Part II A 2009 Module 3D2 Solutions

Q. 1 Solutions Prof M D Bolton

(a) Current in-situ stresses at depth of 10m :

$\sigma_v = 18 \times 10 = 180 \text{ kN/m}^2$

$u = 8 \times 10 = 80 \text{ kN/m}^2$

$\therefore \sigma_v' = \sigma_v - u = 180 - 80 = 100 \text{ kN/m}^2$

$OCR = \dfrac{\sigma_{vo}'}{\sigma_v'} = 2.5$

$\therefore \sigma_{vo}' = 2.5 \times 100 = \underline{\underline{250 \text{ kN/m}^2}}$

$\sigma_{ho}' = 0.55 \times 250 = \underline{\underline{137.5 \text{ kN/m}^2}}$

[3]

(b) Current in-situ stress state in terms of q, p' and p :

$\sigma_v' = 100 \text{ kN/m}^2$ $\quad u = 80 \text{ kN/m}^2$

$\sigma_h' = K_o \sigma_v' = 0.8 \times 100 = 80 \text{ kN/m}^2$

$\sigma_v = 180 \text{ kN/m}^2$

$\sigma_h = \sigma_h' + u = 80 + 80 = 160 \text{ kN/m}^2$

$\therefore q = \sigma_v - \sigma_h = 180 - 160 = \underline{\underline{20 \text{ kN/m}^2}}$

(also $q = \sigma_v' - \sigma_h' = 100 - 80 = 20 \text{ kN/m}^2$)

$p' = \frac{1}{3}\sigma_v' + \frac{2}{3}\sigma_h' = \frac{1}{3} \times 100 + \frac{2}{3} \times 80$

$= 33 + 53 = \underline{\underline{86 \text{ kN/m}^2}}$

$p = p' + u = 86 + 80 = \underline{\underline{166 \text{ kN/m}^2}}$ [3]

(c) $c_u = q_f/2 = 50 \text{ kN/m}^2 \Rightarrow q_f = 2 \times 50 = 100 \text{ kN/m}^2$

$\Delta q = q_f - q = 100 - 20 = 80 \text{ kN/m}^2$

(i) Test A : undrained

Total stress path : $\Delta\sigma_a = 0$ (axis stress constant)

$\therefore \Delta q = \Delta\sigma_a - \Delta\sigma_r = -\Delta\sigma_r$

$\Delta p = \frac{1}{3}\Delta\sigma_a + \frac{2}{3}\Delta\sigma_r = \frac{2}{3}\Delta\sigma_r$

$\therefore \dfrac{\Delta q}{\Delta p} = \dfrac{-\Delta\sigma_r}{2/3\ \Delta\sigma_r} = -3/2$

$q_f = 100\ kN/m^2$, $\Delta q = 8.0\ kN/m^2$

$\Delta p = -\frac{2}{3}\Delta q = -\frac{2}{3} \times 80 = -53\ kN/m^2$

$\therefore$ at failure $p_f = 166 - 53 = 113\ kN/m^2$

~~Ef~~ Effective stress failure criterion $q_f = M p_f'$
$M = 1.0$

$\therefore p_f' = \frac{100}{1.0} = 100\ kN/m^2$

$\therefore$ pore pressure at failure $(u_f) = p_f - p_f' = 113 - 100$
$= \underline{\underline{13\ kN/m^2}}$

[6]

(ii) <u>Test B : drained</u>

Drained test $\Rightarrow$ effective stress path $\frac{\Delta q}{\Delta p'} = -3/2$
(same as for total stress path in Test A)

