Prof I White

3B6 Exam Paper 2009 Cribs

Q1. (a) This question can be answered from bookwork.

In terms of the main processes involved in the interaction between photons and electrons, a good answer should include:

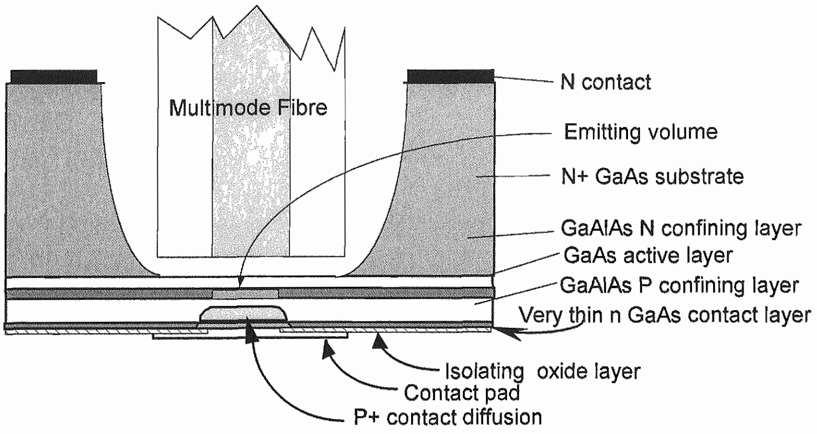
- **Spontaneous Emission:** An electron in a high energy level falls, losing energy which is emitted as a photon – the basis of operation of a light emitting diode (LED).
- **Stimulated Absorption:** An incident photon is absorbed in a material, causing the excitation of an electron to a higher energy level – the basis of operation of a photodiode.
- **Stimulated Emission:** A photon, incident upon an electron in a higher energy level, causes the electron to fall to a lower level thus generating a second photon. This is, therefore, an amplifying action. Two photons are generated from one and, in turn, they can cause the generation of two further photons. Using this method, high optical powers can be generated and this operation is the basis of lasing action. The generated photon has the same frequency and phase as the incident photon and, therefore, very pure monochromatic and coherent light is generated.

The answer should then discuss the operation of the LED, showing the key role of spontaneous emission in generating the optical output. The answer should describe that in its simplest form, a light emitting diode consists of a p-n semiconductor junction at which electrons and holes recombine to generate photons by spontaneous emission. The link between forward voltage, bandgap and wavelength should be made, along with the dependence of output power on current. Good answers also might note the impact of direct and indirect bandgap materials and the role of stimulated absorption in reducing device efficiency

(b) Again this is largely a bookwork section. A good answer would include the following points:

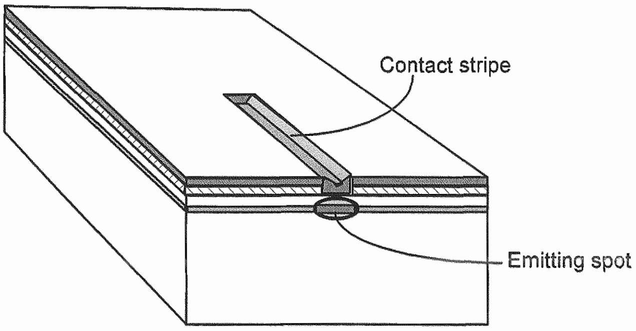
The **Surface Emitting LED** (sometimes called Burrus) diode: Here the emission is normal to the plane of the junction and good emission is often achieved with much of the substrate etched away. A good answer should include detailed comments on the device structure and steps taken to maximise efficiency. It should indicate typical materials involved in construction and pertinent applications.

GaAs based Burrus type high radiance LED



Edge emitting LEDs operate at similar current densities and currents but the emitting spot is much smaller and the use of optical waveguiding increases the maximum brightness available. They are, therefore, often used where the small spot is useful, particularly in high performance fibre optic transmitters, but only when a laser is inappropriate. The answer should describe in detail the structure and operation of the device, steps taken to enhance performance and typical materials used in fabrication. Good answers may indicate the steps taken to avoid stimulated emission.

