ENGINEERING TRIPOS PART IIA

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Wednesday 30 April 2008 9 to 10.30

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Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 A DC electric motor is to be controlled, with armature current $i$ as the input and motor shaft angle $\theta$ as the output. The transfer function from input to output is approximately

$$\frac{\Theta(s)}{I(s)} = \frac{F}{s(Js+B)}$$

where $J$ is the moment of inertia, $B$ is the coefficient of mechanical viscous damping, and $F$ is the torque/current coefficient. The parameters $B$, $F$ and $J$ are all positive. The motor is to be controlled by a negative-feedback system, as shown in Fig.1. The transfer function of the controller is denoted by $K(s)$.

(a) If proportional gain only were to be used, namely $K(s) = k$, sketch the root-locus diagram for $k > 0$ and interpret it. [25%]

(b) A load torque $\tau_L$ can be considered to be an additive disturbance acting at the input of the motor, as shown in Fig.1. If a demanded constant shaft angle $\theta_d$ is to be achieved exactly, despite a constant load torque, show that the controller must include integral action. [25%]

(c) In view of part (b), a 'proportional + integral' controller of the form

$$K(s) = k\frac{s+a}{s} \qquad (k > 0,\, a > 0)$$

is to be used. Sketch the root-locus diagram for this case, assuming that there is no breakaway point other than the origin. [25%]

(d) If the proportional + integral controller is used, as in part (c), show that the closed-loop system will be stable for all $k$ if [25%]

$$a < \frac{B}{J}$$

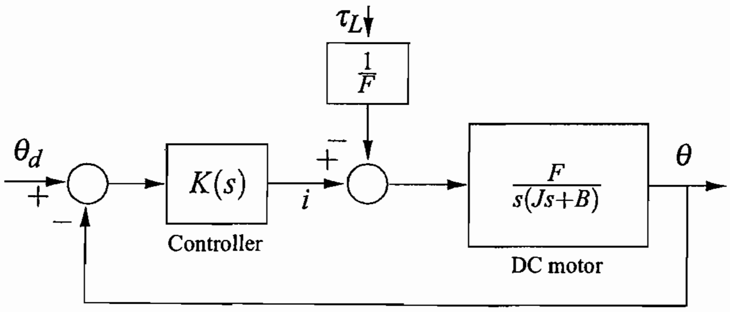


Fig. 1

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2 (a) State the test for *controllability* of a linear state-space system of the form [10%]

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Cx$$

(b) Derive a formula for the transfer function of the system given in part (a). [20%]

(c) Consider the system given in part (a), with the following parameter matrices:

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ b \end{bmatrix}, \qquad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

Verify that this system is not controllable for two values of $b$, and find those values. [30%]

(d) Find the transfer function of the system defined in part (c). [10%]

Relate your answer to your answers to part (c). [30%]

3 The angle $\theta$ of one link of a robot arm is governed by the differential equation

$$J\ddot{\theta} + F\dot{\theta} = u + d$$

where $u$ is the torque produced by a motor located at a joint, $d$ is a disturbance torque which represents various unmeasured effects, and $F, J$ are positive constants.

(a) Obtain a state-space model of the link in the form

$$\dot{x} = Ax + B_1 u + B_2 d$$

using as few state variables as possible. [15%]

Is the open-loop system stable? [10%]

(b) Assuming that both the angle $\theta$ and the angular velocity $\dot{\theta}$ are measured, design a state-feedback control scheme which places both closed-loop poles at some real value $p$, using the motor torque $u$ as the control input. [20%]

Comment on how the required state feedback gains vary with $|p|$, and with the robot parameters $F, J$. [10%]

(c) In order to obtain a desired constant angle $\theta_d$ without error despite the unknown torque $d$ (in the steady state), it is necessary to introduce integral action into the controller. Explain how this can be done by introducing a new state variable, and show how the state-space model is then modified. [25%]

(d) Suppose that it is necessary to track a desired angle $\theta_d$ in the form of a ramp (*ie* of the form $\theta_d(t) = \alpha t + \beta$), without error in the steady state. Suggest how this could be done using the state-feedback framework. [20%]

4 (a) With the aid of a block diagram, explain the structure and purpose of a *state observer* for a linear system of the standard form [20%]

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

(b) If $\hat{x}(t)$ denotes the state estimate produced by an observer, and the state estimation error is $e(t) = x(t) - \hat{x}(t)$, derive a differential equation which governs the evolution of $e(t)$, and hence obtain an expression for $e(t)$ in terms of the initial error $e(0)$. [20%]

(c) A 'compartmental' model used in physiology assumes that the blood pressure $p_k$ and flow rate $q_k$ in the $k^{\text{th}}$ compartment are governed by the differential equations

$$L\dot{q}_k + Rq_k = p_{k-1} - p_k$$
$$C\dot{p}_k = q_k - q_{k+1}$$

for $k = 1, 2, \ldots, N$, where $C$, $L$ and $R$ are constants. Show how these equations can be put into state-space form, treating $p_0$ and $q_{N+1}$ as inputs. How many state variables are required? [20%]

(d) For the model of part (c) with only two compartments, namely $N = 2$, show that $p_1, p_2, q_1, q_2$ can all be estimated from measurements of $p_1$ alone, if $C = L = 1$ and $R = 2$, assuming that $p_0$ and $q_3$ are known. [20%]

(e) For the model of part (c), suppose that $p_0$ and $q_{N+1}$ are known to be constant, but are otherwise unknown. Indicate briefly how they can be estimated using an observer. (You do not need to check observability for this case.) [20%]

**END OF PAPER**