ENGINEERING TRIPOS PART IIA

Module 3C6 Examination, 2009

Answers

1 (a) $\cos\alpha L \cosh\alpha L = -1$, (b) $\omega^2 = \omega_1^2 + \dfrac{E}{12\rho}\left[\dfrac{\int_0^L 3h_0^2 \delta h \left(u_1''\right)^2 dx - \omega_1^2 \int_0^L \delta h\, u_1^2\, dx}{\int_0^L h_0\, u_1^2\, dx}\right]$.

2 (b) $\omega_n = \dfrac{1}{L}\sqrt{\dfrac{E}{\rho}}\left(n - \dfrac{1}{2}\right)\pi$, $u_n(x) = K \sin\dfrac{(n-1/2)\pi x}{L}$; $n = 1, 2, 3...$

(c) $\left.\dfrac{dg}{dt}\right|_{x=L/4} = \dfrac{2}{\rho AL}\sum_{n=1,2..}(-1)^n \sin\dfrac{(n-1/2)\pi}{4}\cos\omega_n t$; (d) $\dfrac{3L}{4}\sqrt{\dfrac{\rho}{E}}$.

3 (a) $V = \dfrac{P}{2L}\left[y_1^2 + (y_2 - y_1)^2 + (y_3 - y_2)^2 + (y_4 - y_3)^2 + y_4^2\right]$

$T = \dfrac{m}{2}\left[\dot{y}_1^2 + 2\dot{y}_2^2 + 2\dot{y}_3^2 + \dot{y}_4^2\right]$

(d) $\alpha = 1.78, -0.28$; $\omega_1^2 = 0.219\dfrac{P}{Lm}$.

4 (a) $\omega_1 = 0$, $u^{(1)} = [1 \quad 1 \quad 1]^T$; $\omega_2 = \sqrt{\dfrac{k}{m}}$, $u^{(1)} = [1 \quad 0 \quad -1]^T$;

$\omega_3 = \sqrt{\dfrac{2k}{m}}$, $u^{(1)} = [1 \quad -1 \quad 1]^T$

(b) $-\Delta$, $-180°$ (c) $\dot{y}(0^+) = \dfrac{I}{4m}$ (d) $y_2\left(\sqrt{\dfrac{m}{k}}\right) = 0.075\dfrac{I}{\sqrt{mk}}$.