Pore pressure remains constant $= u_0 = 80\ kN/m^2$

(q_1, p_1') at start of test
(q_2, p_2') at failure

$q_2 = M p_2' = 1.0 p_2' \Rightarrow p_2' = q_2$

also $q_2 - q_1 = -3/2\,(p_2' - p_1')$

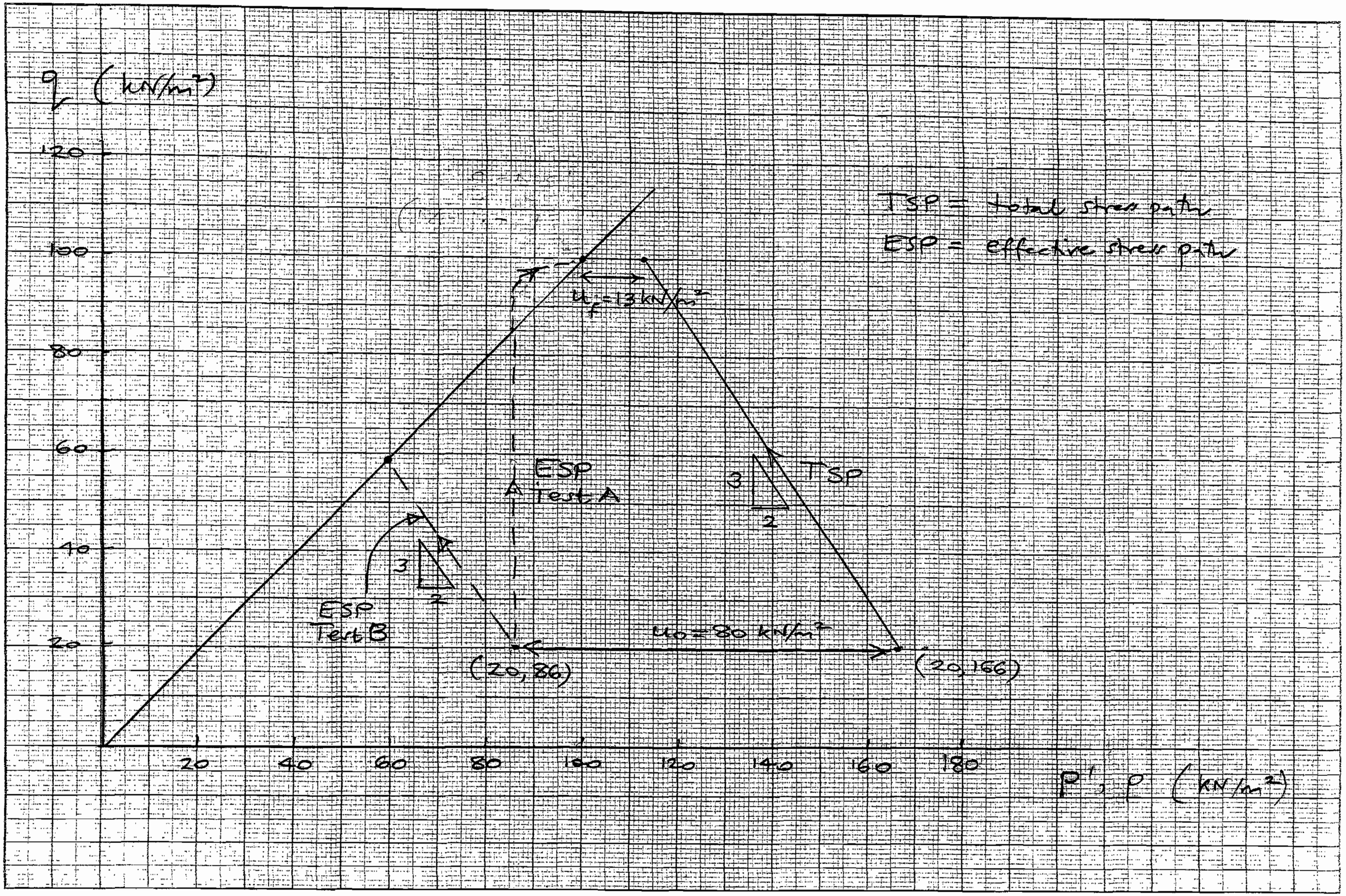
$q_1 = 20\ kN/m^2$, $p_1' = 86\ kN/m^2$

$\therefore q_2 - 20 = -3/2\, q_2 + \frac{3}{2} \times 86$

$\therefore 2.5 q_2 = 129 + 20 = 149\ kN/m^2$

$\therefore q_2 = \frac{149}{2.5} = \underline{\underline{59.6\ kN/m^2}}$ [6]

