3B5 2009 P. Migliorato

Q1 Crib

30%

(a) $\frac{d^2\psi}{dx^2} = -\frac{p^2}{\hbar^2}\exp(ipx) = -\frac{p^2}{\hbar^2}\psi$

$p^2 = 2m[E-V(x)]$

$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi = 0$

(b)

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$-\infty < x < 0 \qquad k_1 = \frac{\sqrt{2mE}}{\hbar}$

$\psi(x) = C_1 \exp(-ik_1x) + C_2\exp(+ik_1x)$

$0 < x < a \qquad k_2 = \frac{i\sqrt{2m(V_0 - E)}}{\hbar} = i\alpha$

$\psi(x) = C_3 \exp(-\alpha x) + C_4\exp(\alpha x)$

$a < x < \infty$

$\psi(x) = C_5 \exp(-ik_1x) + C_6\exp(+ik_1x)$

(c)

C_5=0 as there is no wave from the right.

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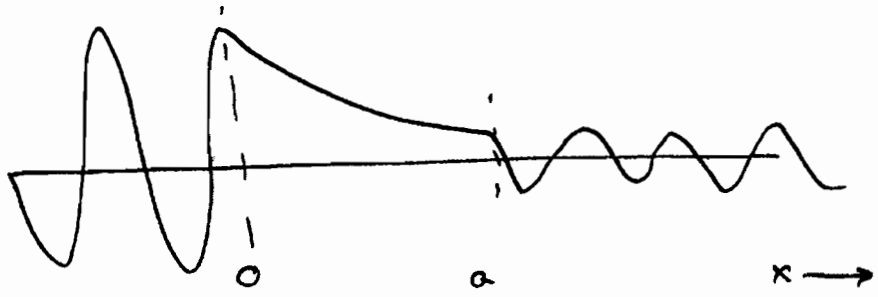


(d)

From the sketch $|\psi|^2$ is different from zero for $a<x<+\infty$. This means that there is a finite probability for the electron to be transmitted through the barrier, in spite of the fact that its energy E is less than the barrier height. This phenomenon has no analogue in classical mechanics and is termed tunnelling. The amplitude of the wavefunction for $-\infty<x<0$ for is larger than for $a<x<+\infty$, since the probability that the particle is reflected is also finite.

10 %

(e)

The amplitude in the region $a<x<+\infty$ depends upon how much the wavefuction decays the in the barrier region, and therefore upon the width and the height of the barrier. This property is put at good use in the Scanning Tunnelling Microscope. This instrument relies on measuring the current between a very sharp metal tip, mounted on a 3D stage and a conducting surface. The current is due to the tunnelling of electrons through the small gap between the two conductors. Due to the aforementioned dependence of the tunnelling probability upon width and height of the potential barrier associated with the gap, one can obtain a nanoscale image of the morphology (width dependence) and chemical composition (heigtht dependence) of the surface. The technique has been employed to obtain images of atoms on a surface using the different spatial distribution of bonding and antibonding wavefunctions.

30 %

Q2 Crib

(a)

i)

15%

The Quasi-Free Electron Approximation

The zero order approximation for an electron in a crystal is that of a free electron, represented by a single wavelength (or monochromatic) wave:

$$\psi_+ = e^{ikx} \qquad (1)$$

We know that waves (whether X-rays or electron waves) can go through a crystal; so monochromatic waves (that is free electrons) may after all represent the truth.

Wave *may* go through a crystal, but not always. There is a strong disturbance when the individual reflections from the crystal planes add in phase; that is under Bragg diffraction conditions

$$n\lambda = 2d\cos\theta, \qquad n = 1,2,3..... \qquad (2)$$

So we may argue that the propagation of electrons is strongly disturbed whenever eqn (2) is satisfied. In one dimension the condition reduces to

$$n\lambda = 2d \qquad (3)$$

Using the relationship between wavelength and wave number the above equation may be rewritten as

$$k = \frac{n\pi}{d} \qquad (4)$$

Thus we may conclude that our free-electron model is not valid when eqn (4) applies.

