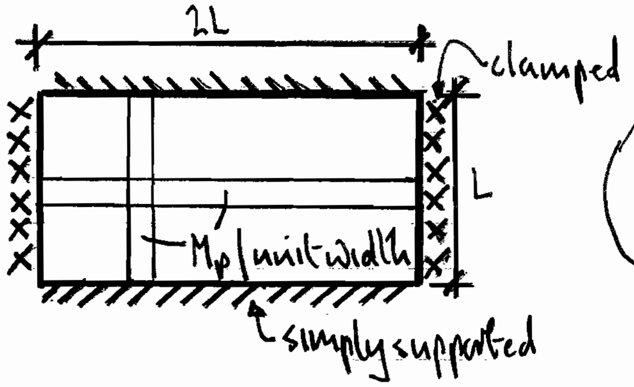
303 | Qu1 | 2008-09 | II A | STRUCT. MAT. DESIGN | K.A. SEFFEN

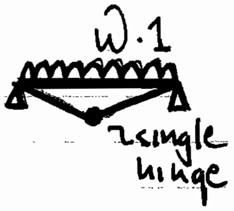
1a)



W carried into plane of page per unit area

(i) If W only carried by strips ∥ to L ⇒

$\Rightarrow M_p = \frac{(W \cdot 1)L^2}{8} \Rightarrow \underline{W_{max} = 8M_p/L^2}$



[Lower bound ≡ upper bound mechanism]

If W only carried by strips ∥ to 2L

$M_p \cdot [\theta + \theta + 2\theta] = (W \cdot 1 \cdot 2L) \underbrace{\frac{L}{2}\theta}_{\text{average c.g. movement}}$ $\Rightarrow M_p = \frac{WL^2}{4} \Rightarrow \underline{W_{max} = 4\frac{M_p}{L^2}}$

Maximum load is $\left(\boxed{\frac{8M_p}{L^2}}\right)$ since safe by Lower Bound, which always tries to maximise load.

(ii) if W/2 carried at the same time on each set of strips, then

- for strips ∥ to L ⇒ $M_p = \frac{W}{2}\frac{L^2}{8}$, $W_{max} = 16M_p/L^2$ — Ⓐ
- for strips ∥ to 2L ⇒ $M_p = \frac{W}{2}\frac{L^2}{4}$, $W_{max} = 8M_p/L^2$ — Ⓑ

Ⓑ is the viable safe load for both sets of strips being active: if $W > 8M_p/L^2$, then Ⓐ works, but Ⓑ fails! Hence, no change from (i).

For unequal sharing, let $\overset{\text{factor}}{\lambda} W$ be carried by strips ∥ to L — Ⓒ
$(1-\lambda)W$ " " " " ∥ to 2L — Ⓓ

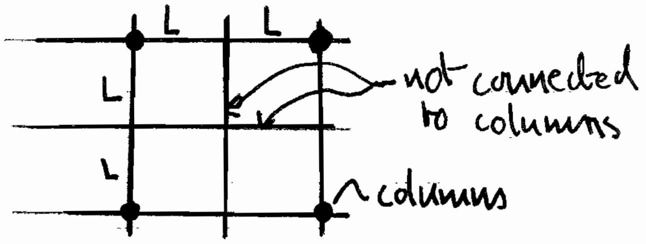
For Ⓒ $\underline{W_{max} = 8M_p/\lambda L^2}$, for Ⓓ, $\underline{W_{max} = 4M_p/(1-\lambda)L^2}$

Optimality when both are satisfied together

$\Rightarrow 8/\lambda = 4/1-\lambda \Rightarrow 8 - 8\lambda = 4\lambda \Rightarrow \lambda = 2/3$,

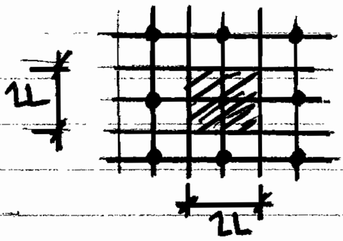
$\Rightarrow \underline{W_{max} = 12M_p/L^2}$

1b)



w everywhere carried in plan per unit area

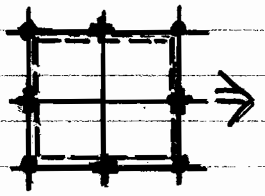
(i)

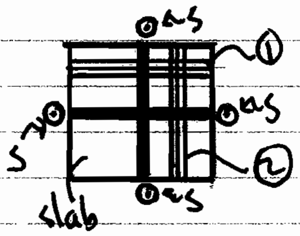


//// force on a given column balances this footprint $w\times(2L)^2$ symmetrical: if no shear forces are present on the edge of region, i.e. simply supported only at beam junctions – viable by Lower Bound

$\Rightarrow$ column force $= 4wL^2$

(ii)



