

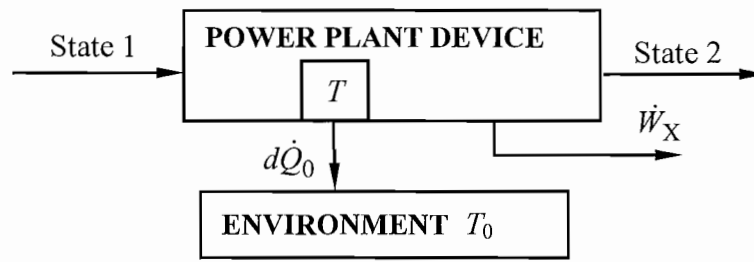
ENGINEERING TRIPOS PART IIA 2009

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

SOLUTIONS TO TRIPOS QUESTIONS

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1. (a)



Neglecting changes in KE and PE between inlet and outlet, the SFEE and SFSE are :

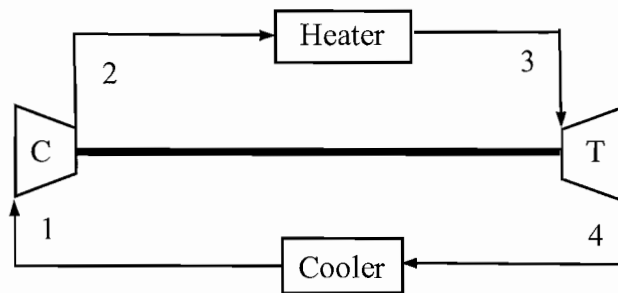
$$\dot{m}(h_2 - h_1) = -\dot{Q}_0 - \dot{W}_X = \int -d\dot{Q}_0 - \dot{W}_X$$

$$\dot{m}(s_2 - s_1) = \int -\frac{d\dot{Q}_0}{T} + \Delta\dot{S}_{irrev}$$

Multiplying the SFSE by T_0 and subtracting the SFEE gives,

$$\dot{m}[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)] = \dot{m}(e_1 - e_2) = \dot{W}_X + T_0 \Delta\dot{S}_{irrev} + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q}_0 \quad [20 \%]$$

(b)



(i) For the compressor (taking property values from Table 1 in the Tripos paper) :

$$[\dot{W}_X]_{comp} = \dot{m}(h_1 - h_2) = 0.25 \times (0.0 - 3062.1) = -765.53 \text{ kW}$$

$$[\dot{W}_{L,Q}]_{comp} = 0 \quad (\text{Compressor is adiabatic})$$

$$[\dot{W}_{L,CR}]_{comp} = \dot{m}T_0(s_2 - s_1) = 0.25 \times 290.0 \times (0.5925 - 0.0) = 42.96 \text{ kW} \quad [10 \%]$$

For the turbine :

$$[\dot{W}_X]_{turb} = \dot{m}(h_3 - h_4) = 0.25 \times (7577.4 - 2335.5) = 1310.48 \text{ kW}$$

$$[\dot{W}_{L,Q}]_{turb} = 0 \quad (\text{Turbine is adiabatic})$$

$$[\dot{W}_{L,CR}]_{turb} = \dot{m}T_0(s_4 - s_3) = 0.25 \times 290.0 \times (4.7603 - 4.1604) = 43.49 \text{ kW} \quad [10 \%]$$

(ii) For the cooler, the SFSE can be written (using the SFEE and $Tds = dh - vdp$),

$$\dot{m}(s_1 - s_4) = \int_4^1 -\frac{d\dot{Q}_0}{T} + \Delta\dot{S}_{irrev} = \int_4^1 \frac{\dot{m}dh}{T} + \Delta\dot{S}_{irrev} = \int_4^1 \dot{m} \left(ds + \frac{v}{T} dp \right) + \Delta\dot{S}_{irrev}$$

For a perfect or semi-perfect (but not imperfect) gas $pv = RT$ and hence,

$$\Delta\dot{S}_{irrev} = -\int_4^1 \dot{m} \frac{v}{T} dp = -\int_4^1 \dot{m} R \frac{dp}{p} = \dot{m} R \ln \left(\frac{p_4}{p_1} \right)$$