Brillouin Zones

1st Brilluin zone: $-\frac{\pi}{d} \le k \le \frac{\pi}{d}$

2nd Brilluin zone: $-\frac{2\pi}{d} \le k \le -\frac{\pi}{d}, \qquad \frac{\pi}{d} \le k \le \frac{2\pi}{d}$

3rd Brilluin zone: $-\frac{3\pi}{d} \le k \le -\frac{2\pi}{d}, \qquad \frac{2\pi}{d} \le k \le \frac{3\pi}{d}$ —o

(ii)

Wavefunctions at zone boundary

For k satisfying Equation 4 the wave is reflected, so the wave function should also contain a term representing a wave in the opposite direction:

$$\psi_1 = Ae^{ikx} + Be^{-ikx} \qquad (5)$$

But, since in a large crystal the directions $+k$ and $-k$ are equivalent, setting:

$$\psi_2 = Be^{ikx} + Ae^{-ikx},$$

we must have:

$$|\psi_1|^2 = |\psi_2|^2 \qquad (6)$$

From Equation 6 we obtain:

$$A = \pm B.$$

Therefore the solutions are standing waves:

$$e^{ikx} + e^{-ikx} \rightarrow \cos kx = \cos n\frac{\pi}{d}x$$

$$e^{ikx} - e^{-ikx} \rightarrow \sin kx = \sin n\frac{\pi}{d}x$$

20%

(6)

These are the best approximation to eigenfunctions of Shroedinger's Equation for the crystal, when Bragg's condition is satisfied.

(iii)

Energy gaps

The two standing waves differ substantially in the location of the nodes and therefore in the maxima and minima of the probability density $|\psi^2|$:

the cosine wave has nodes at $x = n\frac{d}{2}$ for odd values of n, that is midway between the crystal ions;

5%

the sine wave has nodes for $x = n\frac{d}{2}$ for even values of n, that is at the crystal ions positions.

So an electron which is in a state represented by the cosine wave, experiences a *different crystal potential* from that experienced by an electron in a state represented by the sine wave. This is tantamount to saying that the eigenvalues corresponding to the two eigenfunctions must be different. This is the origin of the energy gaps.

(b)

$$v = \frac{1}{\hbar}\frac{dE}{dk} \qquad (1)$$

work done $= dE = Fvdt$ (2)

from (1) $dE = \hbar v dk$ (3)

From (2) and (3) we have:

$$F = \hbar\frac{dk}{dt} \qquad (4)$$

$$a = \frac{dv}{dt} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} \qquad (5)$$

Substituting (5) in (4)

$$F = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

we define:

$$m = \frac{\hbar^2}{\frac{d^2E}{dk^2}} \qquad 30\%$$

(c)

The occurrence of a negative effective mass can be explained in the following way.

Away from the Bragg diffraction condition, that is for $|k| << \frac{\pi}{a}$, an electron in a solid can be regarded in first approximation as a free electron, its wavefunction being:

$$\psi = e^{ikx} \qquad (1)$$

Having assumed that this electron is travelling in the direction of positive x.

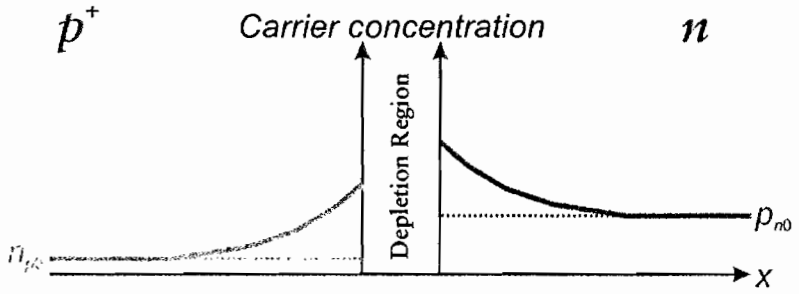
When an external force is applied in the same direction, the electron's momentum increases and therefore k increases too, approaching eventually the value $\frac{\pi}{a}$,

For $k = \frac{\pi}{a}$ the electron wavefunction becomes a standing wave,

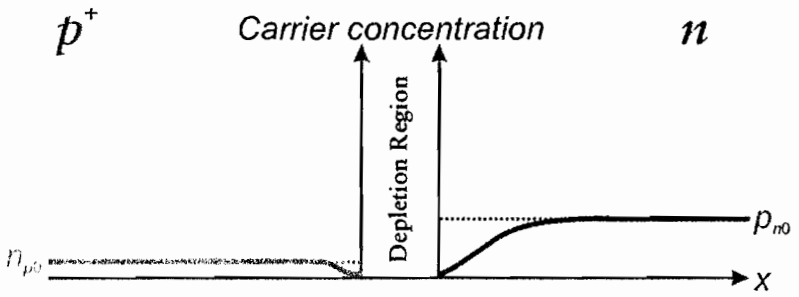
$$\psi = e^{ikx} \pm e^{-ikx}$$

resulting from the superposition of a wave travelling in the direction of +ve x and a reflected wave travelling in the direction of –ve x. Hence the application of a force in the +ve x direction has produced an accelation in the direction of –ve x. This is only consistent with a negative mass for the electron. 30%

3 (a) positive bias applied to the p-type region (forward bias):



negative bias applied to the p-type region (reverse bias):



[20%]

(b) Assumptions: 1) low injection case: the injected minority carrier concentration is much less than the majority carrier concentration
2) no recombination takes place in the depletion region
3) there are negligible fields outside the depletion region.

