

1.(a)

Radiation resistance, R_r :

Power radiated $P_r = \frac{1}{2} I^2 R_r$

\swarrow $\frac{1}{2}$ as $I_{pk} \rightarrow$ rms value

Radiation efficiency, e :

$e = \frac{P_r}{P_{in}}$ where $P_{in} = \frac{1}{2} I^2 (R_r + R_{ohmic})$

Gain, G :

$G = \frac{\text{Max. power radiated/unit area}}{\text{power/unit area from isotropic antenna}} = \frac{P/\text{unit area max.}}{P_{in}/4\pi r^2}$

Directivity, D :

$D = \frac{\text{Gain}}{\text{Radn efficiency}} = \frac{G}{e}$

(gives an indication of how directional an antenna is)

Effective Aperture (Area), A_e :

Power delivered into a matched load by an antenna = $A_e \times$ power density in incident radio wave

Note:

$G = \frac{4\pi A_e}{\lambda^2}$

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ANTENNA EQN.

(b)



Power density at range, $R = \frac{10 \times 10^{-3}}{4\pi R^2}$

Effective aperture, A_e of receive antenna = $\frac{30 \times \lambda^2}{4\pi}$ m²
 (where $\lambda = \frac{3 \times 10^8}{433 \times 10^6} = 0.693\text{m}$)

$\therefore A_e = 1.15\text{m}^2$

\therefore Power into receiver = $\frac{11.5 \times 10^{-3}}{4\pi R^2} = \frac{V^2}{R} = \frac{(10 \times 10^{-6})^2}{75}$

\therefore Max range, $R = \sqrt{\frac{75 \times 11.5 \times 10^{-3}}{4\pi \times 10^{-10}}} = 26.2\text{ km}$

$$(c) \frac{1}{4} \text{ wavelength} = \frac{0.693}{4} \text{ m} = \underline{17.3 \text{ cm}}$$

Gain of $\lambda/4$ dipole is 1.5 \therefore range will increase by $\sqrt{1.5} \Rightarrow \underline{32.1 \text{ km}}$

However, there is no radiation in the direction of the antenna axis (max. radiation is \perp to wire), so if the aircraft were overhead, then the signal may be weaker than expected.

$$(d) \text{ Antenna radiation efficiency} = \frac{R_r}{R_r + R_{ohm}}$$

$$\text{For a simple dipole antenna } R_r = 20\pi^2 \left(\frac{AZ}{\lambda}\right)^2 \Omega$$

$$\text{Hence for } 433 \text{ MHz: } R_r = \frac{20\pi^2}{4} = 49.3 \Omega$$

(AZ is $\lambda/2$ as $\lambda/4$ wire is placed above a reflecting ground plane, which effectively doubles its length)

$$\therefore 0.90 = \frac{49.3}{49.3 + R_{ohm}} \quad \therefore R_{ohm} = 5.48 \Omega \text{ at } 433 \text{ MHz}$$

Assuming the antenna is a thick wire, the current carrying area is prop. to the skin depth, $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

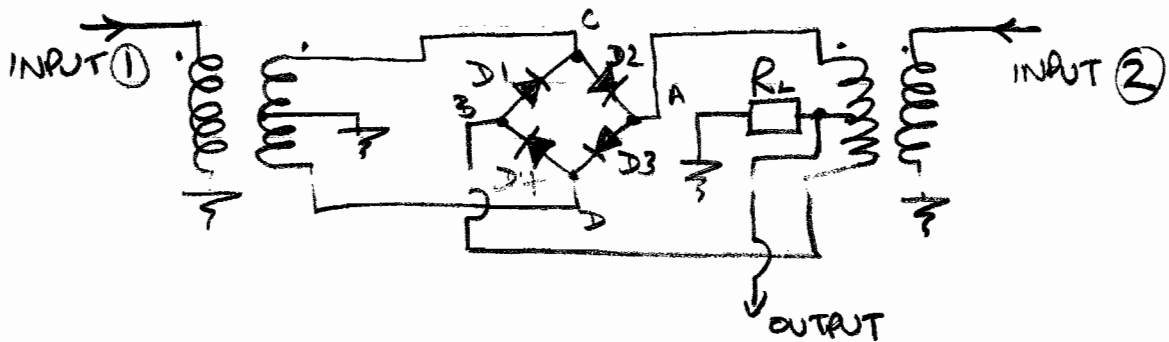
$$\therefore R_{ohm} @ 175 \text{ MHz} = 5.48 \times \frac{\sqrt{175}}{\sqrt{433}} = 3.48 \Omega$$