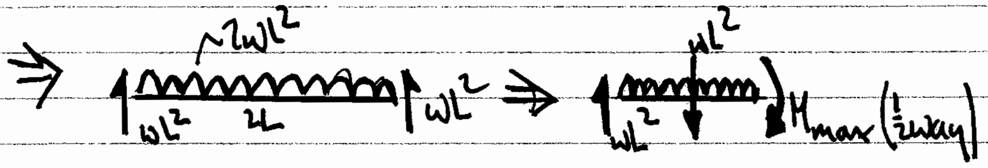


S.F., S, carried at beam-to-beam junctions (+) only

$$\text{Eqlm} \Rightarrow \quad 4S = w\times(2L)^2 \Rightarrow \underline{S = wL^2}$$

Assume w carried by two sets of strips ① + ②, equally. On a given ↔ / ↕ beam (S at the ends), then

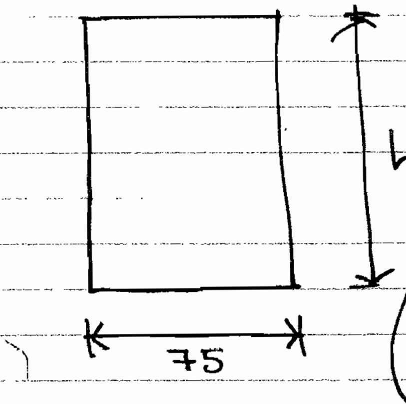
$\left(\frac{w}{2}\times 2L\right)\times 2L$ = total load on one beam $= 2wL^2 \Rightarrow S = wL^2$, as above



$$\Rightarrow \quad M_{max} + wL^2\cdot L - wL^2\cdot 4/2 = 0 \quad \Rightarrow \quad \underline{\underline{M_{max} = wL^3/2}}$$

<u>Mistakes/failings</u>: some candidates forgot that you aim to maximise the LBT solution; but in (aii), if both strips are actively carrying load, the lowest prediction is only viable. Many did not clarify assumptions in (b) in terms of load path; only a few extracted the maximum b.m. above; other, lower solutions were viable as they do work by virtue of the lower bound theorem (but nobody pointed that out).

2(a)



$k_{ls} = 1.1$; $k_h = 1.0$; $k_{mod} = 1.0$

$k_{crit} = 1.0$; $\gamma_m = 1.3$

$f_{m,k} = 18\text{MPa}$; $f_{v,k} = 2\text{MPa}$

① → NO REDUCTION FROM LATERAL BUCKLING DUE TO RESTRAINT PROVIDED BY PLYWOOD

SHEAR

DESIGN SHEAR FORCE V_d (AT SUPPORT) $= \left(2.5\text{kN/m}^2 \times 0.6 \times \frac{4.5}{2}\right) \times 1.5$

$= 5.063\text{kN}$

ELASTIC SHEAR STRESS $v_d = \frac{5.063 \times 10^3}{bh} \times 1.5$ N

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m \geqslant v_d$$

$$\therefore h \geqslant \frac{5.063 \times 10^3 \times 1.5 \times 1.3}{75 \times 1.0 \times 1.1 \times 2}$$

② $h \geqslant 59.85$ mm.

MOMENT

DESIGN BENDING MOMENT M (AT MID SPAN) $= \frac{(2.5\text{kN/m}^2 \times 0.6) \times 4.5^2}{8} \times 1.5$

$= 5.7\text{kNm}$

ELASTIC STRESS (FLEXURAL) $\sigma_{MAX} = \frac{5.7 \times 10^6}{bh^2/6}$

$$f_{m,d} = \frac{k_{mod} \times k_h \times k_{crit} \times k_{ls} \times f_{m,k}}{\gamma_m} \geqslant \sigma_{MAX}$$

$$\therefore h \geqslant \sqrt{\frac{5.7 \times 10^6 \times 1.3 \times 6}{1.0 \times 1.0 \times 1.0 \times 1.1 \times 18 \times 75}}$$

∴ $h \geqslant 172$ mm

② ∴ MINIMUM $h = 172$ mm CONTROLLED BY FLEXURAL STRENGT

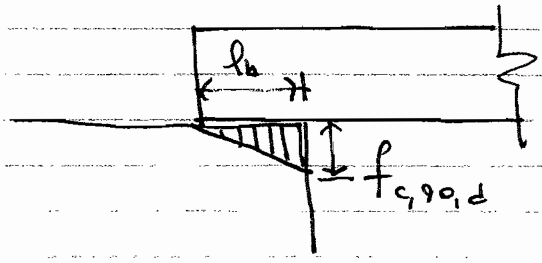
(b) DESIGN BEARING STRESS

$$f_{c,90,d} = k_{ls}\, k_{c,90}\, k_{mod}\, f_{c,90,k} / \gamma_m$$

$$= 1.0 \times 1.1 \times 1.0 \times 4.8 / 1.3$$

① $$= 4.06 \text{ N/mm}^2$$

BUT BEARING PRESSURE IS NOT CONSTANT OVER BEARING LENGTH



$$\left(\frac{1}{2}\, l_b\, f_{c,90,d}\right) \times b \geqslant V_d$$

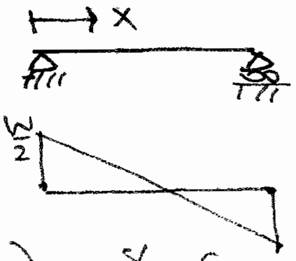
$$\therefore\ l_b \geqslant \frac{5.063 \times 10^3 \times 2}{75 \times 4.06}$$