(d) The drained strength ($q_2 = 59.6\ kN/m^2$) is considerably less than the undrained strength ($q = 2c_u = 100\ kN/m^2$). The long term strength of the clay, if the pore pressure remains unchanged during the very slow excavation, reduces markedly. [2]



q (kN/m²)
120
100
80
60
40
20
20
40
60
80
100
120
140
160
180
p', p (kN/m²)
TSP = total stress path
ESP = effective stress path
u_f = 13 kN/m²
ESP Test A
TSP
3
2
3
2
ESP Test B
u₀ = 80 kN/m²
(20, 86)
(20, 166)

Q.2

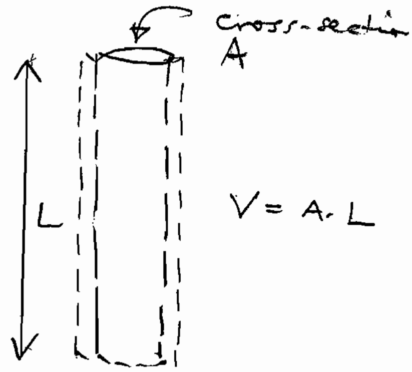
(a) Expansion of pressuremeter

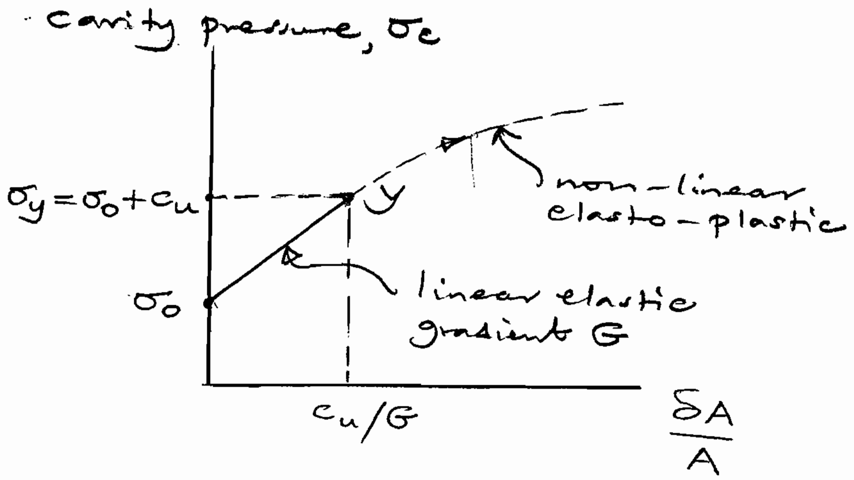
$\delta A = A - A_0$ (and $\delta V = V - V_0$)

caused by increase of pressure

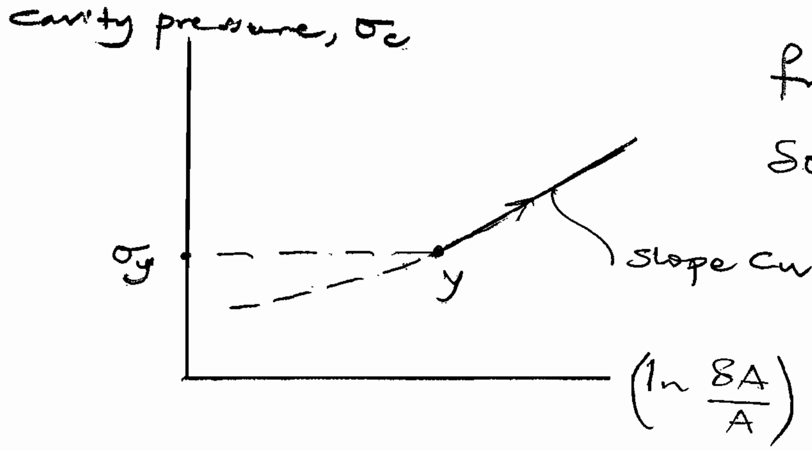
$\delta\sigma_c = \sigma_c - \sigma_0$

where $\sigma_0 = \gamma z$





G determined from gradient of first part of plot (or from unload-reload loop later in test)



