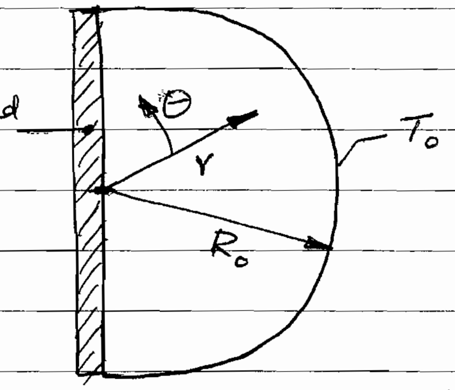
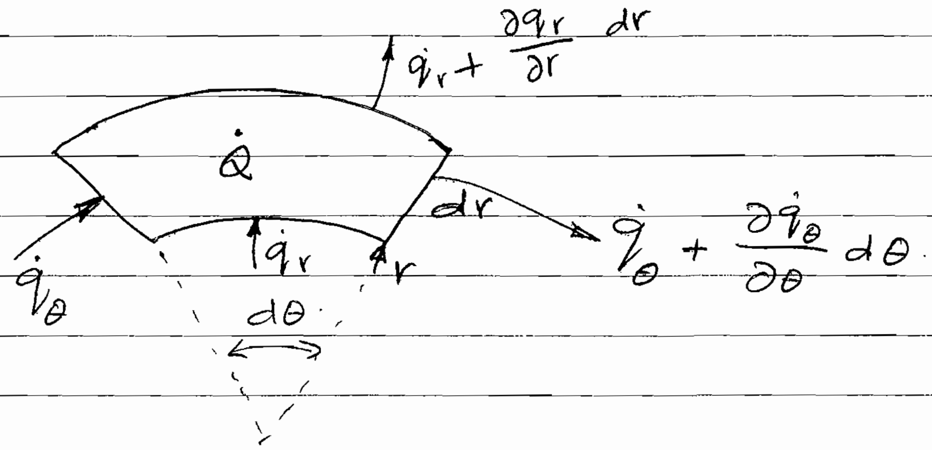
Dr N Swaminathan. **2009- 3A6 CRIBS** 

1)




(a)




Energy balance:

$$\dot{q}_r\, r d\theta - \left(\dot{q}_r + \frac{\partial \dot{q}_r}{\partial r} dr\right)(r+dr)\, d\theta$$

$$+\, \dot{q}_\theta\, dr - \left(\dot{q}_\theta + \frac{\partial \dot{q}_\theta}{\partial \theta} d\theta\right) dr + \dot{Q}\, r\, d\theta\, dr = 0$$

Simplifying & neglecting second order terms:

$$-\dot{q}_r\, dr\, d\theta - \frac{\partial \dot{q}_r}{\partial r}\, r\, dr\, d\theta - \frac{\partial \dot{q}_\theta}{\partial \theta}\, dr\, d\theta$$

$$+\, \dot{Q}\, r\, dr\, d\theta = 0.$$

Divide by $r\,dr\,d\theta$ & note that

$$\dot{q}_r = -k \frac{\partial T}{\partial r} \quad \& \quad \dot{q}_\theta = -k \frac{\partial T}{r\, \partial \theta}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + \frac{\dot{Q}}{k} = 0}$$

(b) The mid plane is insulated &
There is symmetry about horizontal plane
going through the centre.

$$\Rightarrow \quad \frac{\partial T}{\partial \theta} = 0 .$$

(c) Now we have

$$\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right) + \frac{1}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{\dot{Q}}{k} = 0 .$$

$T = T_0$ @ $r = R_0$

Rewrite the above equation as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + \frac{\dot{Q}}{k} = 0$$

$$\Rightarrow \quad r\frac{\partial T}{\partial r} = -\frac{\dot{Q} r^2}{2k} + C_1$$

$$\frac{\partial T}{\partial r} = -\frac{\dot{Q} r}{2k} + \frac{C_1}{r} \; ; \quad C_1 = 0 \quad \because \left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