② $$l_b \geqslant 16.6 \text{ mm}$$

(c) FLEXURAL DEFLECTION $V_f = \dfrac{5W\ell^3}{384EI} = \dfrac{5W\ell^3}{32Ebh^3}$

SHEAR DEFLECTION V_s :

LONGITUDINAL SHEAR STRESS $\tau = \dfrac{SA\bar{y}}{It}$



WHERE $S = \dfrac{W}{2} - \dfrac{Wx}{\ell}$

∴ LONGITUDINAL τ = VERTICAL τ $= \dfrac{3}{2bh}\left(\dfrac{W}{2} - \dfrac{Wx}{\ell}\right) = \gamma_{NA}\, G$
(COMPLIMENTARY SHEAR) — (1)

ALSO $dV_s = \gamma_{NA}\, dx$ — (2)

∴ COMBINING (1) AND (2)

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$$dv_s = \frac{3}{2bhG}\left(\frac{W}{2} - \frac{W}{\ell}x\right)dx$$

$$\therefore\ v_s = \frac{3}{2bhG}\left(\frac{Wx}{2} - \frac{Wx^2}{2\ell}\right) + A$$

AT $x = 0$, $v_s = 0$ $\therefore$ $A = 0$

$\therefore$ SHEAR DEFLECTION AT $x = \ell/2$:

$$v_s = \frac{3W\ell}{2bhG}\left(\frac{\ell}{4} - \frac{\ell}{8}\right) = \frac{3W\ell^2}{16bhG}$$

$\therefore$ TOTAL DEFLECTION AT MID SPAN

$$v_{TOT} = \frac{5W\ell^3}{32Ebh^3}\left(1 + \frac{6Eh^2}{5G\ell}\right)$$

$W = 6.75\,\text{kN}$

$$v_{TOT} = \frac{5\times6.75\times10^3\times4.5^3}{32\times9\times10^9\times0.075\times0.172^3}\left(1 + \frac{6\times9\times10^9\times0.172^2}{5\times0.56\times10^9\times4.5}\right)$$

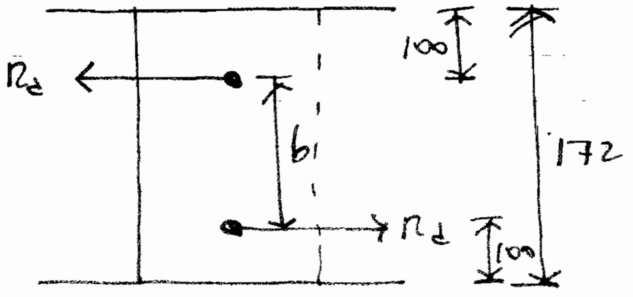
$$= 0.028\,(1 + 0.127)$$

$$= \underline{\underline{31.6\,\text{mm}}}$$

④ THIS IS EQUIVALENT TO $\ell/142$, THIS EXCEEDS

① THE $\ell/200$ RECCOMENDATIONS FOR SIMPLY SUPPORTED BEAMS AND IS LARGER THAN THE $\ell/333$ OR 14 mm REQUIRED TO LIMIT VIBRATIONS.

(d)



$$R_d = \frac{5.7\times10^6\ \text{Nmm}}{b\ \text{mm}}$$

WORK DONE BY LOAD:

$W_E = R_d\,\delta$

②

ENERGY DISSIPATED IN TIMBER:

$W_I = f_{h,1,d}\,t\,d\,\delta$

$\therefore\ R_d = f_{h,1,d}\,t\,d$ — (3)

$$f_{h,0,k} = 0.082(1 - 0.01 \times 50)\,320$$

$$= 13.12\ N/mm^2$$

$$f_{h,1,k} = 13.12\ N/mm^2 \qquad (\alpha = 0^\circ)$$

$$\therefore\ f_{h,1,d} = k_{mod}\, f_{h,1,k} / \gamma_m$$

$$= 10.1\ N/mm^2$$

∴ SUBSTITUTE INTO (3)

$$\frac{5.7 \times 10^6}{b} = 10.1 \times 75 \times 50$$

③ $$\therefore\ \underline{\underline{b = 150\ mm}}$$

IDEALLY SHOULD AVOID THE SPLICE ALTOGETHER A IT IS ~~VERY~~ INEFFICIENT:

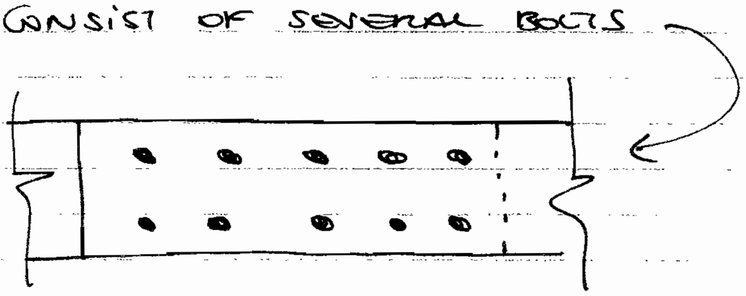
① BEAM DEPTH FROM FLEXURE & SHEAR = 172 mm

" " " SPLICE = 350 mm !!

