2009 IIA 3D5 - Water Engineering Dr D. Liang

Question 1

(a) Infiltration rate: $f = f_c + (f_0 - f_c)e^{-K_f t} = 30e^{-0.2t}$

$f(0) = 30$ mm/hr
$f(1) = 24.6$ mm/hr
$f(2) = 20.1$ mm/hr

During the first hour, all the rainfall infiltrates with zero excess rainfall.
Find equivalent time in the infiltration curve:

$$25 = \int_0^{t_1} f dt = \int_0^{t_1} 30e^{-0.2t} dt = \frac{1}{-0.2} \times 30e^{-0.2t}\Big|_0^{t_1} = \frac{1}{-0.2} \times 30\left(e^{-0.2t_1} - 1\right) = 150 - 150e^{-0.2t_1}$$

So, $t_1 = 0.91$ hr

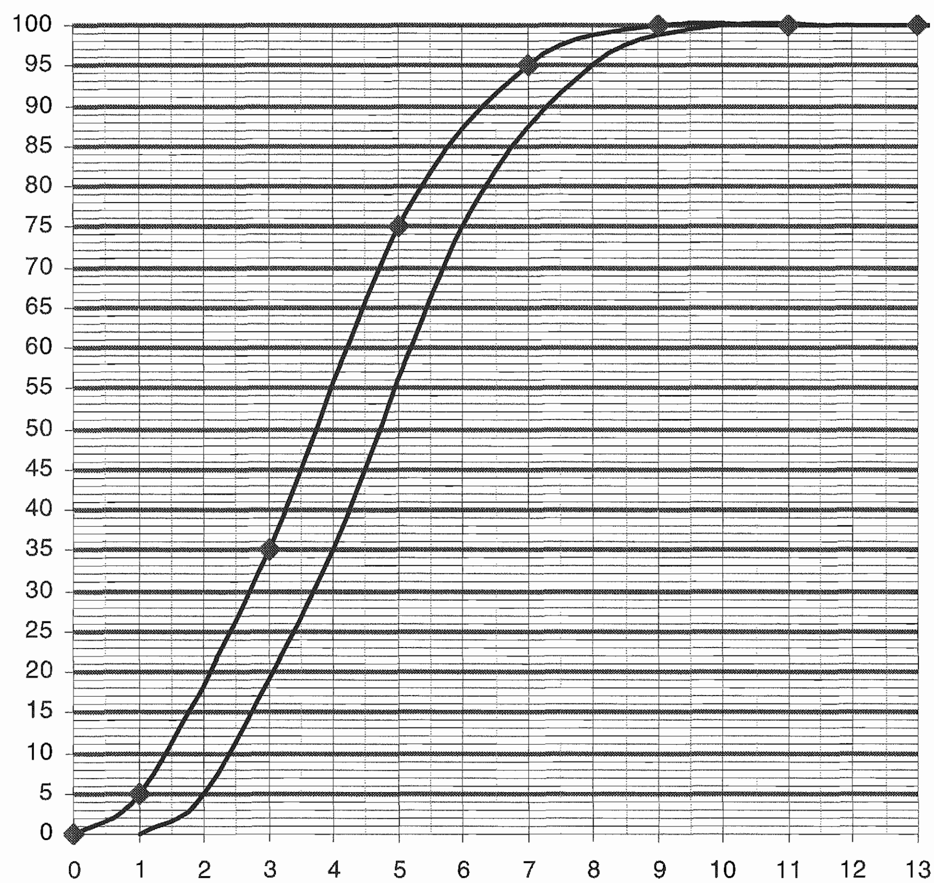
Infiltration in the second hour is:

$$\int_{t_1}^{t_1+1} f dt = \int_{0.91}^{1.91} 30e^{-0.2t} dt = \frac{1}{-0.2} \times 30e^{-0.2t}\Big|_{0.91}^{1.91} = 22.7 \text{ mm}$$

During the second hour, the rainfall excess is 30-22.7 = 7.3 mm.

(b) Plot S curve:

t:	0	1	3	5	7	9	11
%:	0	5	35	75	95	100	100



Readings on S curve at half hour points:

t	0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
%	0	1.5	11.5	26.5	45	66	81.5	91.5	97.5	99.5	100

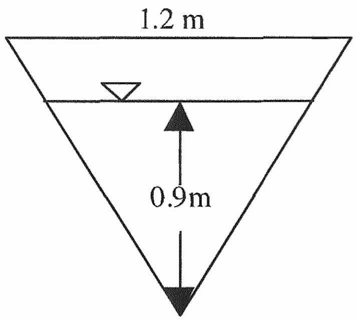
Shift by one hour and subtract:

t	0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
%:	0	1.5	10	15	18.5	21	15.5	10	6	2	0.5

The peak discharge occurs in the fifth hour: $\frac{21\% \times 10 \times 10^6 \times 0.02}{3600} = 11.7 \text{ m}^3/\text{s}$