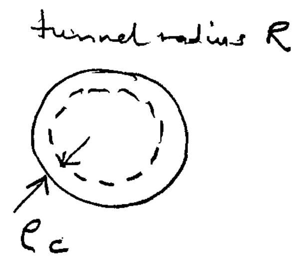
from Data Book:

$$\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$$

~~If~~ the point Y can be estimated from the upper graph (the point at the onset of non-linearity), $\sigma_c = \sigma_0 + c_u = \gamma z + c_u$. Hence c_u can be estimated in principle if linear elastic behaviour is observed. In the lower graph, c_u can be estimated from the slope. (Generally, this is the more reliable method.)

[30%]

(b) $\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$



$\therefore \frac{\delta A}{A} = \frac{c_u}{G}\exp\left(\frac{\delta\sigma_c}{c_u} - 1\right)$

Contraction of cavity with complete unloading

$\therefore \delta\sigma_c = \gamma z$

radial ground movement ρ_c

$\delta A = 2\pi R.\rho_c$

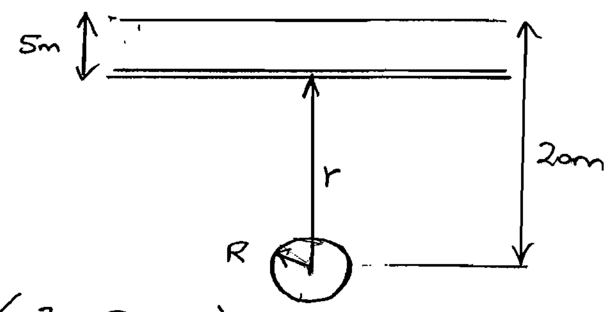
$A = \pi R^2$

$\therefore \frac{\delta A}{A} = \frac{2\pi R.\rho_c}{\pi R^2} = 2\frac{\rho_c}{R}$

$\therefore \rho_c = \frac{1}{2} R \frac{c_u}{G}\exp\left(\frac{\gamma z}{c_u} - 1\right)$

[40%]

(c) Constant volume condition (undrained clay behaviour):



$2\pi r.\rho = 2\pi R.\rho_c$

$\rho_c = \frac{1}{2} \times 2.5 \times \frac{150}{30\times10^3}\exp\left(\frac{20\times20}{150} - 1\right)$