IF SPLICE IS REQUIRED IT SHOULD BE:

- LOCATED AWAY FROM MID-SPAN
- CONSIST OF SEVERAL BOLTS

①

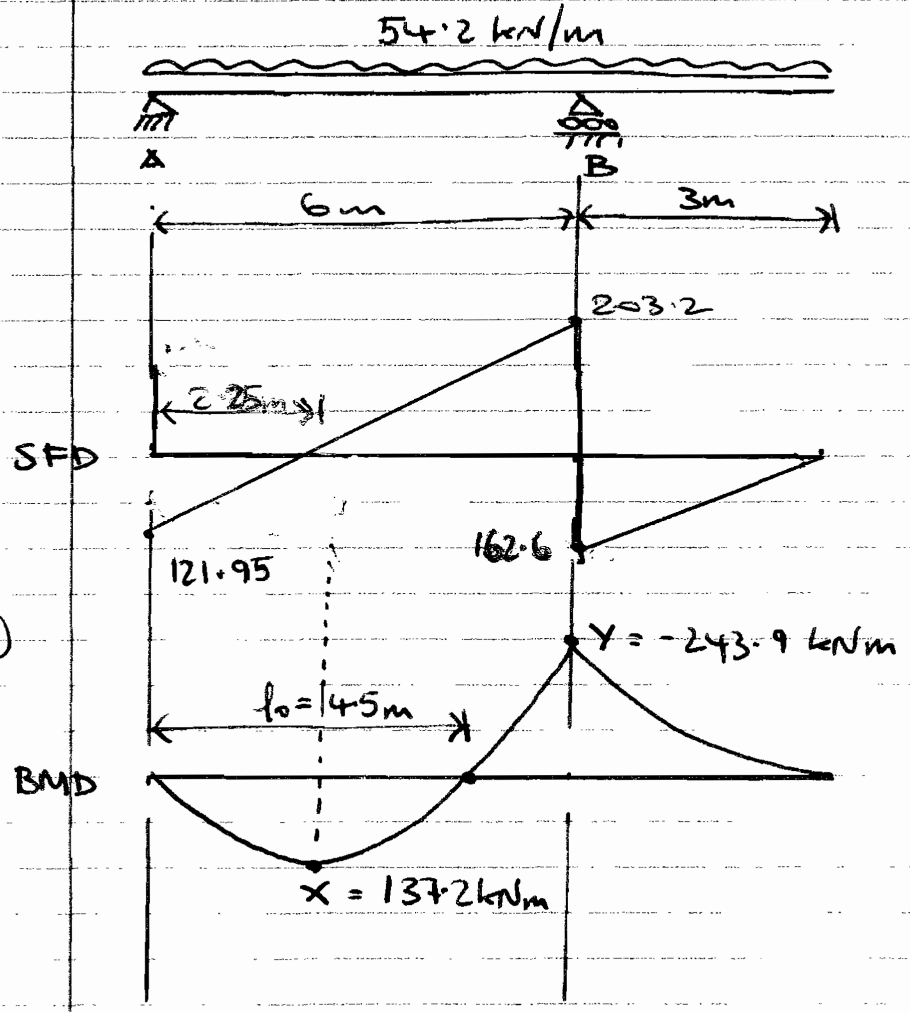


3MS | Qu3 | 2008-09 | IIE | STRUCT. MAT. DESIGN | M. OVEREND

3(a) DESIGN LIVE LOAD = 32 kN/m × 1·6 = 51·2 kN/m

" DEAD LOAD = 24 kN/m³ × 0·5 × 0·25 × 1·4 = 3 kN/m

① TOTAL DESIGN LOAD = 54·2 kN/m



↻ ABOUT B:

$R_A \times 6 = 54{\cdot}2 \times 9 \times 1{\cdot}5$

$\therefore R_A = 121{\cdot}95$ kN

$R_B = 365{\cdot}85$ kN

↻ AT X:

$M_x = 54{\cdot}2 \times 2{\cdot}25 \times \frac{2{\cdot}25}{2}$

$- 121{\cdot}95 \times 2{\cdot}25$

$\therefore M_x = 137{\cdot}2$ kNm

↻ AT Y:

$\therefore M_y = -54{\cdot}2 \times 3 \times 1{\cdot}5$

$= -243{\cdot}9$ kNm

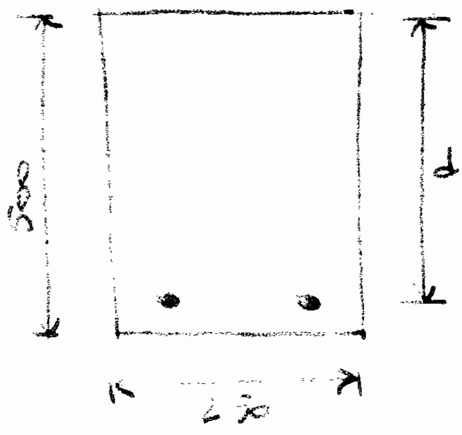
POINT OF CONTRAFLEXURE l_0:
(↻l_0 WHERE MOMENT = 0)