$$\text{and } R_r @ 175 \text{ MHz, where } \lambda = 1.717 \text{ m; } R_r = 20\pi^2 \left(\frac{0.346}{1.717}\right)^2$$

$$R_r = 8.01 \Omega$$

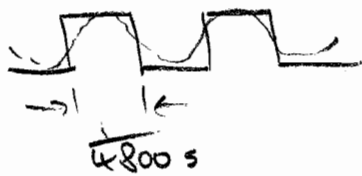
$$\therefore e = \frac{8.01}{8.01 + 3.48} = 0.70 \text{ or } 70\% \quad \text{[and will not be matched]}$$

2. a)



- Mixer selectively inverts, or not, the signal on one input (② in this case) depending on the polarity of the other input (① in this case).
- Hence 2 sine waves in phase will result in a full-wave rectified signal at the output - the components of which are \sim dc. and $2f$ mainly.
- Consider the circuit above, if ① and ② swing +ve, then diodes $D2$ & $D3$ conduct as C goes positive and D negative by the same amount. Hence, point A is a virtual ground and so the output swings -ve.
- As ① and ② swing -ve, then diodes $D3$ and $D4$ conduct causing B to be a virtual ground and so the output again swings -ve. Thus the output signal is a full-wave rectified version of the signal at ②.
- With diode switching, the transition is 'soft', so the higher harmonics are reduced over a true full-wave rectified signal and the major harmonics are just of (d.c.) and $2f$.
- To ensure minimal d.c. in the output, the 2 input waves should be 90° apart i.e. sine and cosine - or the output could be ac coupled through a capacitor.

2(b) choose a Bessel filter to maintain the underlying square wave shape - without ringing.



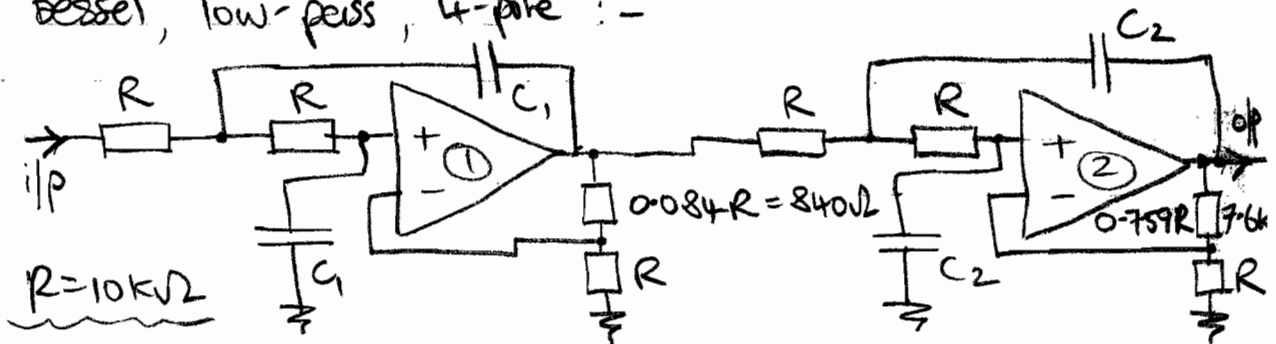
4800 bit/s

fundamental @ 2400 Hz

allow 1st & 3rd harmonic before roll-off \therefore roll-off low pass

filter at say $4 \times 2400 \text{ Hz} = 9600 \text{ Hz}$ (3dB).

