ENGINEERING TRIPOS PART IIA

Wednesday 7 May 2008 2.30 to 4

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Data sheet for Question 2.

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Two uniform cantilever beams have cross-sections as schematically shown in Fig. 1(a). Each beam is subjected to a vertical load acting downwards at its free end and applied at the point marked with X.

(i) Describe qualitatively the response of each beam, paying particular attention to the direction of the tip motion, and any rotation that might occur. Illustrate qualitatively the deformations of the tip cross sections by drawing it before and after the deformation. [30%]

(ii) Explain whether or not a warping restraint would make a significant difference. [5%]

(b) The cross section shown in Fig. 1(b) has been produced by drilling a hole in a square cross section.

(i) Show that the second moments of area of a circular cross section with radius R are $I = \frac{\pi R^4}{4}$. [20%]

(ii) Determine the principal second moments of area for the cross section shown in Fig. 1(b). [45%]

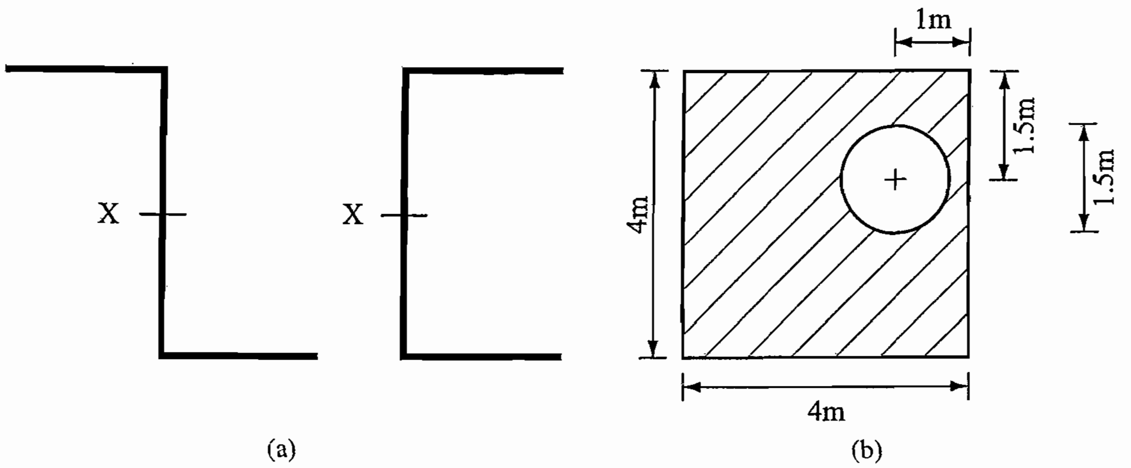

Fig. 1

(cont.

2 The structure shown in Fig. 2 consists of three beams rigidly connected at point B. Joints A and D are built-in and joint C is pinned. All three beams have flexural stiffness $EI = 5 \cdot 10^4\ kNm^2$. The axial stiffness for the beams AB and BC is $EA = 6 \cdot 10^6\ kN$ and for the beam BD the axial stiffness is infinite. The beam AB is loaded with uniform loading $w = 10\ kN/m$.

(a) Determine the number of degrees of kinematical and statical indeterminacy. [10%]

(b) Find the bending moments in the three beams at sections adjacent to point B. (Data sheet is attached.) [70%]

(c) For each of the reactions at support D sketch the influence line corresponding to a transverse rolling point load on AC. [20%]

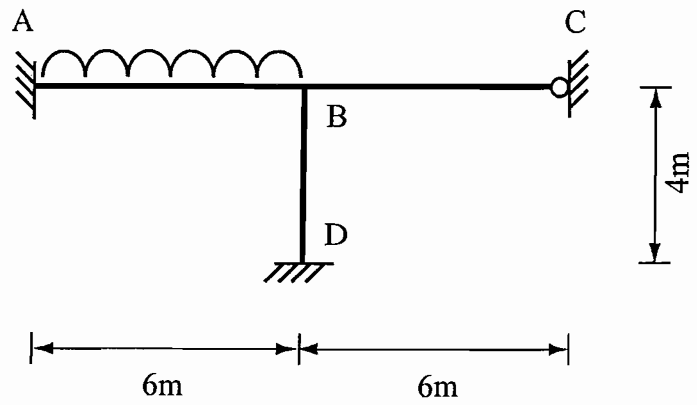

Fig. 2

3 (a) An engineer claims that the mathematical description of any statical elastic stability problem will result in an eigenvalue problem whose eigenvalues are the buckling loads. Explain the limitations of that perspective and give a more general alternative description. Explain the physical meaning of the eigenvalues in your alternative description. [20%]

(b) A beam on an elastic foundation has simply-supported ends a distance L apart and carries a compressive axial load P. The total potential energy function $\Pi(w)$ of the system is given by

$$\Pi(w) = \frac{1}{2}\int_0^L \left\{ EI\left(\frac{d^2w}{dx^2}\right)^2 - P\left(\frac{dw}{dx}\right)^2 + kw^2 \right\} dx \qquad (1)$$

where w is the deflection at any point a distance x along the beam, EI is the bending stiffness of the beam and k is the stiffness of the foundation per unit length.

(i) Assuming a general deflected shape of the form

$$w(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x/L)$$

where n is an integer, determine the stiffness matrix and hence show that $\sin(n\pi x/L)$ is an eigenvector. Determine an expression for the elastic critical load for buckling in the n-th mode. [50%]

(ii) Determine the length L_n that gives the lowest critical load for the n-th mode and determine the corresponding buckling load. [30%]

4 (a) Figure 3 shows a light rigid armature attached to a pin support at B with a rotational spring whose rotational stiffness G creates a restoring moment around B of $G\theta$ when the armature rotates through θ radians. Equal weights at points C and D on the armature each apply a downward force of $W/2$. The locations of the points C and D are defined by the dimensions H and R shown.