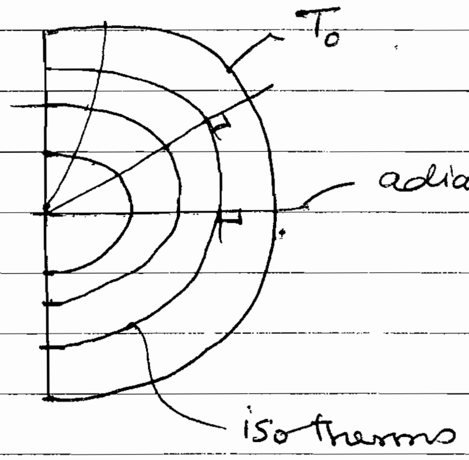
$$\Rightarrow \quad T(r) = -\frac{\dot{Q} r^2}{4k} + C_2$$

$T = T_0$ @ $R_0 = r$ gives $C_2 = T_0 + \frac{\dot{Q} R_0^2}{4k}$

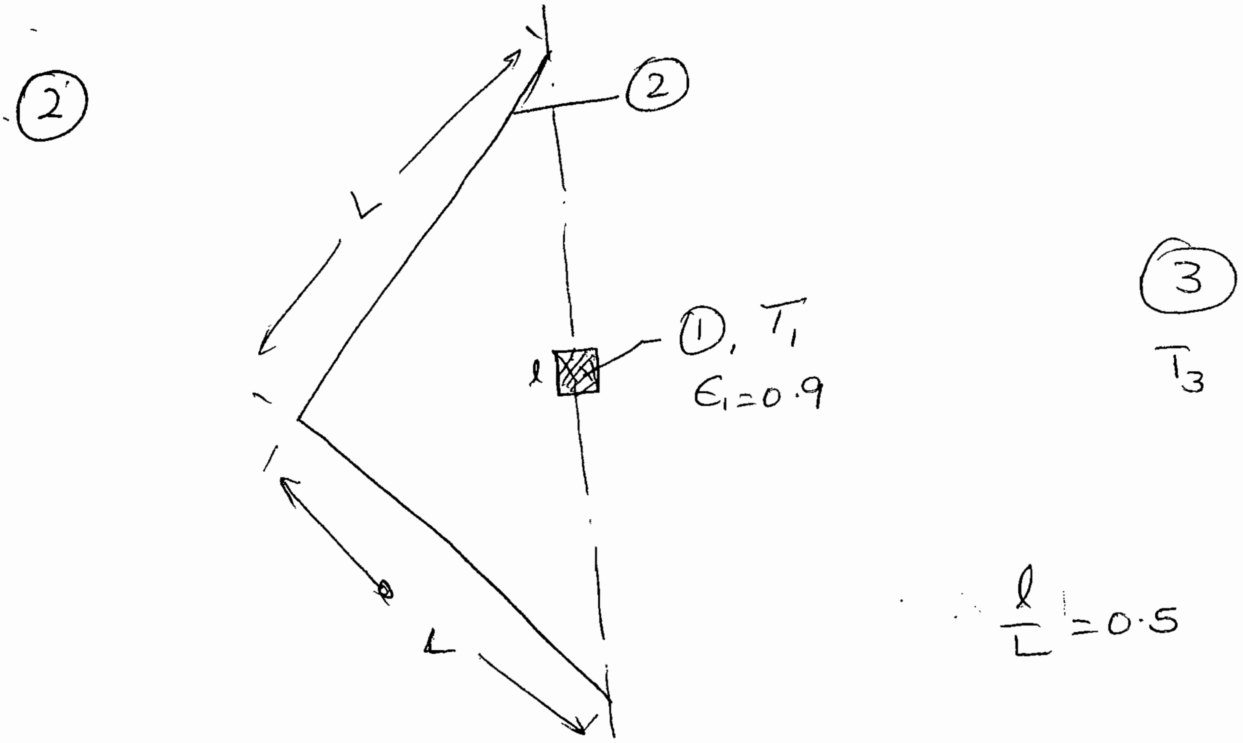
$$\therefore \quad T(r) = T_0 + \frac{\dot{Q} R_0^2}{4k}\left[1 - \left(\frac{r}{R_0}\right)^2\right]$$

$$\boxed{T(r) - T_0 = \frac{\dot{Q} R_0^2}{4k}\left[1 - \left(\frac{r}{R_0}\right)^2\right]}$$

(d)

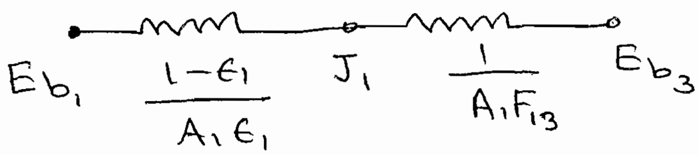



isotherms & adiabats are $\perp r$ to each other at every point.