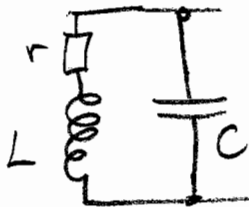
Bessel, low-pass, 4-pole :-



for stage ① $9600 = \frac{1}{1.432 \cdot C_1 \cdot 10^4 \cdot 2\pi} \therefore C_1 = 7.16 \text{ nF}$

for stage ② $9600 = \frac{1}{1.606 \cdot C_2 \cdot 10^4 \cdot 2\pi} \therefore C_2 = 1.03 \text{ nF}$

2(c)



$L = 10^{-5} \text{ H}, C = 3.3 \times 10^{-10} \text{ F} \therefore \omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore f_0 = 2.77 \text{ MHz} (17.4 \times 10^6 \text{ rad/s})$

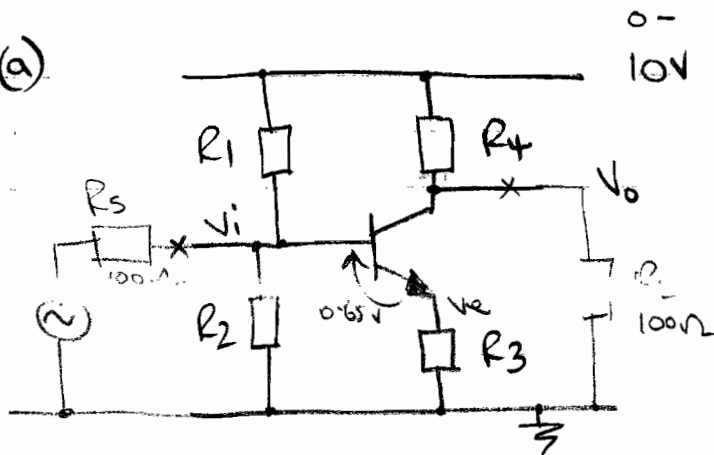
$Q = \frac{\omega L}{r} = \frac{174}{3} = 58 = \frac{R'}{\omega L} \therefore \text{equiv. parallel } R' = 10.1 \text{ k}\Omega$

for $Q=100$, we need $R' = 17.4 \text{ k}\Omega$ - hence we need to shunt the LC tank with $-24 \text{ k}\Omega$ using a negative resistance



N.I.C.

3(a)



For 100Ω output impedance
 $R_4 = 100\Omega$

For 10dB gain, we want a linear gain of $\times 6.5$, including coupling loss of 1 stage. ($2 \times 10^{0.5} \approx 6.5$) $\therefore \times 3.2$ when in ckt.

$$\therefore \frac{R_4}{R_3 + r_e} = 6.5 \quad \text{for } V_0 = 5V \text{ dc, then } I_c = 50\text{mA}$$

$$\therefore r_e = \frac{0.025}{0.05} = \left(\frac{kT/q}{I_c} \right) = 0.5$$

$$\therefore R_3 \approx 15\Omega$$

Then $V_e = 0.75V$ dc. \therefore set base @ $0.75 + 0.65 = 1.4V$ dc.

Choose $R_2 = 200\Omega$, then $R_1 \Rightarrow \frac{200}{R_1 + 200} \cdot 10 = 1.4 \times 1.1$

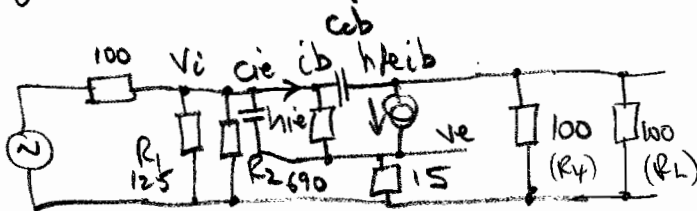
[or 125Ω]

$$\therefore R_1 = 1100\Omega \text{ [or } 690\Omega]$$

allow 10% extra for base load

check i/p impedance = $200 \parallel 100 \parallel 200(15.5) = 160\Omega$ which is ok, but could drop R_2 and R_1 by $(\times \frac{1}{1.6})$ to 125Ω and 690Ω to give 102Ω i/p impedance.

(b)



