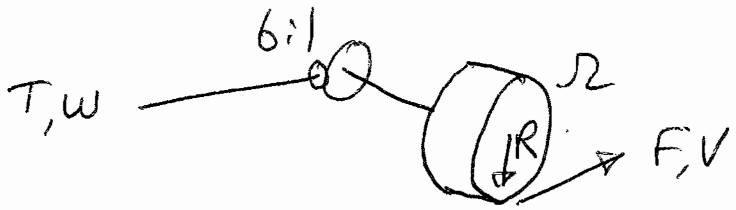
1/ a) see lecture notes

b) convert F–V characteristic to T–ω



$R = 0.3\,m.$

assume no power loss $T.\omega = F.V$

let $\frac{\omega}{\Omega} = G = 6.$

road wheel $V = \Omega R \rightarrow V = \frac{\omega}{G}.R$

and $F.R = T.G \rightarrow F = \frac{T.G}{R}$

replace F and V in the load characteristic equation:

$$\frac{T.G}{R} = 200 + 0.5\,\frac{\omega^2}{G^2}R^2$$

$$T = \frac{R}{G}\left(200 + 0.5\,\frac{\omega^2}{G^2}R^2\right)$$

$$= \frac{R}{G}.200 + 0.5\,\frac{R^3}{G^3}\,\omega^2$$

let $\omega = \frac{2\pi n}{60}$ n is speed in rpm

$$T = \frac{R}{G}200.\quad 0.5\left(\frac{2\pi}{60}\right)^2\frac{R^3}{G^3}n^2$$

n/rpm	0	4000	8000	12 000	16 000	20 000
T/Nm	10	21	54	109	185	284.

Plot on the output characteristic. The lines intercept at the 19,000 rpm rev limit, which is also the maximum output power.

Hence vehicle speed $V = \frac{\omega R}{G} = \frac{2\pi n}{60}\frac{R}{G} = \frac{2\pi}{60}\frac{19{,}000 . 0.3}{6}$

$$\underline{\underline{V = 99.5 \text{ m/s}}}$$

Increasing the speed ratio would move the load characteristic downwards. The veh~~ic~~ vehicle speed would reduce because the engine speed could not exceed 19,000 rpm. Decreasing the speed ratio would move the load characteristic upwards. The operating point would then occur at a lower power, implying lower speed.

(c) (i) Add 50 kW to the engine characteristic. Find where this intercepts the 500 kW power contour. New rev. limit is ~16,300 rpm.

New speed ratio is $6 \times \frac{16.3}{19} = \underline{\underline{5.15:1}}$

(ii) Max. difference between combined output characteristic and the load characteristic is ~ 265 Nm at ~ $\underline{\underline{\text{7500 to 8000}}}$ rpm.

Tractive force is $F = \frac{T.G}{R} = \frac{265.6}{0.3} = 5300\text{ N}$

acceleration $F = ma \therefore a = \frac{5300}{700} = \underline{\underline{7.6 \text{ m/s}^2}}$

(iii) Deceleration torque is ~135 Nm

Hence deceleration is $7.6 \times \frac{135}{265} = \underline{\underline{3.9\ m/s^2}}$