For helium $R = 2.08$ kJ/kg K. Hence,

$$[\dot{W}_{L,CR}]_{cool} = T_0 \Delta\dot{S}_{irrev} = 0.25 \times 2.08 \times 290.0 \times \ln(1.05) = 7.36 \text{ kW}$$

$$[\dot{W}_X]_{cool} = 0 \quad (\text{No shaft power})$$

$$\begin{aligned} [\dot{W}_{L,Q}]_{cool} &= \dot{m}(e_4 - e_1) - T_0 \Delta\dot{S}_{irrev} \\ &= 0.25 \times [(2335.5 - 290 \times 4.7603) - (0)] - 7.36 = 231.39 \text{ kW} \end{aligned} \quad [30 \%]$$

$$\begin{aligned} \text{(iii) Power plus lost power} &= [\dot{W}_X]_{turb} + [\dot{W}_X]_{comp} + \\ &\quad [\dot{W}_{L,CR}]_{turb} + [\dot{W}_{L,CR}]_{comp} + [\dot{W}_{L,CR}]_{cool} + [\dot{W}_{L,Q}]_{cool} \\ &= 1310.48 - 765.53 + 43.49 + 42.96 + 7.36 + 231.39 = 870.15 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Exergy supply rate to heater} &= \dot{m}(e_3 - e_2) \\ &= 0.25 \times [(7577.4 - 290 \times 4.1604) - (3062.1 - 290 \times 0.5925)] = 870.15 \text{ kW} \end{aligned} \quad [15 \%]$$

$$\text{(iv) Rate of heat supply} = \dot{m}(h_3 - h_2) = 0.25 \times (7577.4 - 3062.1) = 1128.83 \text{ kW}$$

$$\text{Thermal efficiency} = \frac{[\dot{W}_X]_{turb} + [\dot{W}_X]_{comp}}{\dot{m}(h_3 - h_2)} = \frac{1310.48 - 765.53}{1128.83} = 0.483 \quad (48.3 \%)$$

$$\text{Rational efficiency} = \frac{[\dot{W}_X]_{turb} + [\dot{W}_X]_{comp}}{\dot{m}(e_3 - e_2)} = \frac{1310.48 - 765.53}{870.15} = 0.626 \quad (62.6 \%)$$

$$\text{Maximum thermal efficiency} = \frac{\dot{m}(e_3 - e_2)}{\dot{m}(h_3 - h_2)} = \frac{870.15}{1128.83} = 0.771 \quad (77.1 \%) \quad [15 \%]$$

2. (a) (i) If the syngas contains only H₂, H₂O, CO and CO₂, the chemical reaction is,



$$\text{Conservation of atomic H} \quad 2a + 2b = 12$$

$$\text{Conservation of atomic C} \quad c + d = 1$$

$$\text{Conservation of atomic O} \quad b + c + 2d = 4$$

The equilibrium equation for Databook Reaction 8, $-\text{CO} - \text{H}_2\text{O} + \text{CO}_2 + \text{H}_2 = 0$, is,

$$\left(\frac{P_{\text{CO}}}{P_0}\right)^{-1} \left(\frac{P_{\text{H}_2\text{O}}}{P_0}\right)^{-1} \left(\frac{P_{\text{CO}_2}}{P_0}\right)^{+1} \left(\frac{P_{\text{H}_2}}{P_0}\right)^{+1} = K_{p8}$$

where p_0 is standard pressure (1 bar). If $n = a + b + c + d$ is the total number of moles and the mixture pressure is p , the equation for K_{p8} becomes,

$$\left(\frac{c p}{n p_0}\right)^{-1} \left(\frac{b p}{n p_0}\right)^{-1} \left(\frac{d p}{n p_0}\right)^{+1} \left(\frac{a p}{n p_0}\right)^{+1} = \frac{ad}{bc} = K_{p8}$$

From the Databook for Reaction 8 at 1200 K, $\ln(K_{p8}) = -0.311$. Hence, $K_{p8} = 0.7327$

From the atom balances, $b = 6 - a$; $b + d = 3$ so $d = a - 3$; $c = 1 - d = 4 - a$

Hence, $n = a + b + c + d = 7$

Assuming $X_{\text{H}_2} = 0.4807$ as given in the question, we find $a = 0.4807 \times 7 = 3.365$

(Note that the mole fraction X_{H_2} is not the same as the number of moles a in the calculation.)