$$A_v = \frac{1}{2\pi C_{ie} r_e} = 22 \times 10^9$$

0.5

$$\therefore C_{ie} = 14.5 \text{ pF}$$

$$C_{cb'} = 0.15 \text{ pF} \times (1 + 3.2)$$

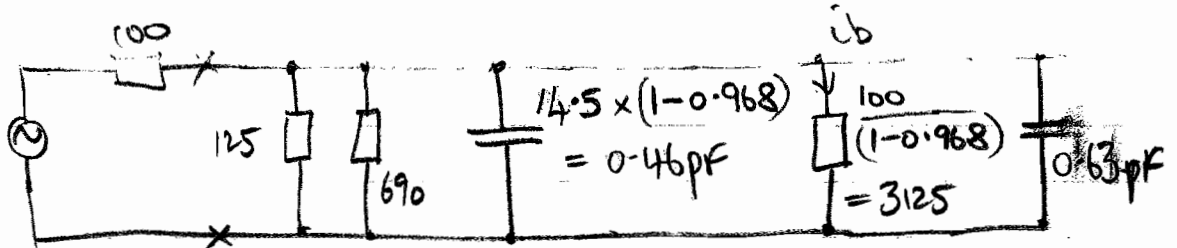
$$= 0.63 \text{ pF}$$

$$h_{ie} = h_{fe} \cdot r_e = 100\Omega$$

$$V_e = \frac{15}{15 + 0.5} \cdot V_i = 0.968 V_i$$

3(b) contd.

SSM @ input



$$\frac{1}{2\pi R' C'} = 1.43 \text{ GHz}$$

or 2.8 GHz
incl. source imped.

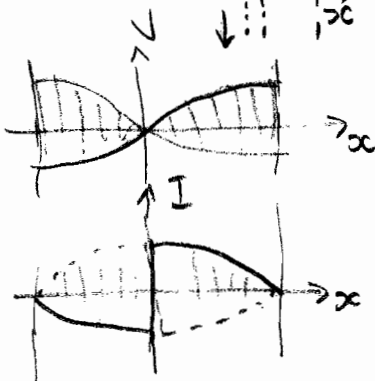
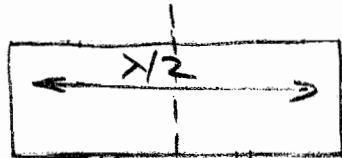
$$R' = 125 \parallel 690 \parallel 3125 \parallel 100$$

$$= 102 \Omega$$

or 50 Ω mic. source imped.

$$C' = 0.63 + 0.46 = 1.09 \text{ pF}$$

(c) Microstrip patch is $\lambda/2$ long and resonates end to end with a standing wave:-



for $f = 1.5 \times 10^9 \text{ Hz}$

and $\epsilon_r = 3$, $\lambda = \frac{3 \times 10^8}{\sqrt{3} \times 1.5 \times 10^9}$
 $= 11.5 \text{ cm}$

$$\therefore \lambda/2 = 5.77 \text{ cm}$$

(d) Amplifier inverts, hence so must patch at resonance, so have 2 'feed' lines - equally spaced each side of the centre-line. Adjust distance ($\pm x$) to get 100Ω impedance. One feed connects to amp. input, the other is output - through at least 1 d.c. block capacitor to avoid upsetting the bias. Feed lines must be v. short (phases)

3(e) $\rightarrow w \leftarrow \rightarrow$ effective width $(w+2t) = A$ / unit length
 due to fringing

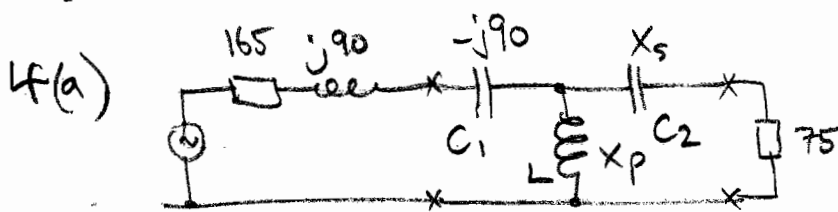


$$C = \frac{A \epsilon_0 \epsilon_r}{t} \quad Z_0 = \sqrt{\frac{L}{C}} \quad \epsilon = \frac{1}{\mu L C}$$

$$\therefore Z_0 = \frac{\sqrt{L}}{\sqrt{C}} = \frac{1/\sqrt{\mu} \sqrt{C}}{\sqrt{C}} = \frac{1}{\sqrt{\mu C}}$$

$$\therefore Z_0 = \frac{t}{A \epsilon_0 \sqrt{\epsilon_r}} \cdot \frac{\sqrt{\epsilon_r}}{C_0} = 100 = \frac{1.6 \times 10^{-3}}{3 \times 10^8 \cdot (w + 3.2 \times 10^{-3}) \cdot 8.85 \times 10^{-12} \cdot \sqrt{3}}$$