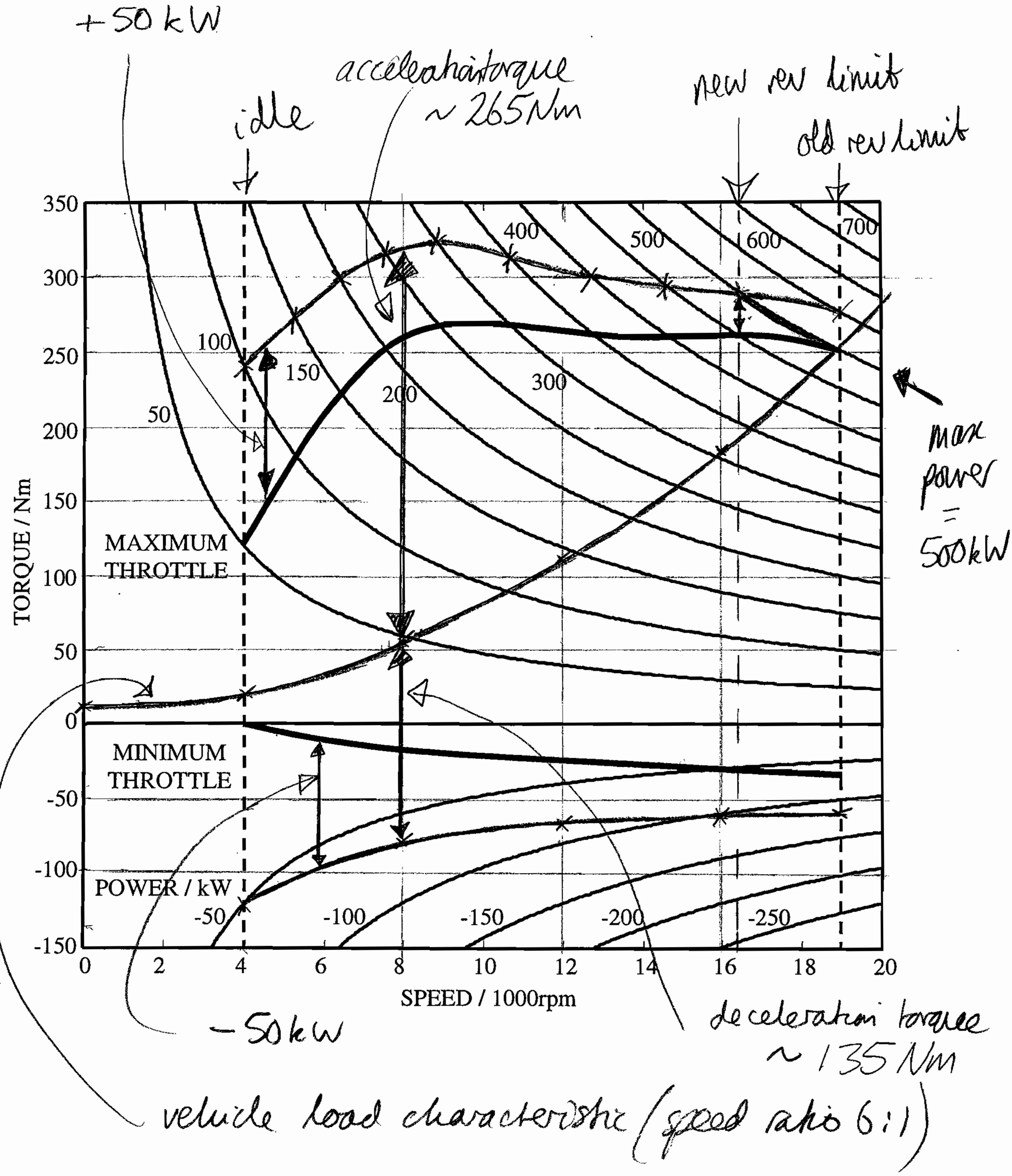
Examiner's comments

Q1 50 attempts. Always a popular topic but this year not very well done. Plotting the load line on the torque/speed axes is essential to part (b). Part (c)(i) was generally OK but (c)(ii) & (iii) produced a variety of curious methods.

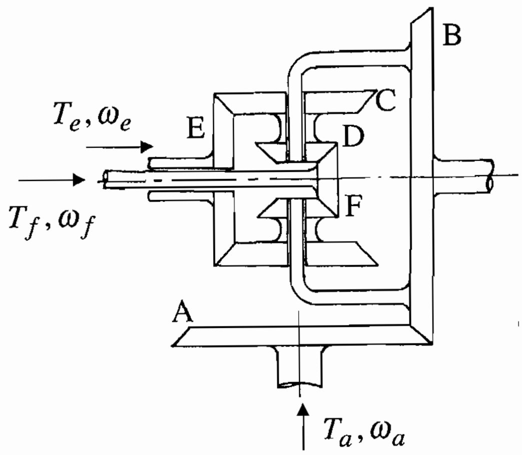
Q2 35 attempts. Part (a) can be done – or at the very least checked – from the displayed equation in part (b) by simply putting $w_e = w_f$. In fact, all can be done from the given equation plus a statement of virtual power. Deriving the given equation was not very well done.

Q3 23 attempts. First part, book work was well done. Because the relation between load and deflection in a Hertzian contact is non-linear it is necessary to do the integration to obtain stored energy – it is not $\frac{1}{2}P\delta$ and certainly not $P\delta$. Quite a few reasonable attempts but few completely correct.