(c) $$\tau_b = \rho g R_h S_b = \frac{C_f}{2}\rho \cdot U^2 \Rightarrow U = \sqrt{\frac{2g}{C_f} R_h S_b} \Rightarrow U = C\sqrt{R_h S_b}$$

(d)



Cross-sectional width at the free surface: $B = 0.9 \times \tan 30^0 \times 2 =$ 1.039 m

Area: $A = \frac{1}{2} \times B \times 0.9 = 0.468 \text{ m}^2$

Wetted perimeter $P_h = 2 \times \frac{0.9}{\cos 30^0} = 2.078$ m

Hydraulic radius $R_h = \frac{A}{P_h} = \frac{0.468}{2.078} = 0.225$ m

Chezy formula $U = C\sqrt{R_h S_b} = 65\sqrt{0.225 \times 0.01} = 3.083$ m/s

Final answer Q = UA = 3.083 × 0.468 = 1.44 m^3/s

Question 2

(a) $Uh = \frac{1}{n} \cdot h^{5/3} \cdot S_b^{\ 1/2} = q$

$\frac{1}{0.012} \cdot h^{5/3} \cdot 0.001^{1/2} = 0.6$

$h = 0.41$ m

U = q/h = 0.6/0.41 = 1.46 m/s

$Fr = \frac{U}{\sqrt{gh}} = \frac{1.46}{\sqrt{9.8 \times 0.41}} = 0.73 < 1$, so the follow is subcritical.

Or $\sqrt{gh_c}\,h_c = q$

$\sqrt{9.8} \cdot h_c^{\ 3/2} = 0.6$

$h_c = 0.33$ m

$h > h_c$, so the flow is subcritical.

(b) Momentum equation: $\sum F = \rho Q(U_{out} - U_{in})$

Consider the unit-width

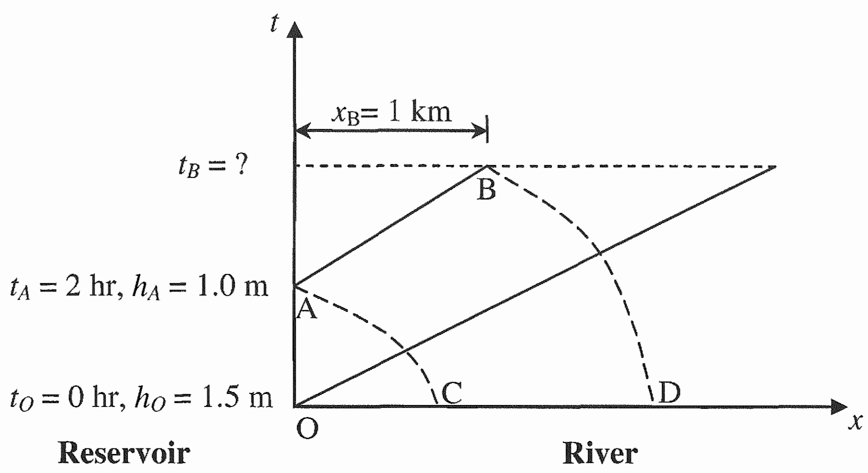
$$\frac{\rho g h_1^2}{2} - \frac{\rho g h_2^2}{2} = \rho q (U_2 - U_1)$$

$$\frac{\rho g h_1^2}{2} - \frac{\rho g h_2^2}{2} = \rho q \left(\frac{q}{h_2} - \frac{q}{h_1} \right)$$

$$\frac{\rho g h_1^2}{2} - \frac{\rho g (2h_1)^2}{2} = \rho q \left(\frac{q}{2h_1} - \frac{q}{h_1} \right)$$

$$h_1 = \sqrt[3]{\frac{q^2}{3g}}$$

(c)



Along negative line AC: $\left(U - 2\sqrt{gh}\right)_A = \left(U - 2\sqrt{gh}\right)_C$

$U_A - 2\sqrt{9.8 \times 1} = -1.2 - 2\sqrt{9.8 \times 1.5}$

U_A = -2.61 m/s

Positive line AB is straight: $\frac{dx}{dt} = U + \sqrt{gh}$

$\frac{x_B - x_A}{t_B - t_A} = \left(U + \sqrt{gh}\right)_A$

$\frac{1000}{t_B - 2 \times 3600} = -2.61 + \sqrt{9.8 \times 1.0}$

t_B = 9121 s =2 hours and 32 minutes= 2.53 hr

Question 3

(a.i)

$$U = C\sqrt{R_h S_b}$$

$$1 = C\sqrt{0.8\times 0.0005}$$

$$C = 50.0$$

$$C = 7.8\ln\left(\frac{12.0\cdot R_h}{k_s}\right)$$

$$50.0 = 7.8\ln\left(\frac{12.0\times 0.8}{k_s}\right)$$

$$k_s = 0.016 \text{ m}$$

(a.ii)

