Prof M D Bolton

Engineering Tripos Part IIA : 2009

Module 3D1 Geotechnical Engineering I

1. a) The time for transient flow is proportional to the quantity of water that will be squeezed out, and inversely proportional to its flowrate. So $t \propto 1/[E_o k]$. We use composite parameter $c_v = E_o k/\gamma_w$ so that we can write dimensionless relationships of the form:

$$R_v = \frac{\rho}{\rho_{ult}} = F(T_v) \quad \text{where } T_v = c_v t/d^2$$

b) Initial increase in total stress, $\Delta\sigma = 100/1 = 100\text{kPa}$

If this is constant for 2m below the footing

$$\frac{\rho_{ult,A}}{2} = \frac{\Delta\sigma}{E_o} = \frac{100}{4000} = 0.025$$

$$\therefore \underline{\rho_{ult,A} = 50\text{mm}}$$

For footing B, $\Delta\sigma = 100/W$ but we want $\rho_{ult,B} = 50$ mm

$$\therefore \frac{0.050}{4} = \frac{100}{W \times 4000}$$

$$\therefore \underline{W = 2\text{m}}$$

c) The clay enjoys double drainage, so $d_A = 1$m and $d_B = 2$m. So $T_{v,A} = 1 \times t\text{years}/1^2 = t\text{years}$, $T_{v,B} = t/4$

Footing A	$t = 1/12$ yr	$t = 1/3$ yr	$t = 1$ yr
$T_{v,A}$	$1/12$	$1/3$	1
$R_{v,A}$	0.33	0.65	0.94
ρ_A	17mm	33mm	47mm

Footing B	$t = 1/12$ yr	$t = 1/3$ yr	$t = 1$ yr
$T_{v,B}$	0.021	0.083	0.25
$R_{v,B}$	0.17	0.33	0.56
$\rho_{,B}$	9 mm	17 mm	28 mm
$\rho_{average}$	13 mm	25 mm	37 mm
ρ_{diff}	8 mm	16 mm	19 mm

Note: R_v @ T_v was obtained from the Table on P6 of the Data Book, based on Fourier series solutions. Solutions based on parabolic isochrones could alternatively have been used, giving slightly different answers ± 1 mm.

d) A calculation is required rather than an estimate based on a few more trials — for which half marks on this section could nevertheless be obtained for a conscientious effort. Use formula for Phase (ii) parabolic isochrone dissipation, with $T_{v,A} = 4\,T_{v,B}$.

$$\rho_{diff} = (R_{v,A} - R_{v,B})\,\rho_{ult}$$

$$= \rho_{ult}\left[\left(1-\tfrac{2}{3}\exp\left(\tfrac{1}{4}-12T_{v,B}\right)\right)-\left(1-\tfrac{2}{3}\exp\left(\tfrac{1}{4}-3T_{v,B}\right)\right)\right]$$

$$= \tfrac{2}{3}\rho_{ult}\left[\exp\left(\tfrac{1}{4}-3T_{v,B}\right) - \exp\left(\tfrac{1}{4}-12T_{v,B}\right)\right]$$

$$\therefore \frac{d\rho_{diff}}{dT_{v,B}} = \tfrac{2}{3}\rho_{ult}\left[-3\exp\left(\tfrac{1}{4}-3T_{v,B}\right)+12\exp\left(\tfrac{1}{4}-12T_{v,B}\right)\right]$$

$$= 0 \text{ when } \exp\left(\tfrac{1}{4}-3T_{v,B}\right) = 4\exp\left(\tfrac{1}{4}-12T_{v,B}\right)$$

So $\rho_{diff,max}$ when $\frac{1}{4}-3T_{v,B} = \ln 4 + \frac{1}{4} - 12T_{v,B}$

$\therefore T_{v,B} = 0.154$, $T_{v,A} = 0.616$: $R_{v,B} = 0.461$, $R_{v,A} = 0.865$