Then : $b = 2.635$, $c = 0.635$, $d = 0.365$

Thus : $K_{p8} = \frac{ad}{bc} = \frac{3.365 \times 0.365}{2.635 \times 0.635} = 0.7340$ (which agrees with the Data Book value)

The mole fractions are therefore,

$$X_{\text{H}_2} = \frac{a}{n} = 0.481, \quad X_{\text{H}_2\text{O}} = \frac{b}{n} = 0.376, \quad X_{\text{CO}} = \frac{c}{n} = 0.091, \quad X_{\text{CO}_2} = \frac{d}{n} = 0.052 \quad [50 \%]$$

(a) (ii) The equilibrium constant for the reforming reaction is given by,

$$\left(\frac{p_{\text{CH}_4}}{p_0}\right)^{-1} \left(\frac{p_{\text{H}_2\text{O}}}{p_0}\right)^{-1} \left(\frac{p_{\text{H}_2}}{p_0}\right)^{+3} \left(\frac{p_{\text{CO}}}{p_0}\right)^{+1} = K_{p(\text{ref})}$$

In terms of the mole fractions,

$$\frac{(X_{\text{H}_2})^3 (X_{\text{CO}})}{(X_{\text{CH}_4})(X_{\text{H}_2\text{O}})} \left(\frac{p}{p_0}\right)^2 = K_{p(\text{ref})}$$

Assuming the mole fraction of CH_4 is so small that the other mole fractions are unaffected,

$$X_{\text{CH}_4} = \frac{(X_{\text{H}_2})^3 (X_{\text{CO}})}{K_{p(\text{ref})} (X_{\text{H}_2\text{O}})} \left(\frac{p}{p_0}\right)^2 = \frac{0.481^3 \times 0.091 \times 2^2}{29.48 \times 0.376} = 0.0037 \quad [20 \%]$$

(b) The vapour is a perfect gas and so its specific enthalpy is a function only of temperature. The liquid specific enthalpy is independent of pressure and so it also is a function only of temperature. The vapour and liquid isobaric specific heat capacities are constant and so the specific enthalpy of evaporation is given by,

$$\begin{aligned} h_{fg}(T) &= h_g(T) - h_f(T) = [h_g(T) - h_g(T_0)] + [h_g(T_0) - h_f(T_0)] + [h_f(T_0) - h_f(T)] \\ &= h_{fg0} + (c_{pg} - c_{pf})(T - T_0) \end{aligned}$$

Neglecting v_f compared to v_g and using $p_s v_g = RT$, the Clausius-Clapeyron equation is,

$$\frac{dp_s}{dT} = \frac{h_{fg} p_s}{RT^2}$$

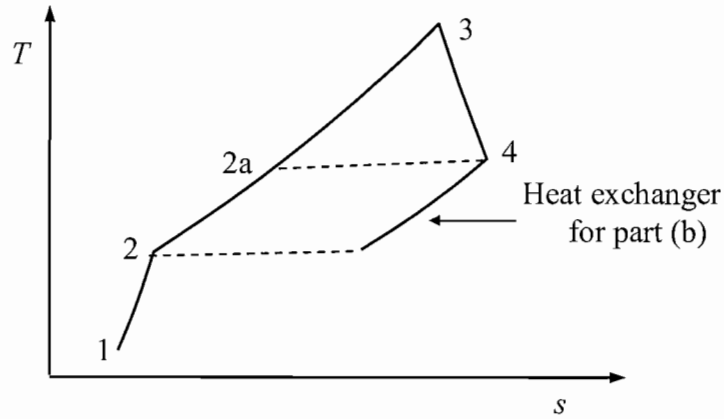
Substituting the expression for h_{fg} and integrating from T_0 to T we have,

$$\int_{p_{s0}}^{p_s} \frac{dp'_s}{p'_s} = \int_{T_0}^T \frac{h_{fg0} + (c_{pg} - c_{pf})(T' - T_0)}{RT'^2} dT'$$