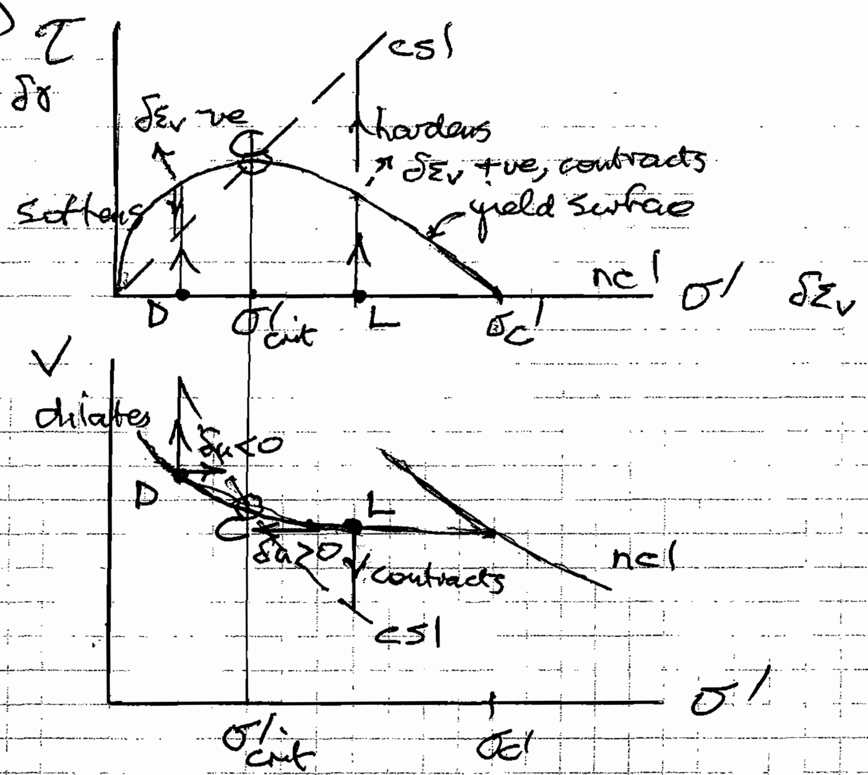
$= 0.033\text{m} \ (33 \text{ mm})$

$\therefore \rho = \frac{R}{r}.\rho_c = \frac{2.5}{15} \times 0.033 = 0.0055\text{m}$

or $\underline{\underline{5.5\text{mm}}}$

[30%]

3. a)



Yielding with $\sigma' > \sigma'_{crit}$ e.g. from L undrained shear test, leads to contraction & hardening.

yielding with $\sigma' < \sigma'_{crit}$ eg. from D undrained shear test, leads to dilation & softening.

The key is that soil must find a critical state after yielding. If undrained it must loose σ' from L or gain σ' from D. Since it is shearing at constant σ, this can only be achieved by generating $\delta u > 0$ from L, $\delta u < 0$ from D. 6

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b) (i) In an undrained test $\tau_{u,max} = \tau_{u,ult}$ if $\sigma' > \sigma'_{crit}$, i.e if $\sigma'_c/\sigma'_0 < E$ (i.e 2.72)

If $\sigma'_c/\sigma'_0 > 2.72$, the yield stress τ_y might possibly exceed $\tau_{u,ult}$.

3 b cont.) (i) $$v_0 = \Gamma + \lambda - \kappa - \lambda \ln \sigma'_c + \kappa \ln \frac{\sigma'_c}{\sigma'_0}$$

But $v_0 = v_{u,crit} = \Gamma - \lambda \ln \sigma'_{crit}$

And $\tau_{u,ult} = \tan\phi_{crit}\, \sigma'_{crit}$

So $$\frac{\tau_{u,ult}}{\sigma'_0} = \tan\phi_{crit} \frac{\sigma'_{crit}}{\sigma'_0}$$

where $$\ln\sigma'_{crit} = \ln\sigma'_c - \frac{\kappa}{\lambda}\ln\sigma'_c + \frac{\kappa}{\lambda}\ln\sigma'_0 - \frac{(\lambda-\kappa)}{\lambda}$$

so $$\sigma'_{crit} = (\sigma'_c)^{1-\frac{\kappa}{\lambda}} (\sigma'_0)^{\frac{\kappa}{\lambda}} E^{-(1-\frac{\kappa}{\lambda})}$$

$$\therefore \frac{\tau_{u,ult}}{\sigma'_0} = \tan\phi_{crit} \left(\frac{\sigma'_c}{E\sigma'_0}\right)^{1-\frac{\kappa}{\lambda}}$$ where E is 2.72

(ii) Find τ_y on the "dry" side and ask if $\tau_y > \tau_{u,ult}$.

