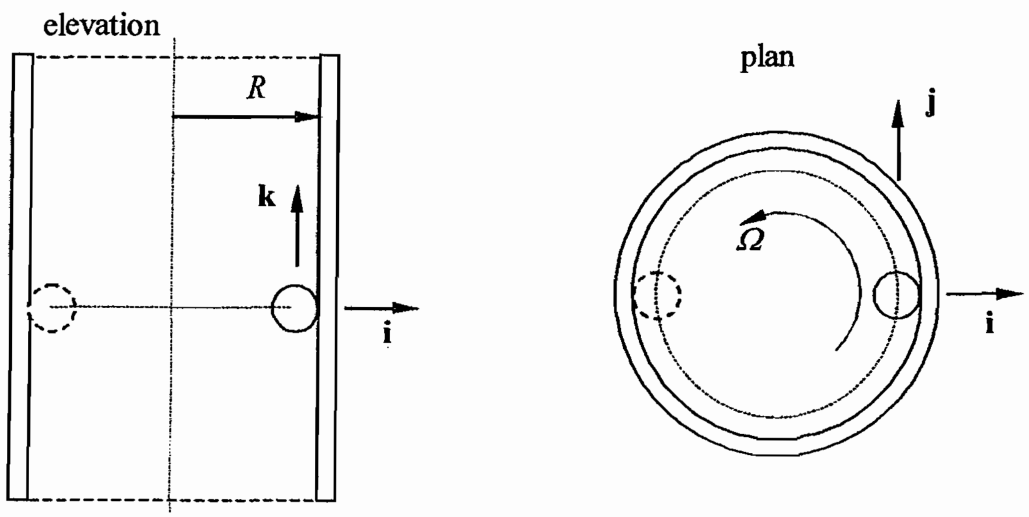

Fig. 3

4 A simplified model of a helicopter rotor blade is shown in Fig. 4. The blade rotates in flap β about a flap hinge , which is offset from the vertical rotor axis by a massless rod of length a. The engine produces a torque Q about the rotor axis, and the rotation angle of the blade about this axis is ψ. The rotor blade is of length L and has uniform mass per unit length m.

(a) Show that the kinetic energy of the rotor blade is

$$T = \frac{1}{6} m L^3 \dot{\beta}^2 + \left(\frac{1}{2} m L a^2 + \frac{1}{2} m L^2 a \cos\beta + \frac{1}{6} m L^3 \cos^2\beta \right) \dot{\psi}^2 .$$ [25%]

(b) Derive expressions for the generalized momenta in β and ψ and discuss the physical meaning of these results. [20%]

(c) By using Lagrange's equation, derive the non-linear equations of motion in β and ψ. Include the effects of gravity, taking the gravitational acceleration to be g, but exclude the aerodynamic forces acting on the blade. [30%]

(d) If the applied torque Q is controlled to enforce a constant rotation rate $\dot{\psi}$, show that the natural frequency ω_n of small oscillations in flap (with $|\beta| \ll 1$) is given by

$$\omega_n = \left[1 + \frac{3a}{2L} \right]^{1/2} \dot{\psi} .$$ [25%]

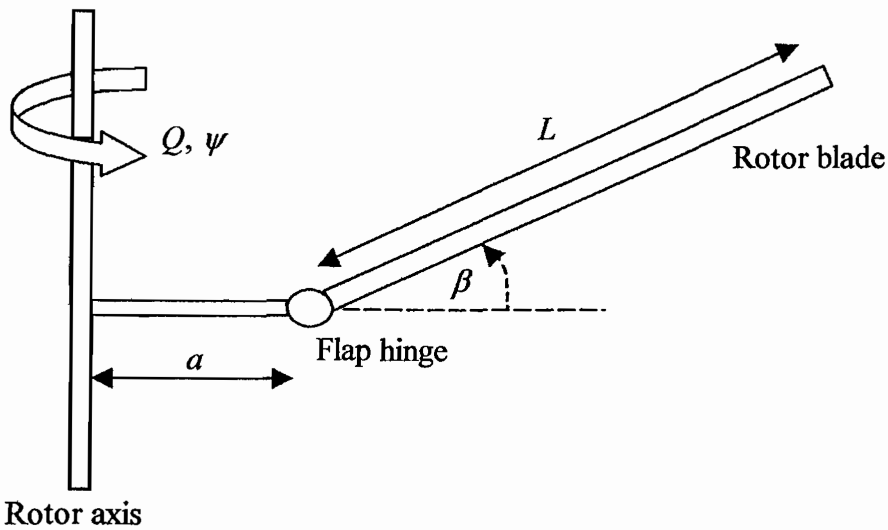

Fig. 4

(TURN OVER

5 A model of a two storey building is shown in Fig. 5. The masses of the floors are represented by point masses M, which are supported on two massless rigid columns which are each of length L. The model has two degrees of freedom, θ_1 and θ_2, representing the rotations of the columns, and rotational springs of stiffness λ are inserted at each of the two rotational joints.

(a) Derive expressions for the potential and kinetic energies of the system, including the gravitational potential energy of the two floor masses. [25%]

(b) By using Lagrange's equation, show that the equations of motion which govern small amplitude vibrations are

$$ML^2\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix}+\begin{pmatrix}2\lambda-2MLg & -\lambda\\ -\lambda & \lambda-MLg\end{pmatrix}\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}.$$ [30%]

(c) Show that the lowest natural frequency is zero if $MLg/\lambda=1-1/\sqrt{2}$. What will happen to the system if $MLg/\lambda>1-1/\sqrt{2}$? [20%]

(d) For the case where $MLg/\lambda\ll 1$, so that the effects of gravity can be neglected, calculate the natural frequencies and mode shapes of the system. [25%]

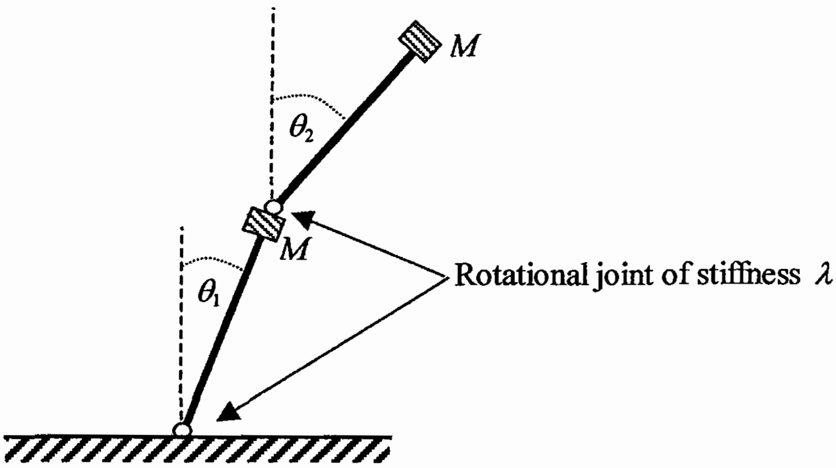

Fig. 5

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