The continuity equation states

$$\frac{\partial(\Delta p)}{\partial t} = \frac{-\Delta p}{\tau_h} - \mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}$$

Under steady state condition and with assumption 3, this becomes

$$D_h \frac{d^2 p}{dx^2} = \frac{p - p_{n0}}{\tau_h}.$$

This differential equation has the general solution

$$p - p_{n0} = A\exp\left(\frac{-x}{L_h}\right) + B\exp\left(\frac{x}{L_h}\right).$$

The minority carrier concentration can only decay away from the depletion edge, hence $B = 0$ and

$$p - p_{n0} = A\exp\left(\frac{-x}{L_h}\right).$$

The constant A can be found by using the boundary condition that at $x = 0$,

$$p = p(0) = p_{n0}\exp\left(\frac{eV}{kT}\right),$$

this gives the solution

$$\underline{p - p_{n0} = p_{n0}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_h}\right).} \qquad [30\%]$$

(c) The emitter injection efficiency γ for p^+np BJT is the fraction of the total emitter current that is due to holes

$$\gamma = \frac{I_{Ep}}{I}.$$

The total emitter current I is the sum of I_{Ep} and the electron component I_{En}, which is due to electrons injected from base to emitter.

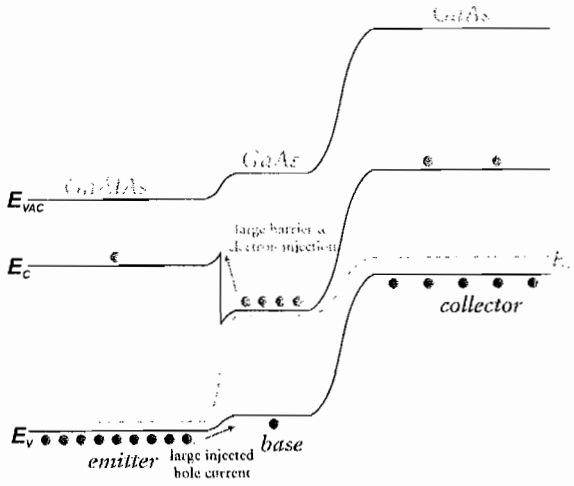
For a BJT, the base width is much less than the hole diffusion length, hence the equation for I becomes

$$I = Ae\left[\underbrace{\left(\frac{D_e}{L_e}\right)\frac{n_i^2}{N_A}}_{\text{electrons}} + \underbrace{\left(\frac{D_h}{W_b}\right)\frac{n_i^2}{N_D}}_{\text{holes}}\right]\left[\exp\left(\frac{eV_{EB}}{kT}\right) - 1\right].$$

Substituting the electron and hole current terms into the expression for γ gives

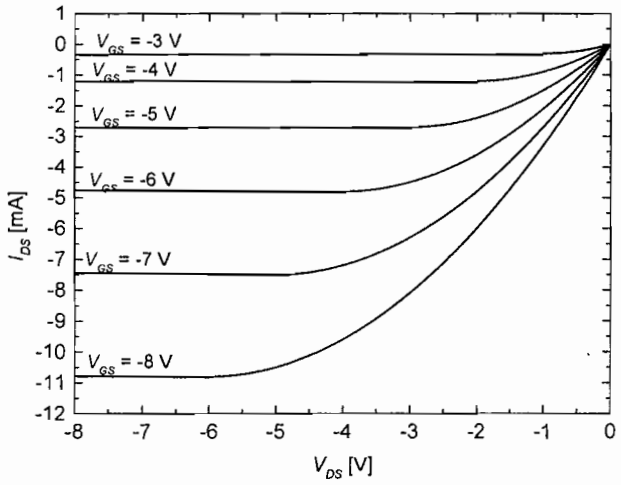
$$\underline{\gamma = \left[1 + \left(\frac{D_e}{D_h}\right)\left(\frac{W_b}{L_e}\right)\left(\frac{N_D}{N_A}\right)\right]^{-1}.} \qquad [30\%]$$

(d) For a HBT, the emitter is fabricated from a semiconductor material with a larger band gap than the material for the base and collector. This creates a large barrier for electron injection from base into the emitter whilst retaining a high doping density in the base. A material system commonly used in HBTs is AlGaAs/GaAs.