Edge emitting LED



Good answers should include comments on the relative performance of the two device structures in terms of output power, beam width and far field, and other features affecting their practical use.

(c) (i) $f = eV_{band\text{-}gap}/h \Rightarrow \lambda = hc/(eV_{band\text{-}gap}) = \boldsymbol{1.31\ \mu m}$

(c) (ii) $\eta_{total} = \eta_{ext}.\eta_{int} = \eta_{ext}\,(1/\tau_r)/(1/\tau_r + 1/\tau_{nr}) = \boldsymbol{2.4\ \%}$

(c) (iii) $I = eP\lambda/(\eta hc) = \boldsymbol{44\ mA}$

(c) (iv) $\tau = 1/(1/\tau_r + 1/\tau_{nr}) = \boldsymbol{1.2\ ns}$

Q.2 (a) This question can be answered from bookwork.

A good answer should describe carefully how the laser is visualised when drawing up the rate equations, and in particular the meaning of the carrier concentration and photon density variable. It will then explain the different terms in the photon rate equation (the rate term, the enhancement of photons in the cavity due to stimulated emission, the loss of photons from the cavity through the facets and due to scattering, and the enhancement of photons in the lasing mode in the cavity due to spontaneous emission). The terms in the electron rate equation should also be defined (the rate term, injection of current, depletion due to spontaneous emission and that due to stimulated emission).

The answer should indicate assumptions made in deriving the equations, namely (i) the carrier, photon and current densities are constant in the diode laser throughout its volume, (ii) that the laser generates purely monochromatic light in one mode, (iii) that the amplification of light by stimulated emission is linear with carrier concentration and, (iv) that temperature effects are negligible.

(b) The derivation of the equations for the carrier concentration should follow conventional bookwork and first consider the photon rate equation:

$$\frac{dP}{dt} = g(n - n_o)P + \beta\frac{n}{\tau_s} - \frac{P}{\tau_p}$$

At steady state, dP/dt = 0 and assuming that β is very small, the photon rate equation becomes,

$$0 = g(n - n_o)P - P/\tau_p$$
$$=> P\{g(n - n_o) - 1/\tau_p\} = 0$$

As P >0, $g(n - n_o) - 1/\tau_p = 0$

$=> n = n_o + 1/(g\tau_p)$ above threshold

However all the terms on the right hand side of the equation are constants. Maintaining a steady state for all values of lasing photon density greater than zero, the carrier constant in the laser is constant. Let this value be also called the threshold carrier density, n_{th}.

Considering the electron rate equation,

$$0 = -g(n - n_o)P - n/\tau_s + I/(eV)$$

Below threshold, when P = 0 (there is no lasing light generated), the electron rate equation becomes simply $n = I\tau_s/(eV)$

As a result the threshold current $I_{th} = eVn_{th}/\tau_s$

$$=> I_{th} = eV[n_o + 1/(g\tau_p)]/\tau_s$$

(c) The photon lifetime of the laser cavity can be readily determined by considering the amplification of laser light as it propagates along the laser cavity.

Assume that stimulated emission encounters a gain per unit length (due to stimulated amplification), G, and a loss per unit length due to scattering and absorption, α, as it passes along the laser. The gain G in practice creates extra photons to compensate for those photons lost as the signal travels over a distance of unit length.