Hence,

$$\begin{aligned} \ln\left(\frac{p_s}{p_{s0}}\right) &= \frac{h_{fg0}}{R} \left(\frac{1}{T_0} - \frac{1}{T}\right) + \frac{(c_{pg} - c_{pf})}{R} \ln\left(\frac{T}{T_0}\right) - \frac{(c_{pg} - c_{pf})T_0}{R} \left(\frac{1}{T_0} - \frac{1}{T}\right) \\ &= \frac{h_{fg0}}{RT_0} \left(1 - \frac{T_0}{T}\right) + \frac{(c_{pf} - c_{pg})}{R} \left[1 - \frac{T_0}{T} + \ln\left(\frac{T_0}{T}\right)\right] \end{aligned} \quad [30 \%]$$

3.



(a) (i) For the compressor,

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(\gamma_a - 1)/(\gamma_a \eta_c)} = 298.15 \times 20^{0.4/(1.4 \times 0.88)} = 788.58 \text{ K}$$

SFEE for the adiabatic combustor (A = air-fuel mass ratio) :

$$(A+1)h_p(T_3) - Ah_a(T_2) - h_f(T_0) = 0$$

From the definition of ΔH_{298}^0 (note that the excess air makes no difference) :

$$(A+1)h_p(T_0) - Ah_a(T_0) - h_f(T_0) = \Delta H_{298}^0$$

Subtracting the second equation from the first gives,

$$(A+1)[h_p(T_3) - h_p(T_0)] - A[h_a(T_2) - h_a(T_0)] = -\Delta H_{298}^0$$

$$(A+1)c_{pp}(T_3 - T_0) - Ac_{pa}(T_2 - T_0) = -\Delta H_{298}^0$$

$$T_3 = T_0 + \left[\frac{-\Delta H_{298}^0 + Ac_{pa}(T_2 - T_0)}{(A+1)c_{pp}} \right]$$

$$= 298.15 + \left[\frac{50 \times 10^3 + 46.2 \times 1.005 \times (788.58 - 298.15)}{47.2 \times 1.100} \right] = 1699.8 \text{ K} \quad [30 \%]$$

(ii) For the turbine (taking $T_3 = 1700 \text{ K}$) :

$$T_{4s} = T_3 \left(\frac{p_4}{p_3} \right)^{(\gamma_p - 1)/\gamma_p} = 1700 \times \left(\frac{1.22}{20} \right)^{0.35/1.35} = 823.3 \text{ K}$$

$$T_4 = T_3 - \eta_t (T_3 - T_{4s}) = 1700 - 0.90 \times (1700 - 823.3) = 910.97 \text{ K}$$

Net work output per kg of air entering compressor :

$$w_{net} = \left(\frac{A+1}{A} \right) c_{pp} (T_3 - T_4) - c_{pa} (T_2 - T_1)$$

$$= \left(\frac{47.2}{46.2} \right) \times 1.100 \times (1700 - 910.97) - 1.005 \times (788.58 - 298.15) = 393.84 \text{ kJ/kg air} \quad [25 \%]$$

(iii) Overall efficiency,

$$\eta_{ov} = \frac{w_{net}}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{393.84}{(1/46.2) \times 50 \times 10^3} = 0.364 \quad [5 \%]$$

(b) The heat exchanger effectiveness is defined by, $\varepsilon_{HX} = \frac{T_{2a} - T_2}{T_4 - T_2}$.

$$T_{2a} = T_2 + \varepsilon_{HX} (T_4 - T_2) = 788.58 + 0.85 \times (910.97 - 788.58) = 892.61 \text{ K}$$

From the SFEE for the combustor,

$$(A+1)c_{pp}(T_3 - T_0) - A c_{pa}(T_{2a} - T_0) = -\Delta H_{298}^0$$

$$A = \left[\frac{-\Delta H_{298}^0 - c_{pp}(T_3 - T_0)}{c_{pp}(T_3 - T_0) - c_{pa}(T_{2a} - T_0)} \right]$$

$$= \left[\frac{50 \times 10^3 - 1.100 \times (1700 - 298.15)}{1.100 \times (1700 - 298.15) - 1.005 \times (892.61 - 298.15)} \right] = 51.30$$

