ENGINEERING TRIPOS PART IIA

Thursday 8 May 2008 9 to 10.30

Module 3C6

VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
3C6 data sheet (6 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 4

1 A uniform beam of length L, mass per unit length m and bending stiffness EI can undergo small transverse bending vibration in one plane, with displacement $w(x,t)$.

(a) For the case where the beam has clamped boundary conditions at both ends, show that the natural frequencies ω are determined by the solutions of the equation

$$\cos\alpha L \cosh\alpha L = 1$$

where

$$\alpha^4 = \frac{m\omega^2}{EI}.$$ [30%]

(b) Obtain a corresponding equation for the natural frequencies when the beam has a clamped boundary at $x = 0$ but is freely pinned to a fixed abutment at $x = L$. [35%]

(c) Sketch graphical solutions to both equations you have derived, and hence show that the natural frequencies of the clamped-clamped beam *interlace* those of the clamped-pinned beam, in the sense that there is exactly one solution of the equation from (a) between each pair of solutions of the equation from (b). [35%]

2 A stretched string with tension P and mass per unit length m has length $2L$, and is fixed at the two ends $x = \pm L$. At the centre of the string ($x = 0$) a transverse spring of stiffness K is fixed to the string, the other end of the spring being attached to a rigid base. The string can execute small vibrations in the plane in which the spring acts, with transverse displacement $w(x,t)$.

(a) Explain carefully why each vibration mode of this constrained string must be either symmetric or antisymmetric. Sketch the first two antisymmetric modes and obtain a formula for the natural frequencies of all the antisymmetric modes. [30%]

(b) Show that at the point $x = 0$ the appropriate boundary condition is

$$P\left[\frac{\partial w}{\partial x}\right]_{0-}^{0+} = Kw .$$

Show that the natural frequencies ω of symmetric modes satisfy the equation

$$\tan\frac{\omega L}{c} = -\frac{2P\omega}{Kc}$$

where $c = \sqrt{P/m}$. [35%]

(c) With the aid of a graphical construction for the equation found in (b), explain the relationship between the symmetric and antisymmetric natural frequencies. Consider the limiting cases (i) $K \to 0$ and (ii) $K \to \infty$ and give a physical explanation for what happens to the natural frequencies in each case. [35%]

3 Figure 1 shows a rigid, uniform, thin bar of mass M and length L, supported by two springs of stiffness k. A point mass m is attached to the centre of the bar by another spring of stiffness k. The bar can move in the vertical direction and rotate in the vertical plane, while the point mass can move in the vertical direction only. The displacements of the ends of the bar and the point mass from equilibrium are denoted x_1, x_2 and x_3 as shown.

(a) Write an expression for the kinetic energy and show that the potential energy is

$$V = \frac{k}{2}\left(\frac{5}{4}x_1^2 + \frac{5}{4}x_2^2 + x_3^2 + \frac{x_1 x_2}{2} - x_1 x_3 - x_2 x_3\right).$$ [20%]

(b) Sketch the mode shapes and write down estimates of the natural frequencies for the cases (i) $m/M << 1$; and (ii) $m/M >> 1$.

For each case state which one of these frequencies is exact. [40%]

(c) For the case in (b)(ii) use Rayleigh's quotient with the mode shape $(x_1, x_2, x_3)^T = (1, 1, \alpha)^T$ to find an exact expression for the remaining two natural frequencies. Compare these frequencies and corresponding mode shapes with your estimates in part (b). [40%]

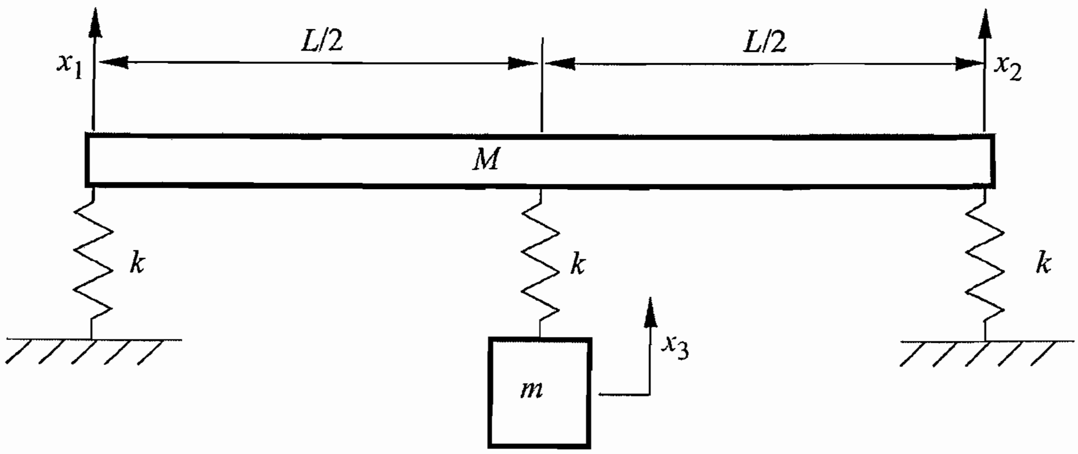

Fig. 1

4 Four equal masses m can move along a horizontal line, constrained by five equal springs of stiffness k as shown in Fig. 2. Displacements of the masses from their equilibrium positions are denoted x_1, x_2, x_3 and x_4, as shown.

(a) Sketch the mode shapes you would expect the system to have in order of increasing frequency. [20%]

(b) Sketch the expected form of the magnitude of the transfer function (on a logarithmic vertical scale) for the velocity of the second mass in response to a sinusoidal force applied to the fourth mass. (Assume a small amount of modal damping for the purposes of sketching this curve.) [20%]

(c) Calculate the mass and stiffness matrices for small vibration of this system. [20%]

(d) Explain why the lowest frequency mode must have a mode shape of the form $(1,\alpha,\alpha,1)^T$, where α is a constant. Find the value of α and hence the lowest natural frequency by minimizing Rayleigh's quotient. [30%]

(e) Are there any other modes found exactly by this analysis? Explain your answer. [10%]

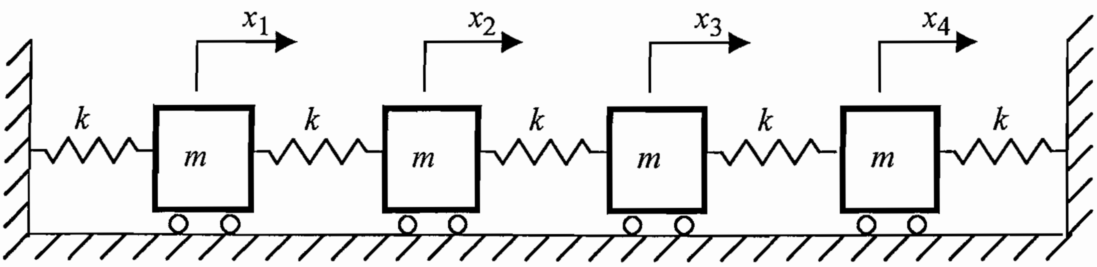

Fig. 2

END OF PAPER