One can write that a lasing signal, A, starting at one facet will be then incident on the opposite facet with an optical power

$$B = \exp\{(G - \alpha)L\} A$$

At that point part of the signal is reflected with a reflectivity R and the signal then passes back amplified by 1 the same amount as above and again reflected by the initial facet. Lasing action occurs when the net round trip gain of the signal is unity;

$$A.\exp\{(G-\alpha)L\}.R.\exp\{(G-\alpha)L\}.R = A$$

$$=> \quad G = \alpha + (1/L)\ln(1/R)$$

The proportion of photons lost per unit time is simply the gain G times the speed of light in the laser material, c/n_r. As a result the average time for which one photon remains in the cavity, the photon lifetime, is given by

$$\tau_p = 1/Gv_g = 1/\{(c/n_r)\{\alpha + (1/L)\ln(1/R)\}\}$$

(d) A good answer should explain how the differential quantum efficiency, η_D, is simply the proportion of photons leaving the cavity through the facets over the total number of photons,

$$\eta_D = \frac{\ln(1/(R_1R_2))/(2L)}{\alpha + \ln(1/(R_1R_2))/(2L)}$$

Q.3 (a) This question can be answered from bookwork.

Answers should include the main features of the photodiodes. In the case of the p-i-n photodiode, the p and n regions of the diode are separated by an intrinsic (i) region. This is chosen to have a width greater than (or equal to) the absorption length. In this case, the applied voltage is dropped only across the i region. This means that, just so long as the depletion region extends across the i region, then a fast response is achieved. Also, the maximum depletion width is the width of the i region, so the problem of very wide depletion widths is avoided. P-i-n photodiodes are probably the most common form of high bandwidth photodiode, with 10GHz bandwidths being readily achievable.

Avalanche photodiodes make use of the property of junction diodes that, if carefully designed and fabricated, begin to *multiply* their reverse leakage or optically generated reverse currents just before they go into full reverse breakdown. Breakdown itself is caused by avalanche processes, where a high energy hole or electron hits an atom in the lattice and ionises it, thereby generating more current. Avalanche multiplication is making use of the same process, but it needs a very special diode, and careful control of the bias voltage.

Good answers should describe the structures of the diodes in detail, indicating the different impact of the layer design (and depletion region) on absorption and transport characteristics of the devices. Trade-offs between capacitance and transport should be explained.

The answer should also comment on the performance of the devices (including their advantages and disadvantages) in terms of responsivity and linearity, sensitivity, speed (including comments on capacitance), dark current, bias requirement and spectral response. Examples should be given of typical applications of each device, along with the materials that might be used in their construction.

(b) The answers here should describe the cause and effect of shot noise and thermal noise in receivers, with additional comments being made on the role of multiplication noise in avalanche photodiodes. Good answers may also make reference to quantum noise and limiting receiver sensitivity.

(c) Noise Power $= 4kTB/R + 2e(\eta(e/(hf))P + I_{dark})B$

$$= (\eta(e/(hf))P)^2 \quad \text{when SNR} = 1$$

$$\Rightarrow 0 = (4kTB/R + 2e\, I_{dark}B) + 2e\eta(e/(hf))BP - (\eta(e/(hf))P)^2$$