The equation of the yield surface is:

$$\frac{\tau_y}{\sigma'_0} = \tan\phi_{crit} \ln \frac{\sigma'_c}{\sigma'_0}$$

Evaluate this, compare with (i), and if greater select this for (ii). Otherwise (ii) ≡ (i)

(iii) $$\frac{\tau_{d,ult}}{\sigma'_0} = \tan\phi_{crit}$$ by definition

(iv) $$\frac{\tau_{d,max}}{\sigma'_0} = \tan\phi_{crit} \ln \frac{\sigma'_c}{\sigma'_0}$$ as per (ii)

For yield on the "dry" side with $\frac{\sigma'_c}{\sigma'_0} > 2.72$.

Otherwise (iv) ≡ (iii)

At OCR = 1, $\sigma'_c = \sigma'_0$

$$\frac{\tau_{u,ult}}{\sigma'_0} = \frac{\tau_{u,max}}{\sigma'_0} = \frac{\tan\phi_{crit}}{E^{1-\kappa/\lambda}} = \frac{\tan\phi_{crit}}{2.72^{0.62}}$$

$$\therefore \frac{\tau_{u,ult}}{\sigma'_0} = \frac{\tan\phi_{crit}}{1.86} = \underline{0.24}$$

$$\frac{\tau_{d,ult}}{\sigma'_0} = \frac{\tau_{d,max}}{\sigma'_0} = \tan\phi_{crit} = \underline{0.445}$$

3 b cont.) At OCR = 10, $\sigma_c' = 10\,\sigma_0'$

$$\frac{\tau_{u,ult}}{\sigma_0'} = \tan\phi_{crit}\left(\frac{10}{2.72}\right)^{0.62} = 2.24\tan\phi_{crit}$$

$$\therefore \frac{\tau_{u,ult}}{\sigma_0'} = 1.00$$

$$\frac{\tau_y}{\sigma_0'} = \tan\phi_{crit}\ \ln 10 = 2.30\tan\phi_{crit}$$

$$\therefore \frac{\tau_{u,max}}{\sigma_0'} = 1.02$$

$$\frac{\tau_{d,ult}}{\sigma_0'} = \tan\phi_{crit} = 0.45$$

$$\frac{\tau_{d,max}}{\sigma_0'} = \frac{\tau_y}{\sigma_0'} = 1.02$$ 10

Whenever $\tau_{max} > \tau_{ult,crit}$, τ may fall further as $\phi_{crit} \rightarrow \phi_{resid}$. This is known as the "residual strength" on "slickensides".

[Marking: the crucial (and difficult) question is whether the student understands the potential fall from peak strength to critical state strength in the case of soils yielding on the "dry" side.

The ultimate undrained and drained strengths is a much more routine question.

Students who miss the algebra but compute all 4 ratios for both OCR 1 and 10 might get 6/10]

c) Mud has $\tau_u < \tau_d$ and can not stand at its "angle of repose". It slumps with temporary excess pore pressures (+ve).

Stiff clay has $\tau_u > \tau_d$ and is strain softening as well as softening due to water ingress satisfying negative excess pore pressures if it is stressed beyond yield. It seems very stable, but this is only temporary. If rupture bands form, ϕ_{crit} drops to ϕ_{res}. 4

4 a) Because grain crushing reduces over-riding and dilatancy which otherwise enhance the shear strength of dense granular materials, we need to define relative crushability

$$I_c = \ln \sigma_c / \sigma'$$

where σ_c is the crushing stress of grains.

Then $I_R = I_D I_c - 1$ correlates with dilatancy giving an extra component $\Delta\phi$ in addition to ϕ_{crit}.

b) Take $\sigma_c = 15000$ kPa from P.10

Constant volume shear demands $I_R = 0$

$\therefore I_c = 1/I_D = 1.33$

$\therefore \sigma'_{crit} = \sigma_c / 3.79 = 3954$ kPa

$\therefore \tau_{crit} = \sigma'_{crit} \tan 32° = 2471$ kPa

This would apply to constant volume shearing of sand in the field. The sand would have to be saturated and sheared very quickly, e.g. by a very fast CPT or explosive. 5

c) $\phi_{max} - \phi_{crit} = 0.8\,\psi_{max} = 5 I_R$

$I_R = 0.75 \times \ln \frac{15000}{\sigma'} - 1$

$= 1.727$ and 3.454

$\Delta\phi =$	$8.6°$	$17.3°$
$\phi_{max} =$	$40.6°$	$49.3°$
$\psi_{max} =$	$10.8°$	$21.6°$

5

d) At $\sigma' = 395$ kPa, $\tau_{max} = 339$ kPa

At $\sigma' = 39.5$ kPa, $\tau_{max} = 46$ kPa

4 d cont.)