Q4 46 attempts. Parts (a) & (b)(ii) generally well done. In b(iii) many candidates did not take into account the change in the effective pressure angle (to 20.64°) brought about by changing centre spacing of the gears.


+50 kW
accelerating torque ~ 265 Nm
idle
new rev limit
old rev limit
max power = 500kW
TORQUE / Nm
SPEED / 1000rpm
MAXIMUM THROTTLE
MINIMUM THROTTLE
POWER / kW
-50kW
deceleration torque ~ 135 Nm
vehicle load characteristic (speed ratio 6:1)

2.



Tooth numbers

$$\begin{cases} A = 20 \\ B = 100 \\ C = 20 \quad D = 15 \\ E = 30 \\ F = 15 \end{cases}$$

	w_a	w_b	$w_{c/d}$	w_e	w_f
① Hold A/B rotate C/D	0	0	1	$-\frac{C}{E}$	$+\frac{D}{F}$
② rotate B	$-B/A$	1	1	1	1
General motion x① + y②	$-\frac{B}{A}y$	y	$x+y$	$-\frac{C}{E}x+y$	$\frac{D}{F}x+y$

If shafts E & F same speed $-\frac{C}{E}x+y = \frac{D}{F}x+y$

ie. $x \Rightarrow 0$ so motion ②

then $\frac{w_a}{w_e} = -\frac{B}{A}$

(b) $w_a = -\frac{B}{A}y$; $w_e = -\frac{C}{E}x + y$; $w_f = \frac{D}{F}x+y$

$$\therefore \begin{cases} \frac{D}{F} w_e = -\frac{C}{E}\cdot\frac{D}{F}x + \frac{D}{F}y \\ \frac{C}{E} w_f = \frac{C}{E}\cdot\frac{D}{F}x + \frac{C}{E}y \end{cases}$$

$$\therefore \frac{D}{F}w_e + \frac{C}{E}w_f = \left(\frac{D}{F}+\frac{C}{E}\right)y = \left(\frac{D}{E}+\frac{C}{E}\right)x - \frac{A}{B}w_a$$

ie. $\frac{E}{C}w_e + \frac{F}{D}w_f + \frac{A}{B}\left\{\frac{E}{C}+\frac{F}{D}\right\} w_a = 0$ —(α)

Check (α) if $w_e = w_f$ then

equ (α) $\left(\frac{E}{C} + \cancel{\frac{F}{D}}\right)\omega_e + \frac{A}{B}\left(\frac{E}{C} \cancel{+} \frac{F}{D}\right)\omega_a = 0$

$$\therefore \quad \frac{\omega_a}{\omega_e} = -\frac{B}{A} \quad \text{as in (a)}$$

(c)(i) Putting in tooth numbers & $\omega_e = 2\omega_f$

(α) becomes: $\frac{\omega_e}{2} \cdot \frac{15}{15} + \frac{30}{20}\omega_e + \omega_a \frac{20}{100}\left(\frac{30}{20} + \frac{15}{15}\right) = 0$

$$\text{ie.} \quad 2\omega_e + \omega_a \cdot \tfrac{1}{2} = 0$$

$$\therefore \quad \frac{\omega_a}{\omega_e} = -4$$

(ii) In general $\frac{3}{2}\omega_e + \omega_f + \frac{1}{2}\omega_a = 0$

V. Power $T_a \omega_a^* + T_b \omega_b^* + T_e \omega_e^* + T_f \omega_f^* = 0$

T_a, T_b, T_e, T_f real torques; $T_b \Rightarrow 0 \because$ freerunning

$\omega_a^*, \omega_b^*, \omega_e^*, \omega_f^*$ any compatible velocities

choose $\omega_e^* = 0$ then $-\frac{2}{3}x + y = 0$ $\quad \frac{x}{y} = \frac{3}{2}$.