$$= a + bP + cP^2 \text{ say}$$

$$\Rightarrow P = (-b+\sqrt{(b^2 - 4ac)})/2c = \boldsymbol{0.117\ \mu W}$$

(d) Noise Power $= 4kTB/R + 2e(\eta(e/(hf))P + I_{dark})BM^{2+x}$

$$= (\eta(e/(hf))P)^2 M^2 \textit{ at receiver sensitivity}$$

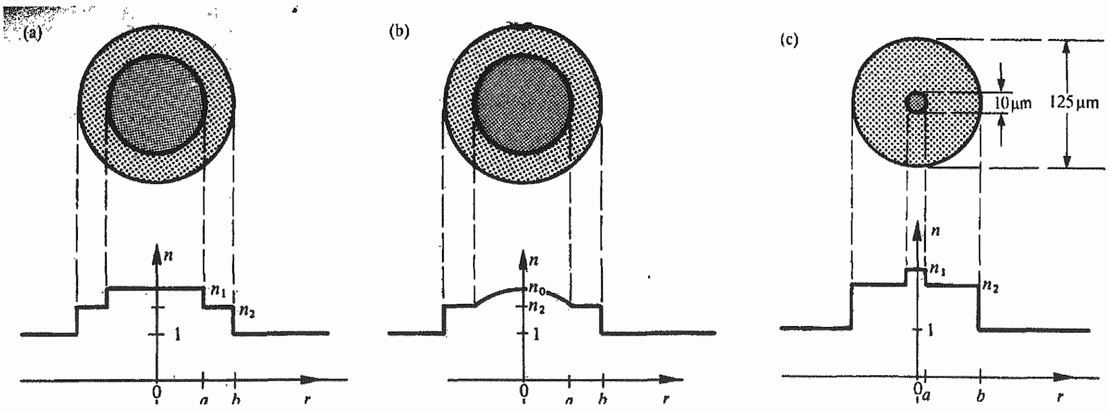
$$\Rightarrow 0 = [4kTB/R + 2eI_{dark}BM^{2+x}] + 2e\eta ePBM^{2+x}/(hf) - (\eta(e/(hf))P)^2 M^2$$

$$\Rightarrow 0 = a + bP + cP^2$$

Solving for the Quadratic gives => $\boldsymbol{P = 16\ nW}$

Q.4 (a) This answer can be based on bookwork.

The answer should describe at least three main types of optical fibre, namely step index (SI) multimode fibre, graded index multimode fibre and step index single mode fibre. Example refractive index profiles should be given (for example as shown below). Comments should include the typical scale of the fibre parameters. For example that multimode fibres have a core diameter typically in excess of 50μm. Step index multimode fibre is relatively cheap, easy to handle and to join together, but it does suffer from high dispersion, and therefore limited bandwidth. Graded index multimode fibre is the most expensive type but it combines the other advantages of SI multimode fibre with greatly reduced dispersion. This is the predominant fibre type used in in-building (up to 550m) applications. Finally, SI single mode fibre has a core diameter <10μm – resulting in only one mode being allowed. This means that dispersion is low but the fibres are quite difficult to handle. Because of the low dispersion, it is the only choice for long distance transmission (>2km – 10,000km). However trade-offs do exist between coupling efficiency and dispersion, and good answers should outline the extent of this and its impact on system design.



Answers should also describe the spectral dependence of attenuation, and together with dispersion should give typical parameters for overall fibre performance when used in transmission applications. The impact of polarisation dependence, nonlinearities, bend loss and environmental effects should also be given in good answers.

(b) Again, this question can be largely based on bookwork, and answers should give detailed descriptions of intermodal dispersion, material dispersion, and waveguide dispersion. Good answers should indicate the relative strengths of the different types of dispersion as a function of fibre type and operating wavelength.

Although single mode fibre, with the lowest dispersion, is used in the longest links, both material and waveguide dispersion are important and have spectral dependence. Answers should indicate that to mitigate this, compensating components including gratings, filters and fibre itself, with the opposite dispersion can be introduced.

Good answers should indicate that electronic techniques can also now be used. Ultimately other techniques such as wavelength division multiplexing can be used to allow higher transmission rates to be achieved.

(c) (i) $NA = (n_{core}^{2} - n_{cladding}^{2})^{1/2} = 0.1 \Rightarrow \boldsymbol{n_{core} = 1.493}$

(c) (ii) $a = 2.4\lambda/(2\pi NA) = 5.7\ \mu m \Rightarrow$ diameter $= \boldsymbol{11.4\ \mu m}$

(c) (iii) $\tau_{disp} = (\tau_{out}^{2} - \tau_{in}^{2})^{1/2} = 300\ ps = D\delta\lambda L = 1/(4B) \Rightarrow \boldsymbol{L = 167\ km}$

(c) (iv) Power received = P_{in} x insertion loss x fibre loss over 167 km
= ***– 54 dBm***

Numerical Answers

Q.1 (c) (i) 1.31 μm, (ii) 2.4 %, (iii) 44 mA, (iv) 1.2 ns
Q.3 (c) 0.12 μW, (d) 16 nW
Q.4 (c) (i) 1.493, (ii) 11.4 μm, (iii) 167 km, (iv) -54 dBm