Suppose you make a linear fit over the last two points. Fit with

$$\tau = c' + \sigma' \tan\phi'$$

Then $c' + 395 \tan\phi' = 339$

$c' + 39.5 \tan\phi' = 46$

$\therefore \tan\phi' = 293/355.5$

So $\tan\phi' = 0.82$

$\phi' = 39.5^\circ$

And $c' = 13$ kPa

The danger is that engineers may come to believe that $\tau_{max} = 13$ kPa at $\sigma' = 0$ for which there is no evidence.

At $\sigma' = 3954$ kPa it gives $\tau_{max} = 3273$ kPa which is much larger than the critical state shear strength we know to be 2471 kPa.

Actually, the envelope is close to a power curve

$$\frac{\tau}{\tau_{crit}} = \left(\frac{\sigma}{\sigma_{crit}}\right)^{\beta} \quad \text{say}$$

Let us calculate two independent values of β from the two estimates:

$0.137 = 0.1^{\beta}$ $\qquad \beta = 0.86$

$0.0186 = 0.01^{\beta}$ $\qquad \beta = 0.86$

and of course it fits exactly at the critical state. Good! 5

Use of ϕ_{crit} absolves the designer from worry over progressive failure, cyclical softening etc that erode the peak strength

Engineering Tripos Part IIA 2009 Module 3D2 Answers

1 (a) 250 kPa and 138 kPa (taking $\gamma_w = 10$ kN/m^3)

(b) $q = 20$ kPa, $p = 166$ kPa, $p' = 86$ kPa

(c) (i) 13 kPa (ii) 60 kPa

(d) the clay in the excavation will soften as time passes, and drainage takes place

2 (c) 5.5 mm

3 (b) (i) $\dfrac{\tau_{u,ult}}{\sigma'_o} = \tan\phi_{crit}\left[\dfrac{\sigma'_c}{2.72\,\sigma'_o}\right]^{1-\kappa/\lambda}$

(ii) Find $\dfrac{\tau_y}{\sigma'_o} = \tan\phi_{crit}\ \ln\left[\dfrac{\sigma'_c}{\sigma'_o}\right]$. Then $\dfrac{\tau_{u,\max}}{\sigma'_o} =$ the greater of $\dfrac{\tau_y}{\sigma'_o}$ and $\dfrac{\tau_{u,ult}}{\sigma'_o}$.

(iii) $\dfrac{\tau_{d,ult}}{\sigma'_o} = \tan\phi_{crit}$

(iv) $\dfrac{\tau_{d,\max}}{\sigma'_o} =$ the greater of $\dfrac{\tau_y}{\sigma'_o}$ and $\dfrac{\tau_{d,ult}}{\sigma'_o}$

For OCR = 1: 0.24, 0.24, 0.45, 0.45

For OCR = 10: 1.00, 1.02, 0.45, 1.02

(c) For OCR = 1, mud slumps to far below its critical state angle.

For OCR = 10, stiff clay initially stands steeply, but it can soften to its residual friction angle, about half its critical state angle, on slip surfaces.

4 (b) 3950 kPa and 2470 kPa (taking $\sigma'_c = 15000$ kPa)

(c) 40.6° and 10.8°; 49.3° and 21.6°

(d) Linear fit: $c' = 13$ kPa, $\phi' = 39.5°$

Power law fit: $\beta = 0.86$ correctly passes through the origin and the critical state.