$$\therefore \quad T_a \omega_a^* + T_e \omega_e^* + T_f \omega_f^* = 0$$

becomes $-5yT_a + 0 + T_f(x+y) = 0$

$$-5T_a + T_f \cdot \frac{5}{2} = 0$$

$$T_f = 2T_a$$

Now choose $\omega_f^* = 0$ then $x + y = 0$ $\quad \frac{x}{y} = -1$

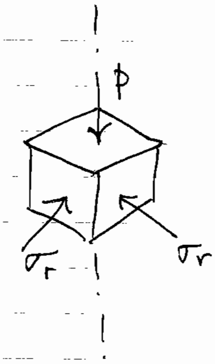
$$\therefore \quad 5yT_a + T_e \cdot \left(-\tfrac{2}{3}x + y\right) = 0$$

$$\therefore \quad T_e = 3T_a$$

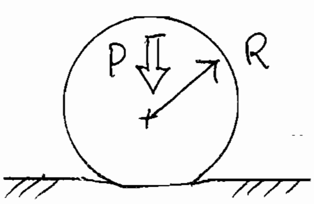
(iii) Powers $\frac{P_e}{P_f} = \frac{T_e \omega_e}{T_f \omega_f} = \frac{3T_a}{2T_a} \cdot 2 = 3:1$

(a) Point loading of an elastic surface generates a set of triaxial stresses. Because of the constraint provided by surrounding material radial stresses will be compressive.

Since yielding depends on stress differences the magnitude of p, the surface pressure, will be greater than Y the uniaxial yield stress.



(b)



If all elastic the WD in pressing surfaces together through "squash" δ

will be $U = \int_0^\delta P\,d\delta$

But at any stage $\delta = \frac{1}{2}\left(\frac{9}{2}\frac{P^2}{E^{*2}R}\right)^{1/3}$ (Data sheet)

So that $d\delta = \left(\frac{9}{16E^{*2}R}\right)^{1/3} \frac{2}{3} P^{-1/3}.\,dP$

$$\therefore \quad U = \frac{2B}{3}\int P.P^{-1/3}\,dP \quad \text{where} \quad B = \left(\frac{9}{16E^{*2}R}\right)^{+1/3}$$

$$\therefore \quad U = \frac{2B}{3}\cdot\frac{3}{5}\left[P^{5/3}\right]_0^P = \frac{2B}{5}P^{5/3}$$

But also from Data sheet $p_0 = \frac{1}{\pi}\left(\frac{6PE^{*2}}{R^2}\right)^{1/3}$

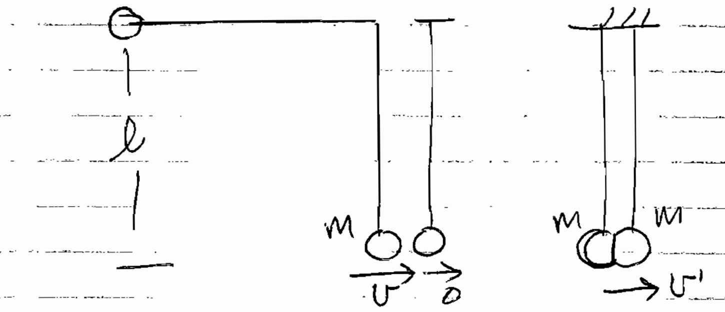
i.e. $P = \pi^3 \frac{R^2}{6E^{*2}}\cdot p_0^3$

So substitute for P, $U = \frac{2}{5}\left(\frac{9}{16E^{*2}R}\right)^{1/3}\cdot\left(\frac{R^2}{6E^{*2}}\right)^{5/3}\pi^5 p_0^5$

$$U \Rightarrow \frac{\pi^5}{60}\frac{p_0^5}{E^{*4}}\cdot R^3$$

Note that since (i) one surface has $R \to \infty$, $R \Rightarrow R_{ball}$
and (ii) one surface rigid i.e. $E \to \infty$
$E^* = \frac{E}{(1-\nu^2)}$

(c) During collision mtm conserved, so equating mtm of first mass just before contact to combined mtm when both masses moving at same speed



$$mv = 2mv'$$
$$\therefore v' = \tfrac{1}{2}v$$

$\therefore$ Stored elastic energy

$$= \tfrac{1}{2}mv^2 - \tfrac{1}{2}(2m)\left(\tfrac{1}{2}v\right)^2$$
$$= \tfrac{1}{2}mv^2 - \tfrac{1}{4}mv^2$$
$$= \tfrac{1}{4}mv^2$$