GaAlAs
GaAs
GaAlAs
E_{VAC}
E_C
collector
E_V
emitter
large injected
hole current
base

[20%]

(a) Output characteristic for p-channel enhancement MOSFET



At the pinch-off condition there is an insufficient voltage drop across the oxide at the drain end of the device to maintain an hole inversion layer.

(b) (i) strong inversion begins at a surface potential of

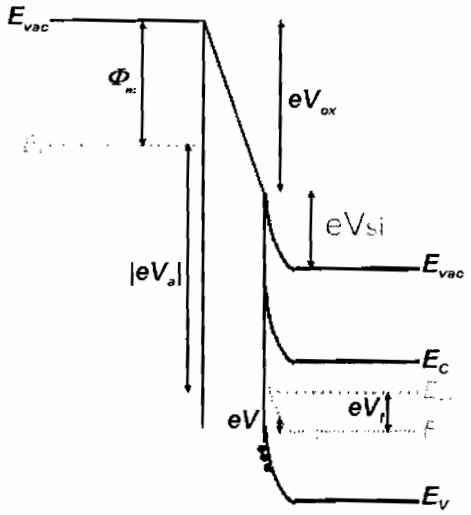
$$V_{si} = \frac{2(E_F - E_i)}{e} = 2V_f \qquad \text{[see band diagram]}$$

in general: $n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

assume all dopants are ionised: $N_D = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

thus $$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$$

and $$V_{si} = 2\frac{kT}{e}\ln\left(\frac{N_D}{n_i}\right) = 0.78 \text{ V}$$



(ii) Calculation of time τ_{inv} required for inversion to occur:

assume all dopants are ionised, i.e. $n_0 = N_D = 5 \times 10^{22}$ m^{-3}

law of mass action: $n_0 \times p_0 = n_i^2$, thus $p_0 = 4.5 \times 10^9$ m^{-3}

at the scattering velocity, v_s, the current density of holes is:

$$J = p_0 e v_S$$

which produces the space charge density at the surface:

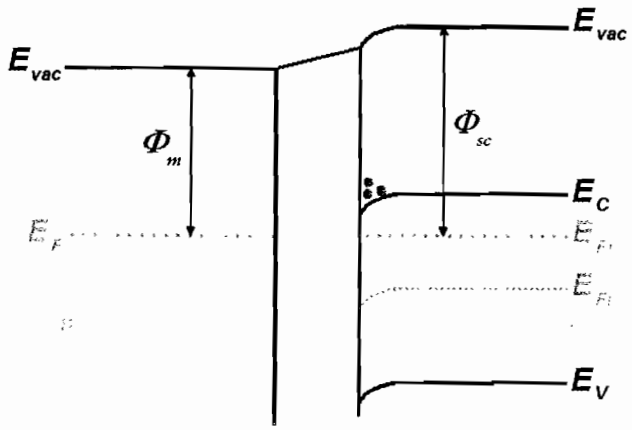
$$Q_S = J\tau = e p_{inv} t_{inv}$$

at strong inversion $n_0 = p_{inv}$, so

$$\tau_{inv} = \frac{e n_0 t_{inv}}{p_0 e v_S}$$

assume $t_{inv} = 5$ nm, which results in $\tau_{inv} = 0.56$ s

(c) n-type MOS capacitor with Φs > Φm



flat band voltage $V_{FB} = \frac{\phi_m - \phi_{sc}}{e}$

In addition to the work function difference, real MOS capacitors are affected by charges in the oxide and at the semiconductor oxide interface. These can be interface trapped charges, fixed oxide charges, oxide trapped charges and mobile ionic charges. In practice the various charges are rolled into an effective charge Q_i, so

$$V_{FBreal} = \frac{\phi_m - \phi_{sc}}{e} - \frac{Q_i}{C_i}$$

where C_i is the insulator, i.e. oxide, capacitance.

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Answers to numerical questions.

Q. 4 b) (i) $V_{si} = 0.78$ V; (ii) $\tau_{inv} = 0.56$s