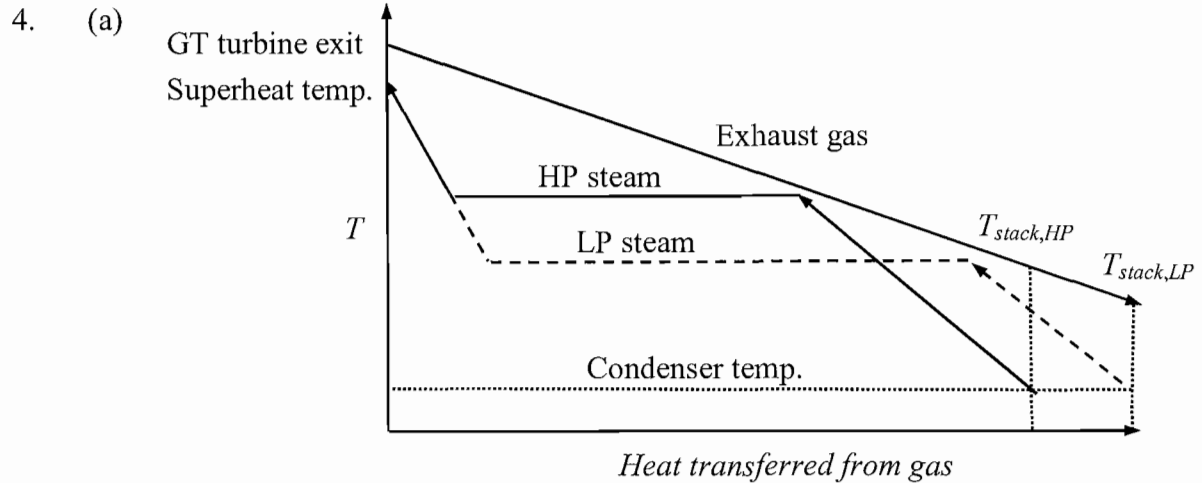
Net work output per kg of air entering compressor :

$$w_{net} = \left(\frac{52.3}{51.3} \right) \times 1.100 \times (1700 - 910.97) - 1.005 \times (788.58 - 298.15) = 391.97 \text{ kJ/kg air}$$

Overall efficiency,

$$\eta_{ov} = \frac{w_{net}}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{391.97}{(1/51.3) \times 50 \times 10^3} = 0.402 \quad [30 \%]$$

(c) Adding an intercooler to a recuperated GT reduces the work input to the compressor without changing the fuel flow rate or the turbine work output. Hence both the overall efficiency and the net work output both increase. [10 %]



As the steam pressure is increased, less heat is transferred from the gas turbine exhaust. The HRSG boiler efficiency, defined by,

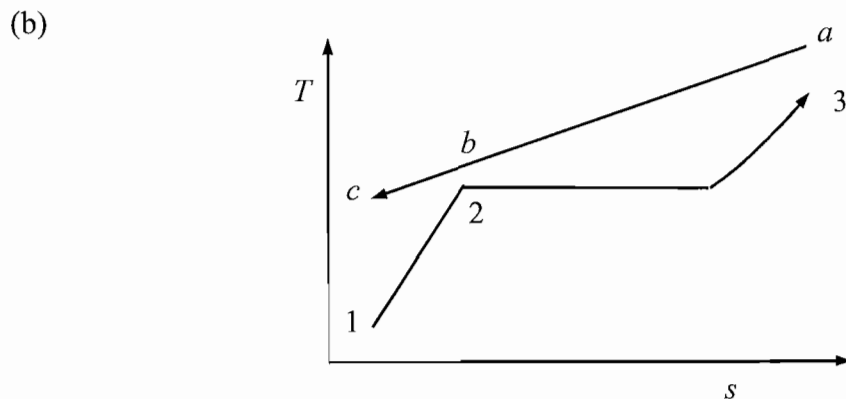
$$\eta_B = \frac{\text{Actual heat transferred}}{\text{Maximum heat transferred (when } T_{stack} = T_0)}$$

therefore decreases. However, the steam cycle efficiency η_{ST} increases because the mean temperature of heat reception for the steam cycle is higher. The CCGT overall efficiency is given by,

$$\eta_{CC} = \eta_{GT} + \eta_B \eta_{ST} (1 - \eta_{GT})$$

so it is not possible to say whether or not η_{CC} increases without further calculation.