But $\tfrac{1}{2}mv^2 = mgl$

$$v^2 = 2gl$$

$\therefore$ Stored elastic energy $= \tfrac{1}{4}m \cdot 2gl$

$$\Delta U_{total} = \frac{mgl}{2}$$

Now, since spheres same size and same material, assume (i) energy into each equal to $\frac{mgl}{4}$, and (ii) interface between them is planar. Thus can treat each sphere as colliding with rigid plane as pt (b)

then $$U = \frac{\pi^5}{60}\frac{p_0^5}{E^{*4}}R^3 = \frac{mgl}{4} = \frac{4\pi}{3}\rho R^3 \frac{gl}{4}$$

where $E^* = E/(1-\nu^2)$

If all stresses to be elastic $\tau_{max} \leq k$

i.e. $0.310\, p_0 \leq k$, so that

$$\frac{\pi^{\cancel{5}\,4}}{60}\frac{k^5}{(.310)^5}\frac{\cancel{R^3}}{E^{*4}} > \frac{\cancel{4}}{3}\cancel{\pi}\cancel{R^3}\rho\frac{gl}{\cancel{4}}$$

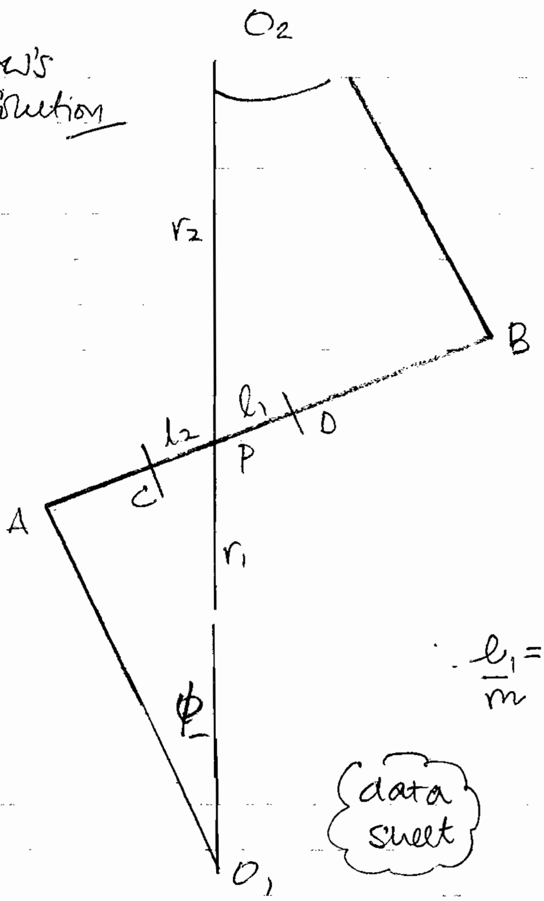
i.e. $$k^5 \geq \frac{60\times(.310)^5}{\pi^4}\rho gl E^{*4}$$

or $$k \geq .281\sqrt[5]{\rho gl E^{*4}}$$

e.g. if $\rho = 7840\,kgm^{-3}$, $g = 9.81\,ms^{-2}$; $l = 0.1\,m$, $E^* = 231\,GPa$ $k \geq 607\,MPa$

JAW's Solution

(a)



$m = \frac{2r}{N}$

$\therefore r_1 = \frac{20 \times 3}{2} = 30\text{mm}$

$r_2 = \frac{60 \times 3}{2} = 90\text{mm}$

$\therefore O_1O_2 = r_1 + r_2 = 120\text{mm}$

$a = m = 3\text{mm} \quad \phi = 20°$

$\therefore \frac{l_1}{m} = [0.02924 \times 20^2 + 20 + 1]^{1/2} - .1710 \times 20$

(data sheet)