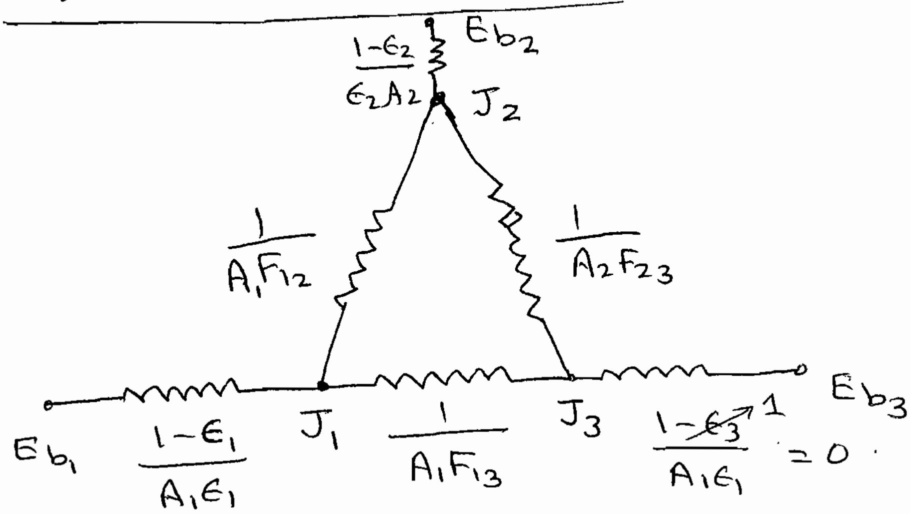



$\frac{l}{L} = 0.5$

(a) Case A without reflector:




Case B - with reflector:



(b) Net heat exchange in Case A.

$$q_0 = \frac{1}{2}\left(\frac{E_{b_1} - E_{b_3}}{R_{total}}\right) = \frac{\sigma\,(T_1^4 - T_3^4)}{R_{total}}$$

$$R_{total} = \frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{13}} \quad ; \quad F_{13} = 1.$$

$A_1 = 4l.$

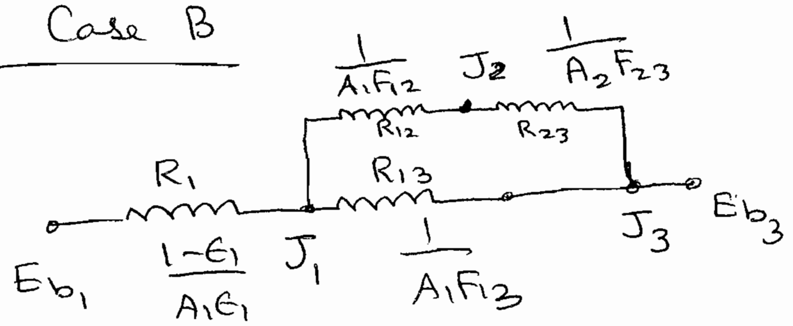
$\epsilon_1 = 0.9$

$$R_{total} = \frac{1}{A_1\epsilon_1}$$

$$\therefore\; q_0 = 2l\,\epsilon_1\,\sigma\,(T_1^4 - T_3^4)$$

$$\boxed{q_0 = 1.8\,l\,\sigma\,(T_1^4 - T_3^4)}$$

(c) in Case B




$$q_R = \frac{(E_{b_1} - E_{b_3})}{R_{total}} \quad ; \quad R_{total} = R_1 + \left[\frac{1}{R_{13}} + \frac{1}{R_{12}+R_{23}}\right]^{-1}$$

$F_{12} + F_{13} + \cancel{F_{11}}^{\,0} = 1.$ By symmetry $F_{12} = F_{13} = 0.5.$