$R_A l_0 - w l_0 \cdot \frac{l_0}{2} = 0$

$\therefore l_0 = \frac{2R_A}{w}$

$l_0 = 4{\cdot}5$m

(b)



$f_{cu} = 40$ MPa ; $f_y = 460$ MPa

$\gamma_m = 1{\cdot}5$; $\gamma_s = 1{\cdot}15$

cover = 40 mm ; ϕ = 25 mm

$\therefore d = h - \phi/2 - \text{cover}$

$= 500 - 12{\cdot}5 - 40 = 447{\cdot}5$ mm

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ASSUME UNDER REINFORCED & SINGLY REINFORCED:

$$M_u = 0{\cdot}225 f_{cu} bd^2 / \gamma_c$$

$$= 0{\cdot}225 \times (40 \times 10^6) \times 0{\cdot}25 \times 0{\cdot}4475^2 / 1{\cdot}5$$

① $= 300\,kNm > \text{MAX. APPLIED MOMENT } M_{App}$

$\therefore$ NO COMPRESSION REINF. REQUIRED

$$M_u = A_s f_y \frac{d}{\gamma_s}\left(1 - \frac{x}{2d}\right) \quad \text{— (1)}$$

WHERE $\frac{x}{d} = \gamma_c A_s f_y / \gamma_s \, 0{\cdot}6 f_{cu} bd$

$$= \frac{1{\cdot}5}{1{\cdot}15} \cdot \frac{460 \times 10^6}{0{\cdot}6 \times 40 \times 10^6} \cdot \frac{1}{0{\cdot}25 \times 0{\cdot}4475} \cdot A_s$$

$$= 224 A_s$$

SUBSTITUTE IN (1):

$$M_u = A_s \cdot 460 \times 10^6 \cdot \frac{0{\cdot}4475}{1{\cdot}15}\left(1 - 112 A_s\right)$$

$$M_u = 179 \times 10^6 A_s (1 - 112 A_s) \quad \text{— (2)}$$

FOR SAGGING MOMENT ($M_{App} = 137{\cdot}2\,kNm$), EQUATION (2) BECOMES:

$$112 A_s^2 - A_s + 7{\cdot}66 \times 10^{-4} = 0$$

SOLVE QUADRATIC FOR LOWER ROOT:

$$A_s = \frac{+1 \pm 0{\cdot}811}{224} = 8{\cdot}45 \times 10^{-4}\,m^2$$

$$\therefore A_s = 845\,mm^2$$

② 1 IN NO. 25mm ϕ BAR $= \frac{\pi 25^2}{4} = 491\,mm^2$

$\therefore$ PROVIDE 2T25 ($982\,mm^2$)

3D8/Qu3/2008-09/IIA/STRUCT. MAT. DESIGN/M. OVEREND

FOR HOGGING MOMENT ($M_{APP} = 243.9$ kNm), EQUATION (2) BECOMES:

$$112 A_s^2 - A_s + 1.363 \times 10^{-3} = 0$$

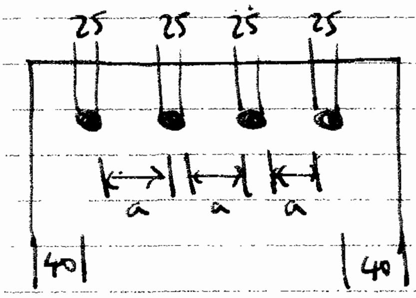
SOLVE QUADRATIC FOR LOWER ROOT:

$$A_s = \frac{+1 \pm 0.623}{224} = 1.68 \times 10^{-3}\ m^2$$

$\therefore A_s = 1680\ mm^2$

② $\therefore$ PROVIDE 4T25 (1964 mm²)

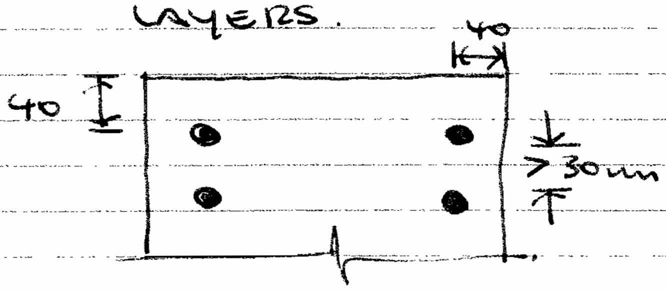
4T25 CANNOT BE PLACED IN TOP LAYER DUE TO SPACING RECOMMENDATIONS:



$a = 23.3$ mm — TOO SMALL.

①

$\therefore$ BARS WOULD BE ~~ARRA~~ PLACED IN TWO LAYERS.