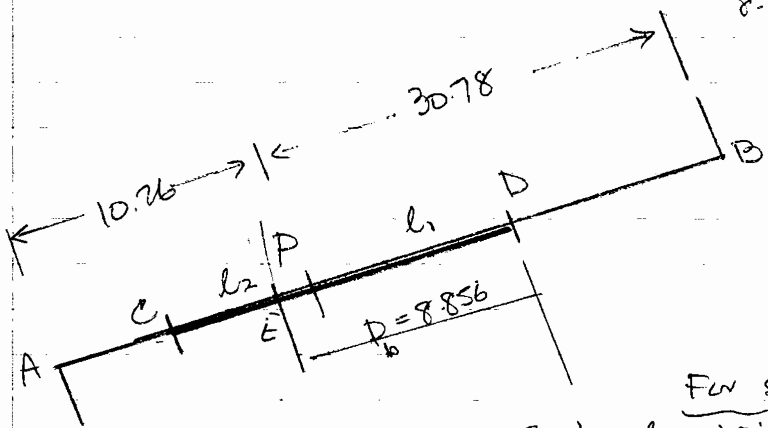
hence $l_1 = 6.89\text{mm}$

and $l_2 = 7.903\text{mm}$

$\therefore l_1 + l_2 = 14.79\text{mm}$

But $p_b = \pi m \cos\phi \Rightarrow 8.856\text{mm}$

Contact ratio $= \frac{14.793}{8.856} = 1.67$ ✓



$PA = r_1 \sin 20 = 10.261$

$PB = r_2 \sin 20 = 30.782$

For single pair contact critical point at E

$R_1 = 10.26 + 6.89 - 8.856 = 8.29$

$R_2 = 30.78 \cancel{+ 0.1583} - 6.89 + 8.86 = 32.75$

$R = 6.62\text{mm}$

Double pair critical point at C

$R_1 = AC = 10.26 - 7.90 = 2.36$

$R_2 = BC = 30.78 + 7.90 = 38.68$

$R = 2.22\text{mm}$

b(i) Increasing the centre distance will increase the pressure angle, move the contact points higher up the teeth – increasing the effective pitch circle – and so decrease the contact ratio.

However the base pitch is unchanged and the gears will still function as involutes giving a constant speed ratio. Backlash will though also be increased.

(ii) Base pitch is given by the spacing of the teeth on the base circle, the fundamental feature defining the tooth geometry. This tooth spacing is unaltered. However the location of the pitch point and hence the pitch radius & circumferential pitch is affected.

To calculate the 'new' pressure angle ϕ'

$$120.5 = r_1' + r_2' = (r_{b_1} + r_{b_2})/\cos\phi' = \frac{120 \cos 20^\circ}{\cos\phi'}$$

hence $\phi' = 20.64^\circ$

$$\therefore \quad \left.\begin{array}{ll} r_1' = 120.5/4 & \text{i.e. increase of } 0.125\text{mm} \\ r_2' = 3 \times 120.5/4 & \quad " \qquad " \quad 0.375\text{ mm} \end{array}\right\}$$

Because the pitch circle radius has increased the module has decreased. Originally addendum $a_1 = a_2 = 3$mm.

$$a_1' \Rightarrow 3 - 0.125 = 2.875 \text{ mm}$$
$$a_2' \Rightarrow 3 - 0.375 = 2.625 \text{ mm}$$

Hence using Data Sheet formula

$$\ell_1' = \{r_1'^2 \sin^2\phi' + a_1'(2r_1' + a_1')\}^{1/2} - r_1' \sin\phi'$$

$$\Rightarrow \left\{\left(\frac{120.5}{4}\right)^2 \sin^2(20.64) + 2.875\left(\frac{2 \times 120.5}{4} + 2.875\right)\right\}^{1/2} - \frac{120.5}{4} \sin 20.64$$

$$\Rightarrow 6.535$$

$$\ell_2' = \left\{\left(\frac{3 \times 120.5}{4}\right)^2 \sin^2(20.64) + 2.625\left(\frac{2 \times 3 \times 120.5}{4} + 2.625\right)\right\}^{1/2} - \frac{3 \times 120.5}{4} \sin 20.64$$

$$\Rightarrow 6.824$$

$$\therefore \quad \text{C-Ratio} = \frac{6.535 + 6.824}{8.86} = 1.51$$

JAW
11-09

Answers

1 (b) 99.5 ms^{-1} (c)(i) 5.15 : 1 (ii) ~ 7.6 ms^{-2} (iii) ~ 3.9 ms^{-2}

2 (c)(i) $\dfrac{\omega_a}{\omega_e} = -\dfrac{B}{A}$ (ii) $T_e = 3T_a$; $T_f = 2T_a$ ((iii) $P_e : P_f = 3:1$

3 (c) $k \geq 0.281 \sqrt[5]{\rho g \ell E^{*4}}$

4 (a) contact ratio = 1.67 single $R_{\min} = 6.62$ mm; double $R_{\min} = 2.22$ mm

$\phi' = 20.64°$, contact ratio = 1.51