$F_{21} + F_{22} + F_{23} = 1.$

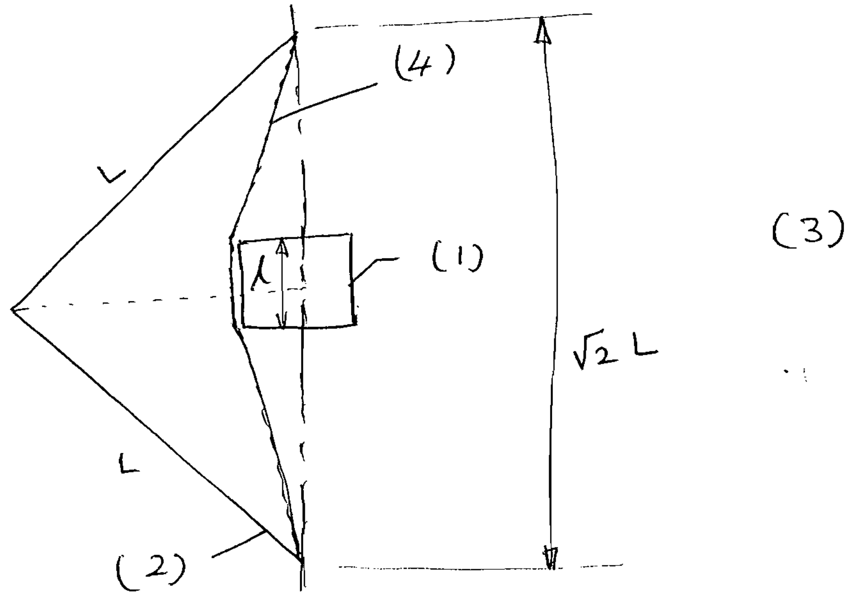
$$F_{21} = \frac{A_1F_{12}}{A_2} = 0.5\,\frac{A_1}{A_2}.$$

$$= \frac{4l}{2\cdot 2L} = \frac{l}{L} = 0.5$$

$F_{23}$ is required

$$\boxed{F_{21} = 0.5}$$

To find $F_{23}$, use imaginary surface (4) as shown below:




$F_{42} = 1$ since 4 - is convex.

Radiation from (2) & leaving via (4) will intercept either (1) or (3)

$$\Rightarrow \quad F_{24} = F_{21} + F_{23}$$

But $A_2 F_{24} = A_4 F_{42} \Rightarrow F_{24} = \left(\frac{A_4}{A_2}\right) F_{42}$

$A_2 = 2L$ per unit length.

$$A_4 = l + 2\left[\left(\frac{\sqrt{2}L - l}{2}\right)^2 + \frac{l^2}{4}\right]^{1/2}$$

$$= L\left\{\frac{l}{L} + 2\left[\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{l}{L}\right)^2 + \frac{1}{4}\left(\frac{l}{L}\right)^2\right]^{1/2}\right\}$$

$$= L\left\{0.5 + 2\left[0.2089 + 0.0625\right]^{1/2}\right\}$$

$$A_4 = 1.54192L$$

$$F_{24} = \frac{1.54192L}{2L} = 0.771$$

$$F_{23} = F_{24} - F_{21}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{4l}{4L}\frac{1}{2} = \frac{l}{L} = 0.5$$

ALSO FROM SYMMETRY arguments.

$F_{11} + F_{12} + F_{13} = 1$

$F_{11} = 0$

$F_{12} = F_{13} = 0.5$

$$\therefore \boxed{F_{23} = 0.271}$$

Now, $$q_R = \frac{(E_{b1} - E_{b3})}{R_{total}} = \frac{\sigma(T_1^4 - T_3^4)}{R_{total}}$$

$$R_{total} = R_1 + \left[\frac{1}{R_{13}} + \frac{1}{R_{12} + R_{23}}\right]^{-1}$$

$$R_1 = \frac{1 - \epsilon_1}{A_1 \epsilon_1} = \frac{0.1}{3.6l} = \frac{0.0278}{l}$$