[25 %]



(i) The SFEE applied to the HRSG down to the pinch point gives,

$$\dot{m}_s (h_3 - h_2) = \dot{m}_g c_{pg} (T_a - T_b)$$

$$\frac{\dot{m}_s}{\dot{m}_g} = \frac{c_{pg} (T_a - T_b)}{(h_3 - h_2)} = \frac{1.10 \times [540.0 - (250.35 + 15.0)]}{(3446.0 - 1087.5)} = 0.1281$$

[15 %]

(ii) The SFEE applied to the whole HRSG gives,

$$\dot{m}_s (h_3 - h_1) = \dot{m}_g c_{pg} (T_a - T_c)$$

$$T_c = T_a - \frac{\dot{m}_s (h_3 - h_1)}{\dot{m}_g c_{pg}} = 540.0 - \frac{0.1281 \times (3446.0 - 121.4)}{1.10} = 152.8 \text{ }^\circ\text{C} \quad [10 \text{ \%}]$$

(iii) Max. work from steam cycle per unit mass of gas ($T_0 = 28.96 + 273.15 = 302.11 \text{ K}$) :

$$\begin{aligned} w_{st,max} &= \frac{\dot{m}_s}{\dot{m}_g} (e_3 - e_1) = \frac{\dot{m}_s}{\dot{m}_g} [(h_3 - T_0 s_3) - (h_1 - T_0 s_1)] \\ &= 0.1281 \times [(3446.0 - 302.11 \times 7.092) - (121.4 - 302.11 \times 0.422)] \\ &= 167.75 \text{ kJ/kg gas} \end{aligned} \quad [15 \text{ \%}]$$

(iv) Max. steam cycle efficiency = $\frac{w_{st,max}}{(\dot{m}_s / \dot{m}_g)(h_3 - h_1)}$

$$= \frac{167.75}{0.1281 \times (3446.0 - 121.4)} = 0.394 \text{ (39.4 \%)} \quad [10 \text{ \%}]$$

(c) (i) For the 150 bar steam pressure level the maximum steam cycle work is,

$$\begin{aligned} w_{st,max} &= 0.1180 \times [(3310.8 - 302.11 \times 6.348) - (121.4 - 302.11 \times 0.422)] \\ &= 165.09 \text{ kJ/kg gas} \end{aligned}$$

$$\text{Max. steam cycle efficiency} = \frac{165.09}{0.1180 \times (3310.8 - 121.4)} = 0.439 \text{ (43.9 \%)} \quad [10 \text{ \%}]$$

(ii) The fuel input and the GT work output per unit mass of gas passing through the HRSG will be the same for both cycles. Hence, the cycle with the highest CCGT overall efficiency will be the one with the larger value of $w_{st,max}$. This is the cycle with the 40 bar steam pressure level (despite the fact that the cycle with the 150 bar pressure level has the higher steam cycle efficiency). [15 %]

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MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

ANSWERS

1. (b) (i) Compressor : -765.5 kW , 43.0 kW , 0.0
Turbine : 1310.5 kW , 43.5 kW , 0.0
(ii) Cooler : 0.0 , 7.4 kW , 231.4 kW
(iii) Heater : 870.2 kW
(iv) 48.3% , 62.6% , 77.1%
2. (a) (i) $X_{\text{H}_2} = 0.481$, $X_{\text{H}_2\text{O}} = 0.376$, $X_{\text{CO}} = 0.091$, $X_{\text{CO}_2} = 0.052$
(ii) $X_{\text{CH}_4} = 0.0037$
3. (a) (ii) 393.84 kJ/kg air , (iii) 0.364
(b) 51.3 , 0.402
- 4 (b) (i) 0.1281
(ii) $152.8 \text{ }^\circ\text{C}$
(iii) $167.8 \text{ kJ/kg of gas}$
(iv) 39.4%
(c) 43.9% , The 40 bar pressure level cycle.