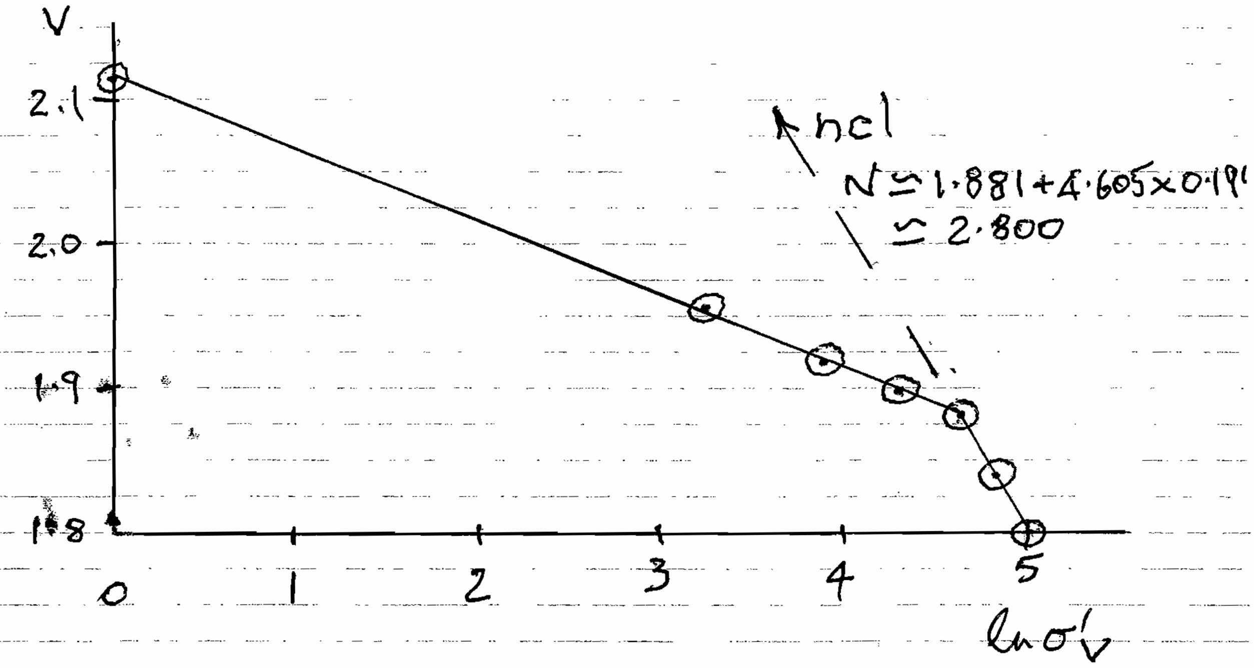
$\rho_{diff} = 20$ mm at $t = 0.616$ y $= 7.4$ months

2. a) At 1 kPa, $e = wG_s = 0.42 \times 2.65 = 1.113$

$\therefore v = 1 + e = 2.113$

Using $v \propto h$ we get:

σ_v' kPa	1	25	50	75	100	125	150
$\ln \sigma_v'$	0	3.219	3.912	4.317	4.605	4.828	5.011
v	2.113	1.955	1.918	1.895	1.881	1.836	1.800



$$\kappa = \frac{(2.113 - 1.881)}{(4.605 - 0)} = \underline{0.0504}$$

$$\lambda = \frac{(1.881 - 1.800)}{(5.011 - 4.605)} = \underline{0.1995}$$

$\Gamma + \lambda - \kappa = 2.800$ so $\Gamma = \underline{2.65}$

$\underline{\sigma'_{v,max} = 100 \text{ kPa}}$

b) Initially, guess $\gamma = 18 \text{ kN/m}^3$

$\therefore \sigma_v' \simeq 5 \times 8 \simeq 40 \text{ kPa}$ so $\ln \sigma_v' = 3.6889$

$\therefore v \simeq 2.113 - 0.0504 \times 3.689 = 1.927$

$\therefore \rho = (2.65 + 0.927) \times 1000 / 1.927 = \underline{1856 \text{ kg/m}^3}$

$\therefore \underline{\gamma = 18.2 \text{ kN/m}^3}$ OK

c) Dry $\gamma = 1650 \times 9.81/1000 = 16.2\ kN/m^3$

Saturated, and assuming $G_s = 2.65$,

$v = 2.65/1.65 = 1.606$ so $e = 0.606$

$\therefore \gamma'_{sat} = 1.65 \times 9.81/1.606 = 10.1\ kN/m^3$

So $\Delta\sigma = 3 \times 16.2 + 1 \times 10.1 = 58.7\ kPa$

So under 1D settlement conditions, at 5m depth:

σ'_v goes 40 kPa to 99 kPa

$\therefore \Delta v = 0.0504 \ln(99/40) = 0.0457$

$\therefore \varepsilon_v = 0.0457/1.927 = 0.0237$

Taking this as representative of all 10m of clay:

<u>settlement $\simeq$ 0.24m</u>

d) New $\Delta\sigma = 4 \times 16.2 = 64.8\ kPa$

So at 5m, σ'_v goes 99 kPa to 105.1 kPa

$\therefore \Delta v = 0.0504 \ln(100/99) + 0.1995 \ln(105/100)$

$= 0.0102$

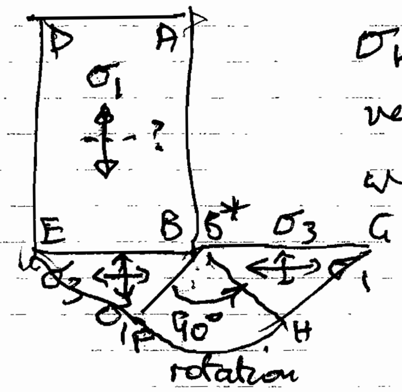
$\therefore \varepsilon_v = 0.0102/1.881 = 5.4 \times 10^{-3}$

<u>extra settlement $\simeq$ 54 mm</u>

3 a) (i) If Fig 2b is lower bound, the major principal stress in Δ BFE must be vertical. So AB, DE and BE must be frictionless.

(ii) A stress fan mobilises the full shear strength on radial lines, so BC and B^*C^* must do likewise, and that wall segment must be assumed to be perfectly rough.

(iii)



ABED: props will tend to let σ_h relax, so assume σ_1 is vertical, consistent with zero wall friction.

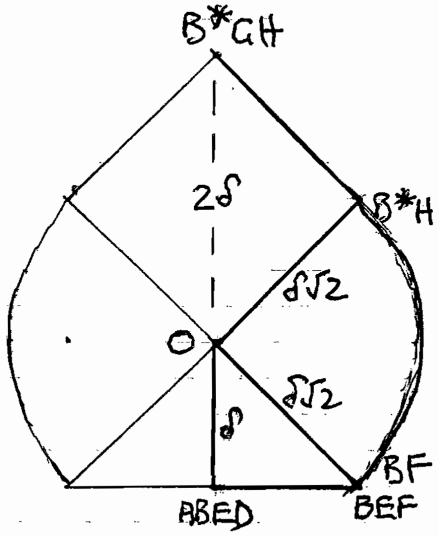
BFE: BE taken frictionless, soil is squeezing into the excavation, so σ_1 is vertical.

B^*GH: B^*G is unloaded & heaving, so σ_1 is horizontal.

(iv) $\gamma z_f = c_u(1 + \pi + 1)$

$\therefore z_f \geqslant 5.14\, c_u/\gamma$ if wall is rough

b) (i) Soil on ABC drags down on the wall. Soil on C^*B^* drags up. It is unlikely that the base resistance on CC^* would carry the difference. Better discount friction on AB, to guarantee vertical equilibrium of the wall b)(ii)



(iii) Work equation gives

$$2\gamma Z_f X \delta = c_u\left[2\,DE\,\delta + 2EB\,\delta + 2EF\,\delta\sqrt{2} + 2B^*H\,\delta\sqrt{2} + 2\cdot\frac{\pi}{2}\cdot BF\cdot \delta\sqrt{2}\cdot 2\right]$$

$$\therefore \gamma Z_f X = c_u\left[Z_f + X + X + X + \pi X\right]$$

$$= c_u\left[Z_f + (\pi+3)X\right]$$

$$\therefore Z_f(\gamma X - c_u) = c_u X(\pi+3)$$

So $Z_f = (\pi+3)\dfrac{c_u}{\gamma}\cdot\dfrac{1}{(1-c_u/\gamma X)}$

This was an upper bound, so

$$Z_f \leq (\pi+3)\frac{c_u}{\gamma}\cdot\frac{1}{(1-c_u/\gamma X)}$$

Wall friction had no extra effect compared with the lower bound solution. Factor $(\pi+2) \to (\pi+3)$ because of soil/soil shear on BE.