THIS REQUIRES A RE-CALCULATION OF EQUATION (2) AS x/d HAS NOW CHANGED.

HOWEVER BY INSPECTION WE ARE PROVIDING 280 mm² MORE REINF. THAN IS REQUIRED WHICH IS SUFFICIENT TO COMPENSATE $\therefore$ NO FURTHER CALCULATIONS REQUIRED.

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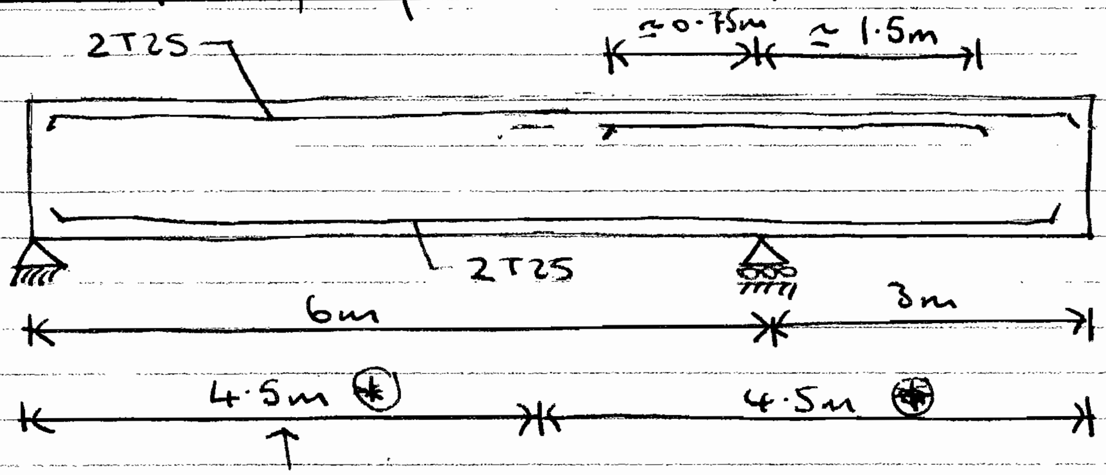


② ✱ BOTTOM STEEL COULD BE CONFINED TO THIS ZONE + TOP STEEL COULD BE CONFINED TO THIS ZONE

HOWEVER A MIN. OF TWO 'FRAMING' BARS ARE REQUIRED FOR SHEAR REINFORCEMENT – IT IS THEREFORE SENSIBLE TO ~~USE STEPS~~ USE THE 2T25's TOP AND BOTTOM THROUGHOUT.

ALSO SHOULD ONE OF THE SPANS BECOME UNLOADED THERE WOULD BE SOME CAPACITY TO DEAL WITH THE CHANGE.

(c) $V_{MAX} = 203.2 \text{ kN}$

$$V_{Rd,c} = \frac{0.18}{\gamma_c} . \underset{\downarrow}{k} \left[100 \rho_1 \cdot f_{ck} \right]^{1/3} . bd$$

$$\therefore \frac{V_{Rd,c}}{bd} = \frac{0.18}{1.5} \left(1 + \sqrt{\frac{200}{447.5}} \right) \left[100 \times \frac{1964}{250 \times 500} \times \underbrace{0.8 \times 40}_{f_{ck} = 0.8 f_{cu}} \right]^{1/3}$$

$$\frac{V_{Rd,c}}{bd} = 0.738 \text{ N/mm}^2$$

$$\frac{V_{MAX}}{bd} = \frac{203.2 \times 10^3}{250 \times 447.5} = 1.816 \text{ N/mm}^2 \nless \frac{V_{Rd,c}}{bd}$$

$\therefore$ SHEAR REINF. REQUIRED. AT MAX SF.

②

Shear reinforcement is required here :

[illegible] > [illegible] 1.08 N/mm²

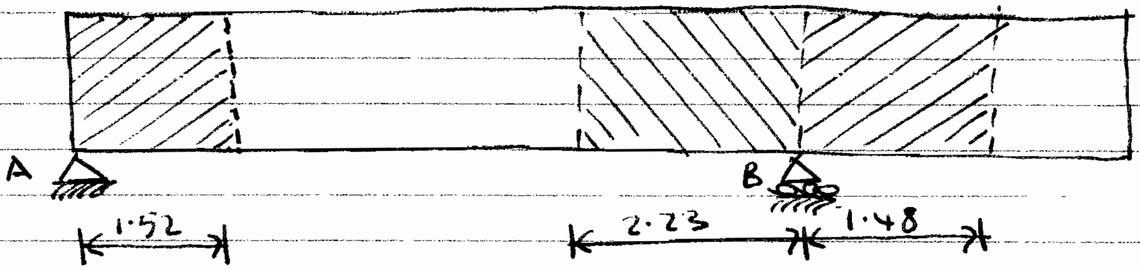
[illegible]