$$\therefore \underline{w = 0.28 \text{ mm}}$$



$$\omega = 2\pi \cdot 500 \times 10^6$$

$$= \pi \times 10^9 \text{ rad/s}$$

Firstly cancel reactance at $-j90 = j\omega C_1$

$$\therefore C_1 = 3.54 \text{ pF} \quad R_{hi} = 165, \quad R_{lo} = 75$$

$$\text{Then } Q = \sqrt{\frac{165}{75} - 1} = 1.095 = \frac{165}{X_p} = \frac{X_s}{75}$$

$$\therefore X_p = 150 = \omega L \quad \therefore L = 4.77 \times 10^{-8} \text{ H} = 48 \text{ nH}$$

$$X_s = 82.1 = \frac{1}{\omega C_2} \quad \therefore C_2 = 3.88 \times 10^{-12} \text{ F} = 3.9 \text{ pF}$$

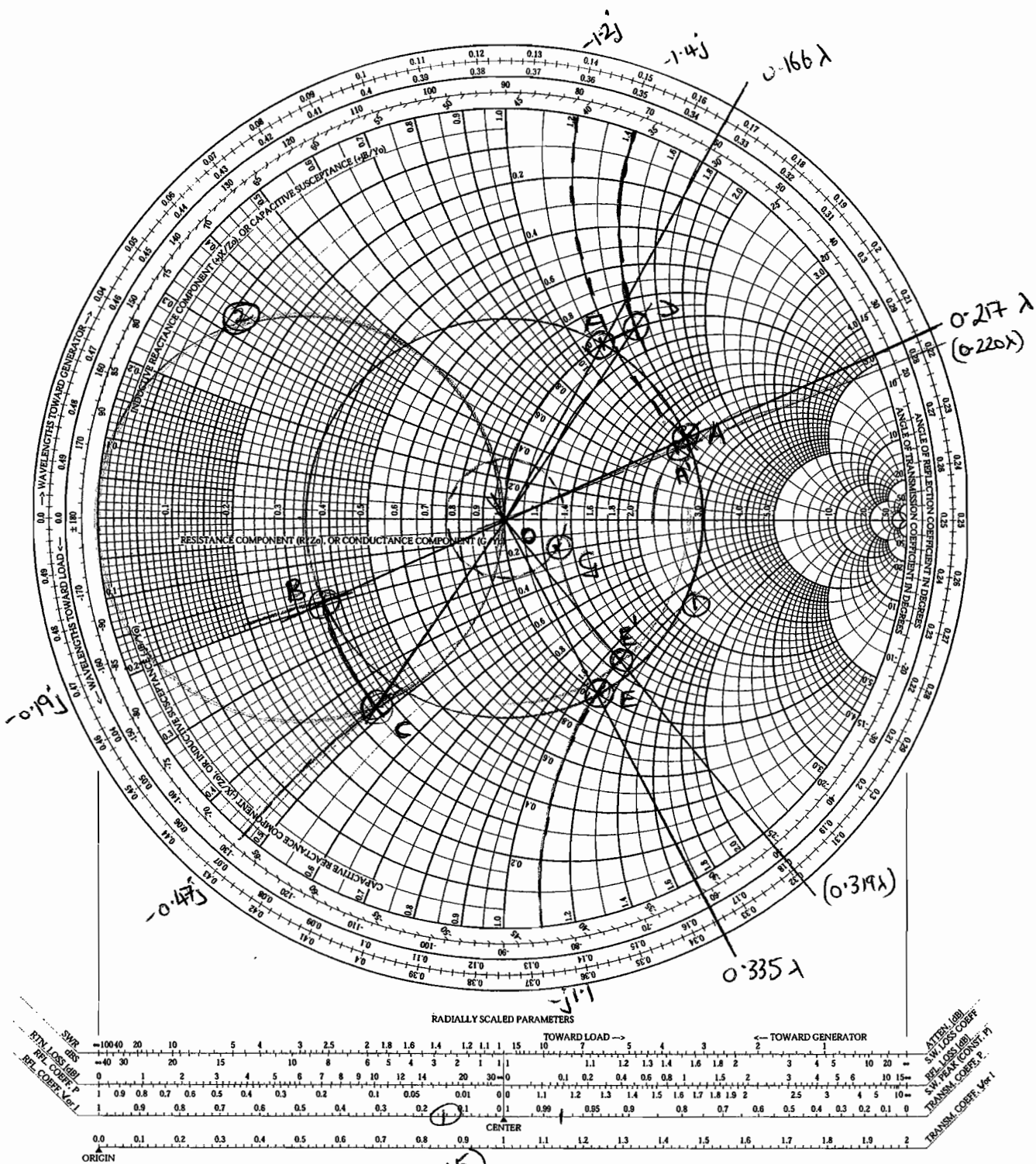
(b) Normalise $\frac{165 + j90}{75} = 2.2 + j1.2$ plot point 'A'