$$R_{13} = \frac{1}{A_1 F_{13}} = \frac{0.5}{l}$$

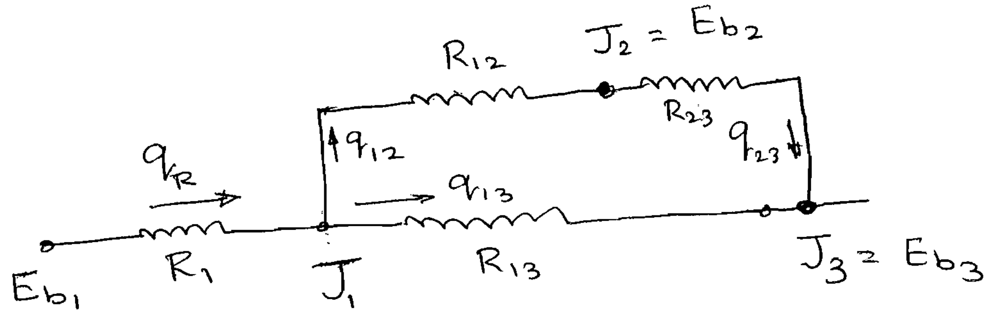
$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{0.5}{l}$$

$$R_{23} = \frac{1}{A_2 F_{23}} = \frac{1}{2L * 0.271} = \frac{1.845}{L} = \frac{0.923}{l}$$

$$R_{total} = \frac{0.0278}{l} + \frac{0.370}{l} = \frac{0.3978}{l}$$

$$\therefore \boxed{q_R = 2.514\, l\, \sigma \left(T_1^4 - T_3^4\right)}$$

(d)




$$q_R = q_{13} + q_{12} \quad ; \quad q_{12} = q_{23}$$

$$q_{13} = \frac{J_1 - J_3}{R_{13}} \quad ; \quad q_{12} = \frac{J_1 - J_2}{R_{12}}$$

$$q_{23} = \frac{J_2 - J_3}{R_{23}}$$

$$q_{12} = q_{23} \Rightarrow \frac{J_1 - J_2}{R_{12}} = \frac{J_2 - J_3}{R_{23}}$$

$$\frac{J_2}{R_{23}} + \frac{J_2}{R_{12}} = \frac{J_1}{R_{12}} + \frac{J_3}{R_{23}} \quad \text{— (A)}$$

$$J_2 = \left[\frac{J_1}{R_{12}} + \frac{J_3}{R_{23}}\right]\left[\frac{R_{12} R_{23}}{R_{12} + R_{23}}\right]$$

But $J_1 = E_{b1} - q_R R_1 = \sigma T_1^4 - 2.514 * 0.0278\, \sigma \left(T_1^4 - T_3^4\right)$

From Eq. A

$$1.083 l J_2 + 2 l J_2 = 2 l J_1 + 1.083 l J_3$$

$$\Rightarrow \quad 3.083 \sigma T_2^4 = 2 J_1 + 1.083 J_3$$

But $J_1 = \sigma T_1^4 - 0.0699 \sigma (T_1^4 - T_3^4)$

$$= 0.93 \sigma T_1^4 + 0.07 \sigma T_3^4$$

$$\therefore \boxed{T_2 = \left(0.603 T_1^4 + 0.397 T_3^4\right)^{1/4}}$$

(e) $q_o = 1.81 \sigma (T_1^4 - T_3^4)$

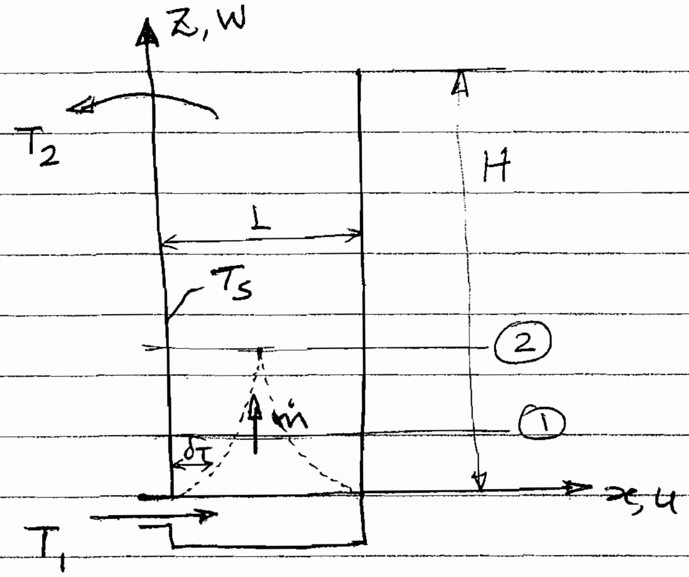
$q_R = 2.514 l \sigma (T_1^4 - T_3^4)$

$$\Rightarrow \quad \frac{q_o}{q_R} = 0.716 \qquad \boxed{q_R > q_o}$$ as it is aimed.