∴ WHERE $V_{APPLIED} > 0.738 \times 250 \times 447.5$

$V_{APPLIED} > 82.56\ kN$

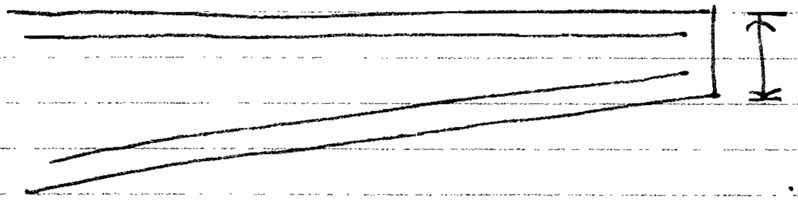
FROM S.F.D. THIS OCCURS IN THE FOLLOWING REGIONS:

③



(d) BY INSPECTION OF B.M.D AND EQUATION (1), h MAY BE REDUCED TO ZERO – THIS IS NOT POSSIBLE IN PRACTICE.

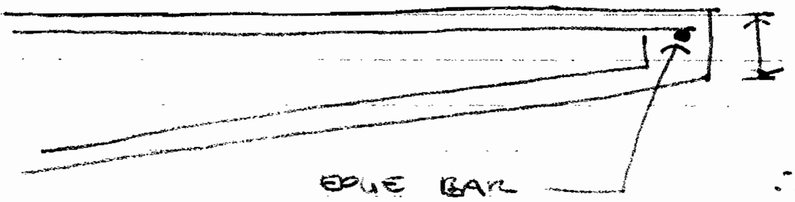
'h' IS THEREFORE LIMITED BY COVER AND SPACING OF BARS:



h = 40 + 25 + 30 + 25 + 40

∴ h = 160 mm

A POSSIBLE ALTERNATIVE IS:



h = 40 + 25 + 25 + 40

∴ h = 130 mm

③

OR

WITHOUT EDGE BAR → h = 105 mm

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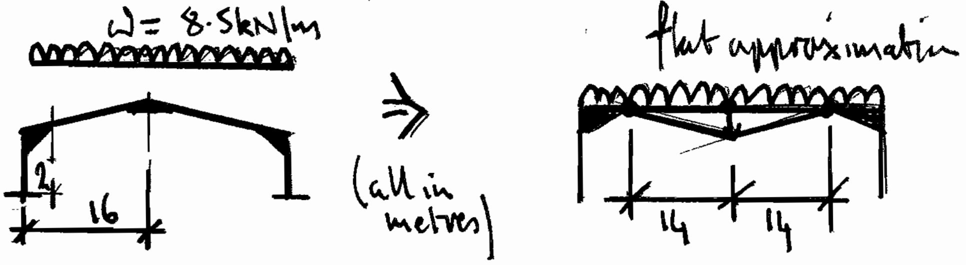
4a)



⇒ M_p for three hinge collapse ⇒ $M_p = \frac{wL^2}{16}$, L = 14m ⇒ $\frac{8500 \times 14^2}{16} =$ 466.5kNm

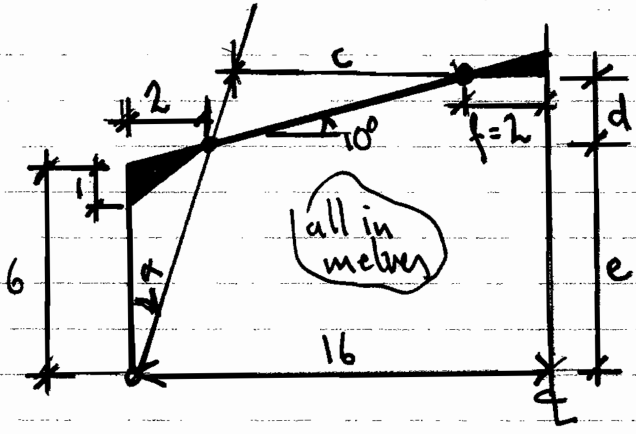
⇒ $Z_p = M_p / \sigma_y \sim$ S275 ≡ 275MPa ⇒ $46900 / 275 \times 10^6$ = 1514.5cm³

Z_p of 457×191×74 UB (suggested) = 1653cm³, ∴ sufficient.

However, ω after self-wt = 8.5 + 73.4 kg/m × 9.81 m/s² kN/m = 9.22kN/m

⇒ revised $M_p = \frac{\overset{9.22\text{kN/m}}{w} L^2}{18}$ = 451.8kNm ⇒ reqd Z_p = 1643cm³ ∴ suggested UB just OK

4b) Using the geometrical layout from lectures, for the indicated collapse mechanism



c, d, e and f (given) are needed to calculate revised Mp

$e = 6 + 2\tan 10° = 6.35$m

$d = 12 \tan 10° = 2.12$ m

$c = 16 - 2 - 2 - d\tan\alpha$

$\tan\alpha = 2/e$

$c = 12 - 2.21 \cdot \frac{2}{6.35} = 11.33$m

From notes $M_p = \frac{wc^2}{4} \frac{[1 + 2f/c]}{[1 + d/2e]}$ } known or derived from 1st principles

$$\Rightarrow M_p = \frac{9.22 \times 10^3 \times 11.33^2}{4} \frac{[1 + 2 \times 2/11.33]}{[1 + 2.12/2 \times 6.35]} = 343.1\text{kNm}$$

Reduction of 18% c.f. (a)

⇒ A new beam can be chosen with Z_p = 343.1kN/m / 275MPa = 1247cm3

⇒ 457×152×60 (1287cm³) OK, just or 457×152×67 (1453cm³) too high, perhaps.