The strength of the wall has been ignored. Its plastic shear strength S_p could have given extra work, for both walls of $2S_p\delta\sqrt{2}$, increasing Z_f by $S_p\sqrt{2}/\gamma X$

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4. a) Initial volume $= \pi 50^2 \times 100/4 = 196350 \text{ mm}^3$

Assume sample stays cylindrical with area A mm^2

Then $A = (196350 + 1000\,\Delta V_{ml})/(100 - \Delta H_{mm})$

Q N	184	258	361	505	510	419	320
A mm^2	1966	1971	1983	2014	2044	2072	2166
q kPa	94	131	182	251	250	202	148

Pick $q_{max} = 251$ kPa at $\sigma_3' = 50$ kPa

$\therefore K_{p,max} = 301/50 = 6.02$

$\therefore \phi_{max} = 45.7°$

Pick $q_{crit} = 148$ kPa

$\therefore K_{p,crit} = 198/50 = 3.96$

$\therefore \phi_{crit} = 36.6°$

The silt was medium-dense and gained volume on being sheared due to dilation, requiring to increase its voids ratio to a critical state at which it could shear without further change of volume.

The estimate of ϕ_{crit} will be inaccurate because:

- a shear band will probably occur just after peak strength, so the geometry will not be cylindrical as we assumed
- the sample may still be softening, so critical conditions may not have been reached.

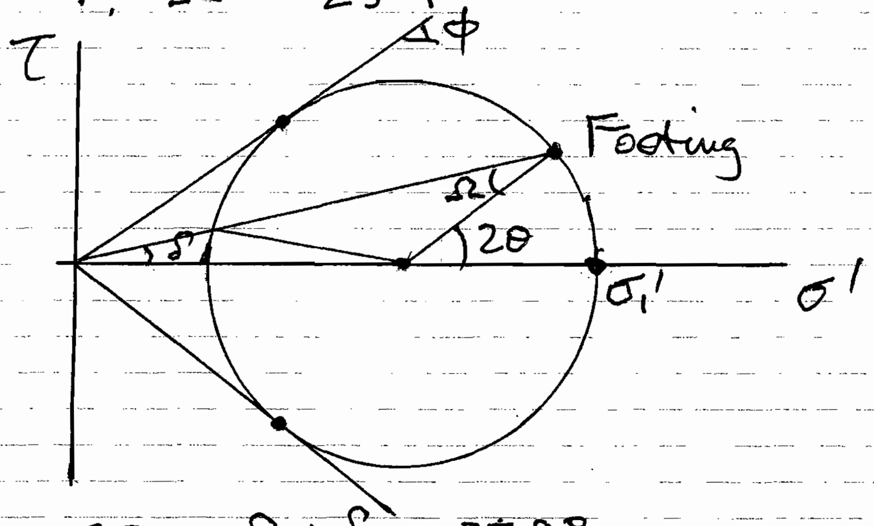
4 b) (i) Since the loading is repetative, it would cause cyclical dilation and softening if we attempted to mobilise more than ϕ_{crit}. Furthermore, there is progressive failure to worry about if $\phi > \phi_{crit}$.