LC { Admittance is point 'B' on circle ①.
 Plot circle ② to find point 'C' where 'C-D' gives impedance
 point 'D' where Re part is unity
 B → C is a parallel inductor $j(0.47 - 0.19) = j0.28$ susceptance
 or $j\frac{1}{0.28}$ reactance $= \frac{75}{j0.28} = j268 = j\omega L \quad \therefore L = 85.3 \text{ nH}$
 D → O is a series capacitor $-j1.4 = -j105 = \frac{1}{j\omega C} \quad \therefore C = 3.03 \text{ pF}$

Coax. + L { on circle ①
 A → E use $(0.335 - 0.217) \times = 0.118 \times$ then E → O
 use series inductor to cancel $-j1.1 \quad \therefore \omega L = 1.1 \times 75 = 82.5$
 $L = 26.3 \text{ nH}$

Coax. + C { on circle ①
 A → F use $(0.5 - (0.217 - 0.166)) \times = 0.449 \times$ then F → O
 use series capacitor to cancel $j1.12 \quad \therefore \frac{1}{\omega C} = 1.12 \times 75 = 84$
 $C = 3.79 \text{ pF}$

Chart for question 4; to be detached and handed in with script.



4(c) If $\epsilon_r = 2.2$ then $v = \frac{3 \times 10^8}{\sqrt{2.2}} = 2.02 \times 10^8$ m/s

⊙ 500 MHz, $\lambda = 0.405$ m

(L) cost = 10 + 2 = 12 p

(coax. + L) cost = 10 + 0.118 × 0.405 × 20 = 11 p

(coax. + C) cost = 2 + 0.449 × 0.405 × 20 = 5.6 p = cheapest.

(d) For $f: 500 \text{ MHz} \rightarrow 420 \text{ MHz}$

output impedance falls from $165 + j90$ to $165 + j76$: point A'

Then, length of line is $\frac{420}{500} \times 0.118 \lambda = 0.099 \lambda$: point E'

Then, indicator of normalised impedance 1.1 $\rightarrow 1.1 \times \frac{420}{500} = 0.92$
only $\therefore -0.176$ reactance is left.

So, impedance is point 'G'. Reading radius off Ref. Coeff. scale gives $\rho = 0.15$.

3B1 – 2009 Numerical answers

1 (b) 26.2 km (1.33 pW)

1 (c) 17.3 cm each half. Range increases to 32.1 km

1(d) 70%

2 (b) Use a low pass Bessel filter (for pulse shape) @ 9600 Hz. $C_1 = 1.16 \text{ nF}$, $C_2 = 1.03 \text{ nF}$

2(c) $Q = 58$ with equiv. $R = 10.1 \text{ k}\Omega$. For $Q=100$, $R' = 17.4 \text{ k}\Omega$, hence shunt with $-24 \text{ k}\Omega$

3 (a) $R_4 = 100 \text{ }\Omega$, $R_3 = 15 \text{ }\Omega$, $R_2 = 120 \text{ }\Omega$, $R_1 = 680 \text{ }\Omega$

3 (b) 1.46 GHz (or 2.9 GHz incl. source impedance of $100 \text{ }\Omega$)

3 (c) 57.7 mm

3 (e) 0.28 mm

4 (a) $C_{\text{series}} = 3.54 \text{ pF}$, $C = 3.9 \text{ pF}$, $L = 48 \text{ nH}$

4 (b) (i) series 3.03 pF , parallel 85.3 nH
(ii) 26.3 nH and 0.118λ
(iii) 3.79 pF and 0.449λ

4 (c) coax. + C = 5.6 pence

4 (d) Voltage refl. coeff. = 0.15