1) Increasing $\epsilon_1$ will increase $q_R$ further

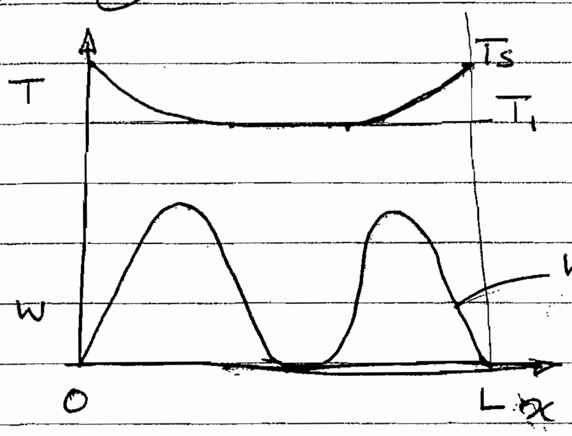
2) Emissivity of the reflector will not play a role since the reflector is insulated, irradiation is equal to reradiation.

3

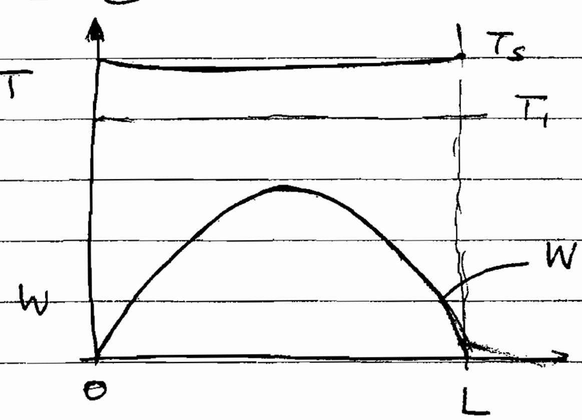



(a) location 2 is fully developed. This means that the boundary layers will interactio as shown in the figure above.

@ Location 1




@ Location 2



(b) Fully developed flow.

momentum Equation: $\nu \frac{d^2 w}{dx^2} = - g\beta(T-T_1)$

energy equation: $w \frac{\partial T}{\partial z} \approx \alpha \frac{\partial^2 T}{\partial x^2}$

Physical meanings

momentum Equation — Viscous forces are balanced by bouyancy forces.

energy Equation — Convection along z direction is balanced by diffusion in x-direction.

(c) $x \sim \delta_T \qquad z \sim H$

momentum Equation: $\nu \frac{w}{\delta_T^2} \sim g\beta\Delta T$

$$\Rightarrow \boxed{W \sim \left(\frac{g\beta\Delta T\delta_T^2}{\nu}\right)}$$ as required

(d) Energy Equation:

$$\frac{W\Delta T}{H} \sim \alpha \frac{\Delta T}{\delta_T^2} \Rightarrow \frac{W}{H} \sim \frac{\alpha}{\delta_T^2}$$

But $\delta_T^2 \sim \alpha\tau$ & using the above scaling for W

$$\boxed{\tau_s \sim \left(\frac{\nu H}{g\beta\Delta T\alpha}\right)^{1/2}}$$

(e)

$$(T - T_1) \approx (T_s - T_1)$$

$$\Rightarrow \frac{d^2 w}{dx^2} = -\frac{g\beta}{\nu}(T_s - T_1) = -\frac{g\beta}{\nu}\Delta T$$

B.Cs are $w = 0$ @ $x = 0$ & $L$

$\frac{dw}{dx} = 0$ @ $x = L/2$.

