## ENGINEERING TRIPOS PART IIA

Wednesday 7 May 2008 9 to 10.30

Module 3D3

## STRUCTURAL MATERIALS AND DESIGN

Answer not more than three questions.

All questions carry the same number of marks.

The **approximate** number of marks allocated to each part of a question is indicated in the right margin.

Attachments: Special datasheets (10 pages).

STATIONERY REQUIREMENTS Single-sided script paper SPECIAL REQUIREMENTS Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator. 1 (a) A fixed end beam of span L in linear-elastic/perfectly-plastic material with equal tensile and compressive yield strength  $\sigma_f$  is to carry, without collapsing, a uniformly distributed total load  $\gamma_f W$  using  $\sigma_f$  to the full. The central deflection of the beam (one-fifth of that for a simply-supported beam) under load cW is not to exceed L/F. Derive a formula for the maximum span-to-depth ratio, L/d. Explain why L/d plays an important role in the initial sizing of beams.

(b) Figure 1 shows a schematic plan view of a typical symmetrical 10 m square bay ABCD, plus parts of identical and adjacent bays, of an extensive floor structure supported on a grid of columns. The structure is to carry uniformly and everywhere a downward load of 23 kN/m<sup>2</sup> at the ultimate limit state, including the self-weight and partial safety factors. The upper part of the structure is a continuous floor slab. In regions such as AWZDZ'W'A, this slab may be regarded as acting as a series of beam strips running parallel to W'W or the equivalent, whilst regions such as WXYZ may be regarded as two sets of beam strips running parallel to WX and WZ, respectively. These strips (not shown in the figure) are supported by light main beams such as WX and WZ, and light cantilevers such as AW and BX. The joints such as W can transmit vertical forces only and no couples; while the joints at points such as A, B *etc.* can transmit couples as well as forces.

(i) Regarding the beam strips as simply supported on the main beams and cantilevers, determine the maximum bending moment and shear force in a typical main beam such as WX, the maximum bending moment in a typical cantilever such as AW, and the total downward force transmitted to each column at a point such as A.

(ii) Evaluate the maximum moments per unit width in the beam strips and discuss why it is desirable for the distance WX to be approximately  $\sqrt{2}$  times the distance W'W particularly if the beam strips are to be treated as continuous over the main beams. Discuss briefly whether the beam strips may be considered as being continuous over the cantilevers?

[25%]

[35%]

(contd.)

[25%]

(c) In which circumstances may simplified equilibrium systems of the above kind be used in the design of structures. [15%]
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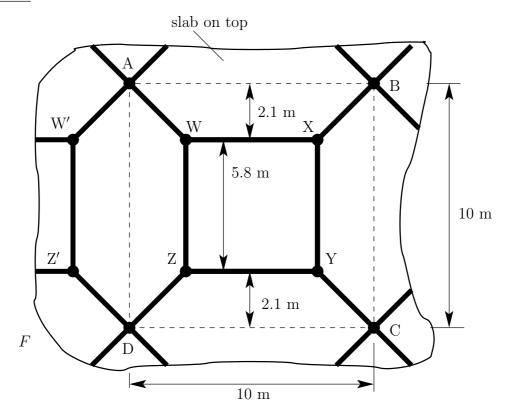


Fig. 1

It is proposed to weld from S355 steel plate a uniform I-section girder of overall depth 1.2 m, to span 25 m in a simply supported manner. The two flanges would be 450 mm wide and 25 mm thick, and the web thickness would be 15 mm. The material partial safety factor  $\gamma$  is to be 1.05.

(a) Assuming fully plastic behaviour show that a uniformly distributed load of approximately 1970 kN at ULS (including any specified load factors) would exhaust the bending resistance at mid-span.

(b) Discuss what calculations would be required for regions near each support where there would be a bearing below the bottom flange, in order to demonstrate that this loading could indeed be carried at ULS and suggest what stiffener(s), if any, would be required. What further checks might be needed at mid-span and what main force(s) would be considered in designing the welds between the plates?

(c) Consider the top flange of the beam as a column under axial compression only, and effectively pin-ended at points where the flange has lateral supports. Using the steel data sheet, estimate the maximum spacing of lateral supports needed for this flange at mid-span if only 90% of the full yield stress needs to be attained at mid-span.

(d) It is suggested that it would be better to build a rectangular box of the same overall dimensions and flange thickness but with two web plates each of thickness 7.5 mm, resulting in an increased second moment of area  $I_{yy}$ , negligible warping factor  $C_w$  and a basic torsion constant increased to approximately  $3 \times 10^5$  cm<sup>4</sup>. Estimate what maximum moment at mid-span such a beam could be expected to attain at ULS without any lateral supports. [25%]

[25%]

[25%]

3 A simply supported reinforced concrete beam has a span of 10 m. It is required to carry a central vertical force of 50 kN and its self-weight. The concrete cube strength is 30 MPa and a cross-section of width 300 mm and depth 600 mm is specified with 50 mm cover. The reinforcement bars have diameter 30 mm and a yield stress of 460 MPa. The partial safety factors for concrete and steel are 1.5 and 1.15 respectively, and the load factors for dead and live loads are 1.4 and 1.6 respectively.

(a) Sketch the bending moment and shear force diagrams according to the design loads, indicating salient values.[20%]

(b) Choose a layout of longitudinal reinforcement ensuring that spacing requirements are satisfied. [35%]

(c) Determine whether or not shear reinforcement is required. [20%]

(d) The live load is increased to 100 kN but the original cross-section is retained in the design. Choose another layout of longitudinal reinforcement. [25%] 4 (a) A thin-walled box section is made from a high strength carbon fibre/epoxy resin. The overall height and width are 150 mm and 100 mm, and the wall thickness everywhere is 5 mm. The epoxy matrix has  $E_m = 5$  GPa and the carbon fibres, of volume fraction 0.6, have  $E_f = 230$  GPa.

(i) Calculate the equivalent Young's modulus and the bending stiffnessof the composite cross-section. [15%]

(ii) If the resulting composite has limiting stresses  $\sigma_{LT} = 600$  MPa and  $\sigma_{LC} = 500$  MPa , calculate the maximum moment capacity of the section. Compare your result to an identical steel box section with yield stress 355 MPa . [20%]

(b) The box section in (a) is used as a cantilever beam of length 0.5 m, which carries a vertical tip force of 50 kN and an axial torque of 10 kNm. The corresponding tip deflection and rotation must be less than 5 mm and 0.03 rad, respectively.

(i) Find the minimum value of  $E_{xx}$  and  $G_{xy}$  for the beam in which the deformation limits are just satisfied. [15%]

(ii) One fifth of the plies are oriented at 90° but the relative proportions of 0° plies and  $\pm 45^{\circ}$  plies are not yet known. Construct a table showing the variation of  $E_{xx}$ ,  $G_{xy}$ , tip deflection and rotation when the proportion of 0° plies is 0%, 20%, 40%, 60% and 80%. Therefore estimate a viable range for  $E_{xx}$  and  $G_{xy}$  that satisfies the stiffness requirements. [30%]

(iii) Select a preferred lay-up for the results found in (b.ii). Compare the maximum longitudinal strain under loading to the permissible strain: what do you conclude? [20%]

## END OF PAPER