Determine the equilibrium paths of the system, and sketch these as graphs of W as a function of θ. Determine the stability of the various equilibrium paths, and the value of W at which the system will undergo a rotational instability. [40%]

(b) A light straight rod of length L on simple supports has Euler buckling load P_E under axial compression. The same rod now has its ends rigidly welded to the armature at B and a support at A as in Fig. 4. Point B of the armature is restrained against horizontal movement by a pin-roller support.

When $L = 4H$, determine the critical ratio W/P_E at which this system will undergo a rotational instability. A graph of s and c stability functions is provided in Fig. 5. [50%]

(c) Explain briefly how you would proceed with the analysis of part (b) if the roller support at B was not present. [10%]

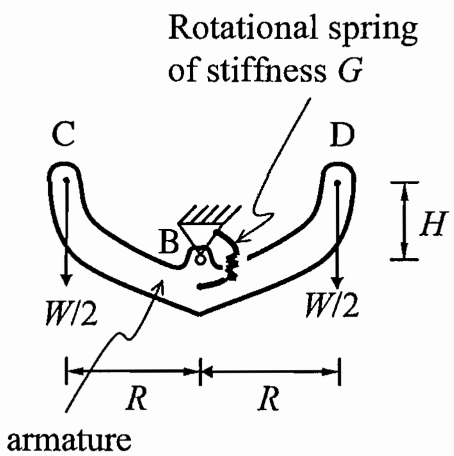

Fig. 3

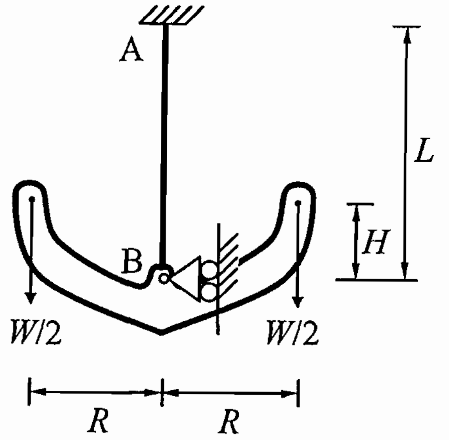

Fig. 4

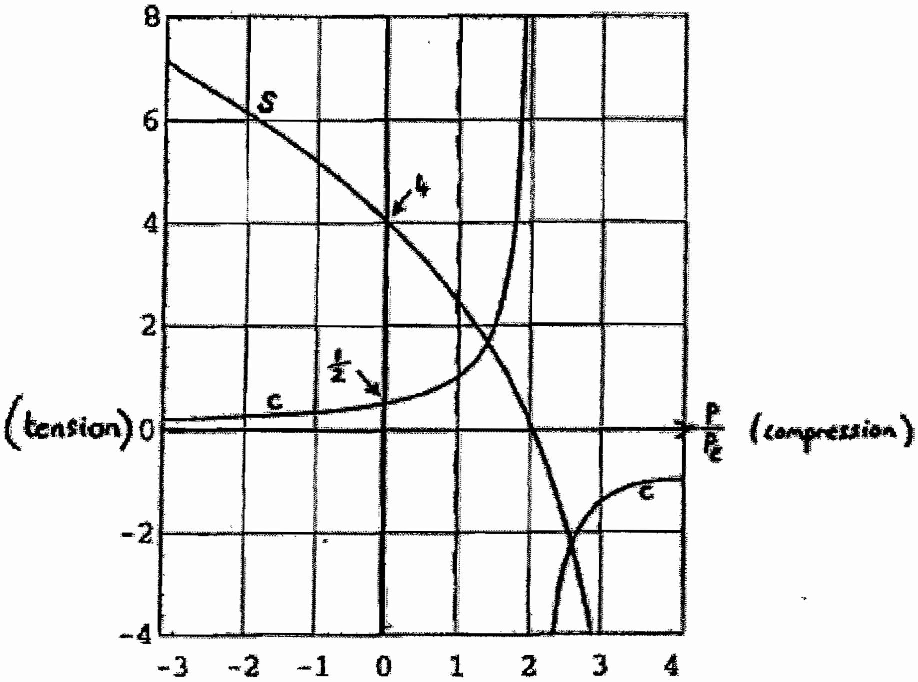

Fig. 5

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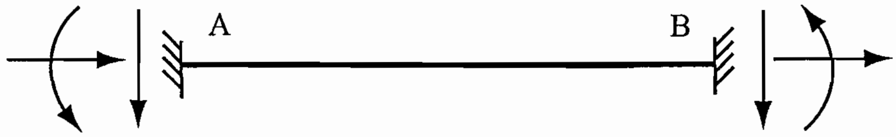
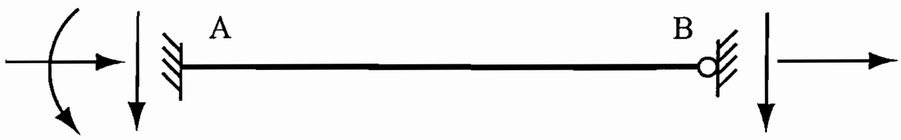
Data Sheet for Question 2: Stiffness Matrices.

Beam type I



$$\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}$$

Beam type II



$$\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}$$