Integrate the above momentum Equation with the above B.C.s.

$$w(x) = \frac{g\beta\Delta T L x}{2\nu}\left(1 - \frac{x}{L}\right)$$

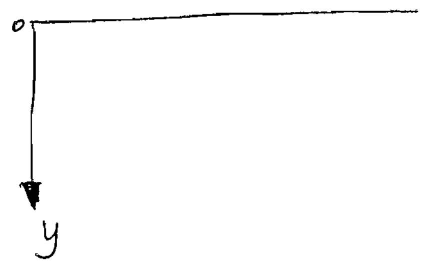
But $$\dot{m} = \int_0^L \rho\, w(x)\, dx$$

$$\Rightarrow \boxed{\dot{m} = \frac{\rho g \beta \Delta T L^3}{12\nu}} \quad \text{kg/sec-m.}$$

Heat gained by the air is

$$\boxed{\dot{Q} = \dot{m} C_p (T_2 - T_1)} \quad \text{Per unit depth.}$$

(4)




$C_{in} = 0.1\%$

$C_o = 1\%$

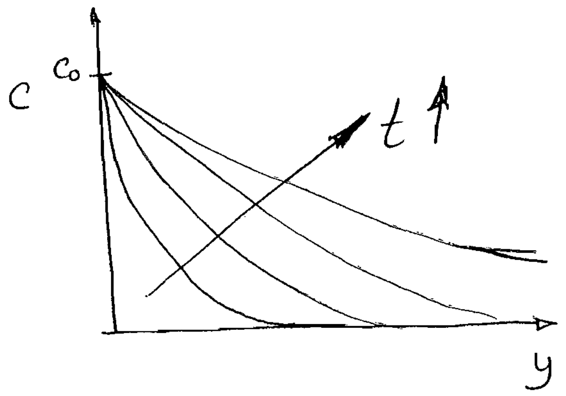
$T = 950 K$

$D = 2.67 \times 10^{-5} \exp\left(-\frac{17400}{T}\right)$

$= 2.965 \times 10^{-13} \ m^2/sec$

$$\frac{C - C_o}{C_{in} - C_o} = erf(w), \quad w = \frac{y}{\sqrt{4Dt}}$$

(a)




(b) @ $y = 0.5 \times 10^{-3} m$ $\quad C = 0.00456$

$$\Rightarrow \quad erf(w) = \frac{0.00456 - 0.01}{0.001 - 0.01} = 0.604$$

from Table $\quad w = 0.6 = \dfrac{y}{\sqrt{4Dt}}$

$$\Rightarrow \quad t = \frac{y^2}{0.36 * 4 * D} = 6.78$$

$$\boxed{t = 6.78 \text{ days}}$$

(c) now $y = 1\times10^{-3}$ m.

$$\therefore \boxed{t = 27.11 \text{ days.}}$$

Since $t = \dfrac{y^2}{4w^2D}$ ; $y$ is twice $\Rightarrow$ $t$ is 4 times larger.

(d) $$D = \frac{y^2}{4w^2t} = 2.67\times10^{-5} \exp\left(-\frac{17400}{T}\right)$$

$$2.67\times10^{-5} \exp\left(-\frac{17400}{T}\right) = \frac{(0.5\times10^{-3})^2}{1.44\,t}$$

$t = 3.39$ days

$$\Rightarrow \boxed{T = 987.3 \text{ K.}}$$

(e) $$t = \frac{y^2}{4w^2D}$$

if T is kept the same

t can be reduced by increasing W which can be achieved by

1) increasing the surface concentration Co

2) Using a steel with higher carbon content (Cin)

# List of Answers

1. (c) $T(r) - T_o = \dfrac{\dot{Q}R_0^2}{4k}\left[1 - \left(\dfrac{r}{R_0}\right)^2\right]$

2. (b) $q_0 = 1.8l\sigma(T_1^4 - T_3^4)$

   (c) $q_R = 2.514l\sigma(T_1^4 - T_3^4)$

   (d) $T_2 = (0.603T_1^4 + 0.397T_3^4)^{1/4}$

3. (d) $\tau_s \sim \left(\dfrac{\nu\, H}{g\,\beta\,\Delta T\,\alpha}\right)^{1/2}$

   (e) $\dot{m} = \dfrac{\rho\, g\, \beta\, \Delta T\, L^3}{12\,\nu}$ and $\dot{Q} = \dot{m}c_p\,(T_2 - T_1)$

4. (b) $t = 6.78$days

   (c) $t = 27.11$days

   (d) $T = 987.3$K