(ii) Use $\phi = \phi_{crit} = 36.6^\circ$

$$\delta = \tan^{-1} 1/4 = 14.0^\circ$$

Referring to Data Book p. 16: $\sin\Omega = \sin 14^\circ / \sin 36.6$

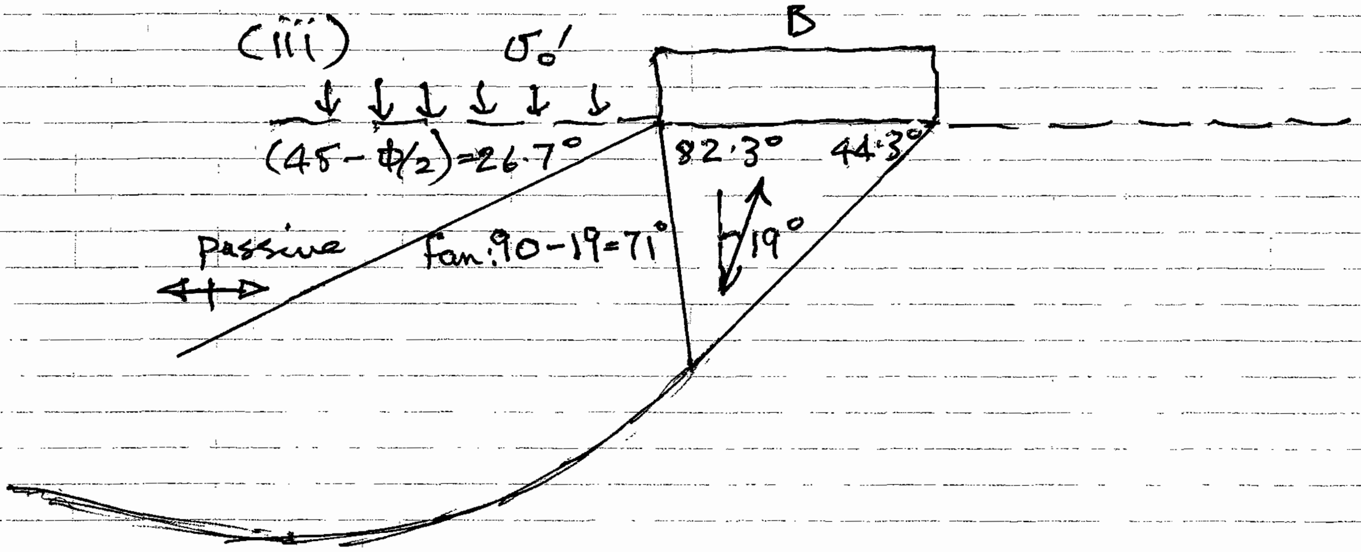
$\therefore \ \Omega = 23.9^\circ$



$$2\theta = \Omega + \delta = 37.9^\circ$$

$\therefore \ \theta = 19^\circ$

(iii)



4 b) (iii) cont.

In the passive zone $\sigma_1' = K_p \sigma_0'$

So under the footing, having rotated 71°;

$\sigma_1' = K_p \sigma_0' \exp(2\Delta\theta \tan\phi')$

$$\frac{\sigma_1'}{\sigma_0'} = 3.96 \times \exp\left(2 \times \frac{71}{180} \times \pi \times \tan 36.6°\right) = 24.9$$

$$\therefore \frac{\sigma_3'}{\sigma_0'} = 24.9 / K_p = 6.3$$

$$\text{So } \frac{\sigma_F'}{\sigma_0'} = \frac{(24.9+6.3)}{2} + \frac{(24.9-6.3)}{2}\cos 38°$$

$$= 15.6 + 9.3 \times 0.788$$

$$\therefore \frac{\sigma_F'}{\sigma_0'} = 22.9$$

(iv) $\sigma_F' = 100 / B_{min}$

$\therefore B_{min} = 100 / (22.9\, \sigma_0')$

$z = 0$, $\sigma_0' = 10$ kPa, $B_{min} = 0.44$ m

$z = 1$, $\sigma_0' = 20$ kPa, $B_{min} = 0.22$ m

$z = 2$, $\sigma_0' = 30$ kPa, $B_{min} = 0.15$ m

assuming the silt carries at least 10 kPa of capillary suction.

Engineering Tripos Part IIA 2009 Module 3D1 Answers

1	(b)	50 mm, 2.0 m	
	(c)	average settlements:	13 mm, 25 mm, 37 mm
		differential settlements:	8 mm, 16 mm, 19 mm
	(d)	20 mm at 7.4 months	
2	(a)	2.65, 0.200, 0.050; 100 kPa	
	(b)	40 kPa, 1856 kg/m^3	
	(c)	roughly 237 mm	
	(d)	an extra 54 mm	

3 (a) (i) zero friction; (ii) perfectly rough (iii) vertical, vertical, horizontal

(iv) $Z_f \geq (\pi + 2)\, c_u/\gamma$

(b) (i) vertical equilibrium

(iii) $Z_f \leq (\pi + 3)\,(c_u/\gamma) \;/\; (1 - c_u/\gamma X)$

4 (a) 45.7°, 36.6°

(b) (ii) 19° (iv) 0.44 m, 0.22 m, 0.15 m

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