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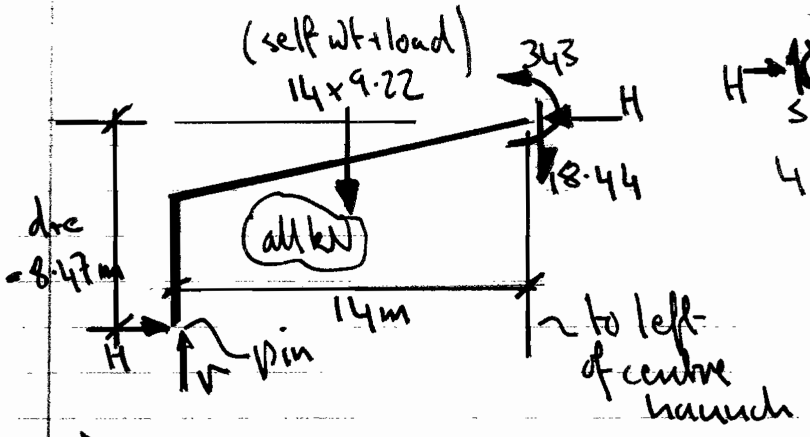
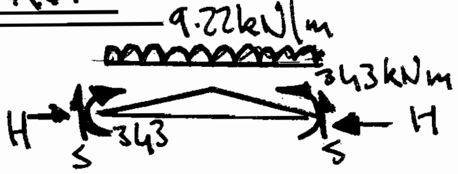




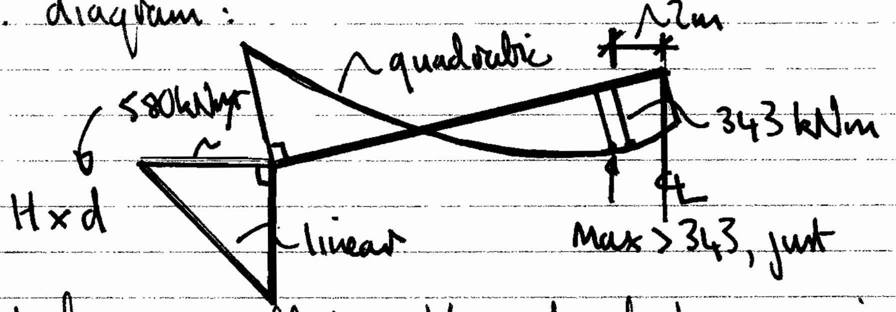
$4 \times 9.22 = 2S \Rightarrow S = 18.44\text{kN}$

$V = 14 \times 9.22 + 18.44 = \underline{147.5\text{kN}}$: moments about pinned foot $H \times 8.47 + \overset{M_p \downarrow}{343} - 18.44 \times 14 - 14 \times 9.22 = 0$

$\Rightarrow H = \underline{96.7\text{kN}}$

4c) Axial force in rafter $= H\cos \overset{\text{pitch}}{10^\circ} + V\sin 10^\circ = \underline{\underline{120.8\text{kN}}}$

B.M. diagram:



Axial force in rafter ⇒ M_p reduced by removing purely compressive core of depth d, according to

$\underset{0.0081\text{m}}{\overset{\uparrow}{b}}d \cdot \sigma_y = \underset{\text{axial force}}{\underline{120.8\text{kN}}} \Rightarrow d = 54\text{mm}$

Lost M_p portion $= \frac{bd^2}{4} \cdot \sigma_y = 1.64\text{kNm} < 0.5\%$ of plastic capability in bending

Thus if choosing a 457×152×60 UB, then there is still enough residual Z_p, to carry axial force (if the Z_p decreases by 0.5%).

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MODULE 3D3, STRUCTURAL MATERIALS AND DESIGN

1 (a.i) 8 Mp / L^2; (a.iii) 12 Mp / L^2; (b.i) 4 w L^2; (b.ii) shear force = w L^2; maximum moment = w L^3 / 2

2 (a) h_min = 172 mm; (b) l_b >= 16.6 mm; (c) v_tot = 31.6 mm, b = 150 mm

3 (a) moment range -243 kNm to 137 kNm; (b) sagging reinforcement area = 845 mm^2; hogging reinforcement area = 1680 mm^2; (c) V_max = 203.2 kN; (d) h = 160 mm

4 (a) Mp = 416.5 kNm, revised Mp after self-weight included = 451.8 kNm; (b) Mp = 343.1 kNm, chose a 457 X 152 X 60 UB (Zp = 1287 cm^3); reaction components at pinned feet, 147.5 kN and 96.7 kN; (c) axial force in rafter = 120.8 kN, resulting in 0.5% loss of Mp.