ENGINEERING TRIPOS PART IIA

Monday 5 May 2008 9 to 10.30

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: Final

1 In December 2007 Porterhouse college interviewed 24 applicants for the Engineering tripos. The decision to offer a place was made on the interviewer's score alone. The admission office wants to see how closely the interview score relates to the A-level score of the candidates. The data gave the following table, where x denotes the A-level score and y the interview score.

x	y
99.22	9.00
95.22	8.50
98.89	8.00
96.33	8.00
94.44	8.00
97.71	7.50
96.22	7.50
95.56	7.50
91.44	7.50
95.44	7.00
94.22	7.00
91.89	7.00

x	y
97.22	6.50
93.33	6.50
92.00	6.50
91.78	6.50
88.67	6.50
91.33	6.00
91.11	5.50
86.89	5.50
84.78	5.50
80.22	5.00
82.50	4.50
80.11	4.50

(a) Assume the A-level score is the independent vairable and the interview score the dependent variable. Apply the least squares linear regression model $y = a + bx + \varepsilon$ to derive expressions for a and b. Use the data in the table find their values. [35%]

(b) Explain the meaning of the correlation coefficient R and calculate its value. Comment on your result. [20%]

(c) Explain what is meant by the standard error S_e. [10%]

(d) Explain why a and b are random variables. Explain the significance of their means? Calculate their variances and thus find their distributions. [35%]

Hint: You might find the following data useful:

$\sum x_i = 2206.52$, $\sum x_i^2 = 203568.55$

$\sum y_i = 161.50$, $\sum y_i^2 = 1120.75$

$\sum x_i y_i = 14983.07$, $S_e = 0.607502$

Version: Final

2 On the island of Faraway, an investor can only invest in four risky assets with expected rates of return $\{6\%, 7\%, 8\%, 9\%\}$ and covariance matrix:

$$\sigma_{ij} = \begin{pmatrix} 0.2 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.5 \end{pmatrix}$$

(a) Find the weights corresponding to the minimum-variance portfolio S consisting of these four assets and calculate the expected rate of return and variance of this portfolio. [40%]

(b) Assume, for now, that there is a riskless asset with a 5% rate of return. Find the weights of the portfolio P of risky assets, such that any efficient portfolio can be constructed as a combination of P and the riskless asset. [30%]

(c) Explain what is meant by the term *market portfolio*. Determine whether P is a market portfolio. [10%]

(d) If no riskless asset is available, explain how any portfolio on the efficient frontier can be constructed. Hence find the weights of the portfolio with the largest expected return, that lies on the efficient frontier and has no asset shorted. [20%]

Hint: You might find the following data useful:

$$(\sigma^{-1})_{ij} = \begin{pmatrix} 6.76 & -1.62 & -1.08 & -0.81 \\ -1.62 & 4.19 & -0.54 & -0.41 \\ -1.08 & -0.54 & 2.97 & -0.27 \\ -0.81 & -0.41 & -0.27 & 2.30 \end{pmatrix}$$

3 Your local district general hospital is receiving complaints from impatient patients who have been attending the eye department. The hospital managers have been called to a board meeting to decide on official waiting times for all the patients attending the eye casualty service.

All patients first see the nurse who spends on average μ_N^{-1} minutes per patient, confirming personal information. The patients then wait to be seen by the doctor who spends on average $\mu_D^{-1}(> \mu_N^{-1})$ minutes, diagnosing and treating each patient.

Assume that both service times are exponentially distributed. Further assume that the patient arrival rate is $\lambda(< \mu_D)$.

(a) Justify that we can model the patient inter-arrival time as being exponentially distributed. [10%]

(b) By considering a *birth and death* process, derive the steady state probabilities of patients waiting to see the nurse. [40%]

(c) What is the average number of patients waiting to see the nurse? [10%]

(d) What is the average time patients need to wait in order to see the nurse? [10%]

(e) What is the average time taken by a patient to be both seen by the nurse and the doctor? [20%]

(f) In order to speed up the process at which patients can been discharged, would it be better to increase the number of nurses or the number of doctors? Justify your answer. [10%]

4 Consider the Markov chain with state space $\{1, 2, 3, 4, 5, 6\}$ and transition matrix:

$$P = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

(a) Determine the communicating classes of the chain, and for each class indicate whether it is closed or not. [20%]

(b) Suppose that the chain starts in state 3; determine the probability that it is in state 6 after n transitions as $n \to \infty$. [40%]

(c) Suppose that the chain starts in state 4; determine the probability that it ever reaches state 6. [40%]

END OF PAPER