$$C' = 7.8\ln\left(\frac{12h}{k_s'}\right) = 7.8\ln\left(\frac{12\times 0.8}{0.002}\right) = 66.12$$

$$\theta = \frac{\tau_b}{g(\rho_s-\rho)d} = \frac{R_h S_b}{(s-1)d} = \frac{0.8\times 0.0005}{(2.65-1)\times 0.002} = 0.1212$$

$$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 8\left[\left(\frac{C}{C'}\right)^{1.5}\theta - 0.047\right]^{1.5}$$

$$\frac{q_b}{\sqrt{9.8\times(2.65-1)\times 0.002^3}} = 8\left[\left(\frac{50.0}{66.12}\right)^{1.5}\times 0.1212 - 0.047\right]^{1.5}$$

$$q_b = 1.7\times 10^{-5} \text{ m}^2/\text{s} = 0.045 \text{ kg/(m·s)}$$

(b.i)

$$C = 7.8\ln\left(\frac{12h}{k_s}\right) = 7.8\ln\left(\frac{12\times 1.6}{0.1}\right) = 41.0$$

$$\tau_b = \rho g\frac{U^2}{C^2}$$

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{g}\frac{U}{C} = \sqrt{9.8}\times\frac{0.5}{41.0} = 0.038 \text{ m/s}$$

$$d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.0002\cdot\left(\frac{9.8\times(2.65-1)}{10^{-12}}\right)^{1/3} = 5.06$$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049\cdot d_*^3} - 10.36\right] = \frac{10^{-6}}{0.0002}\left[\sqrt{10.36^2 + 1.049\cdot 5.06^3} - 10.36\right] = 0.026 \text{ m/s}$$

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{\bar{c}(0.3)}{\bar{c}(0.6)} = \left(\frac{1.6-0.3}{0.3}\cdot\frac{0.6}{1.6-0.6}\right)^{\frac{0.026}{0.4\times 0.038}}$$

$$\bar{c}(0.3) = 5.12\cdot\bar{c}(0.6) = 5.12\times 0.1 = 0.512 \text{ kg/m}^3$$

(b.ii)

$$D_x = D_{tx} + D_L = (0.15 + 5.93)hu_* = 6.08 \times 1.6 \times 0.038 = 0.37 \text{ m}^2\text{/s}$$

$$c(x,t) = \frac{M_1 / A}{\sqrt{4\pi D_x t_1}} \exp\left(-\frac{(x-u_0 t_1)^2}{4D_x t_1}\right) + \frac{M_2 / A}{\sqrt{4\pi D_x t_2}} \exp\left(-\frac{(x-u_0 t_2)^2}{4D_x t_2}\right)$$

$$= \frac{100/(20\times1.6)}{\sqrt{4\pi\times0.37\times70\times60}} \exp\left(-\frac{(2000-0.5\times70\times60)^2}{4\times0.37\times70\times60}\right) + \frac{50/(20\times1.6)}{\sqrt{4\pi\times0.37\times65\times60}} \exp\left(-\frac{(2000-0.5\times65\times60)^2}{4\times0.37\times65\times60}\right)$$

$$= \frac{3.125}{139.71} \exp\left(-\frac{(-100)^2}{6216}\right) + \frac{1.5625}{134.63} \exp\left(-\frac{50^2}{5772}\right)$$

$$= 4.47\times10^{-3} + 7.53\times10^{-3}$$

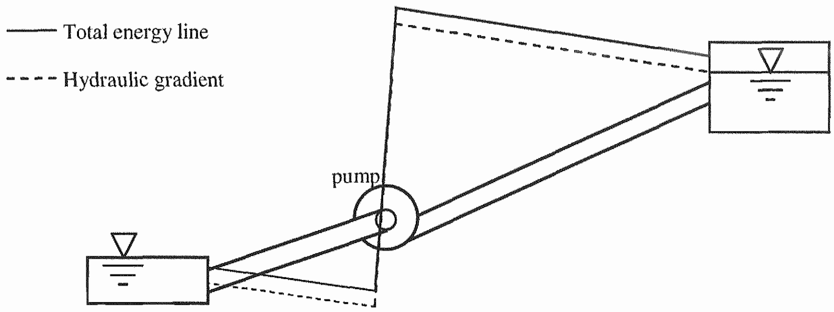
$$= 12.0\times10^{-3} \text{ kg/m}^3$$

Question 4

(a)
The flow over dunes are subcritical, thus the water surface profile is in phase with the bed profile. The flow over antidunes are normally supercritical, thus the water surface profile is out of phase with the bed profile.

The sediment transport is related to the flow velocity. Larger amount of sediment is carried over the crest than the trough of the dunes, thus scour occurs on the front side of the dunes (as the flow accelerates from trough to crest) and deposition occurs on the lee side of the dunes (as the flow decelerates from crest to trough). For antidunes, larger amount of sediment is carried at the trough than at the crest, thus deposition occurs on the front side and scour occurs on the lee side.

(b)

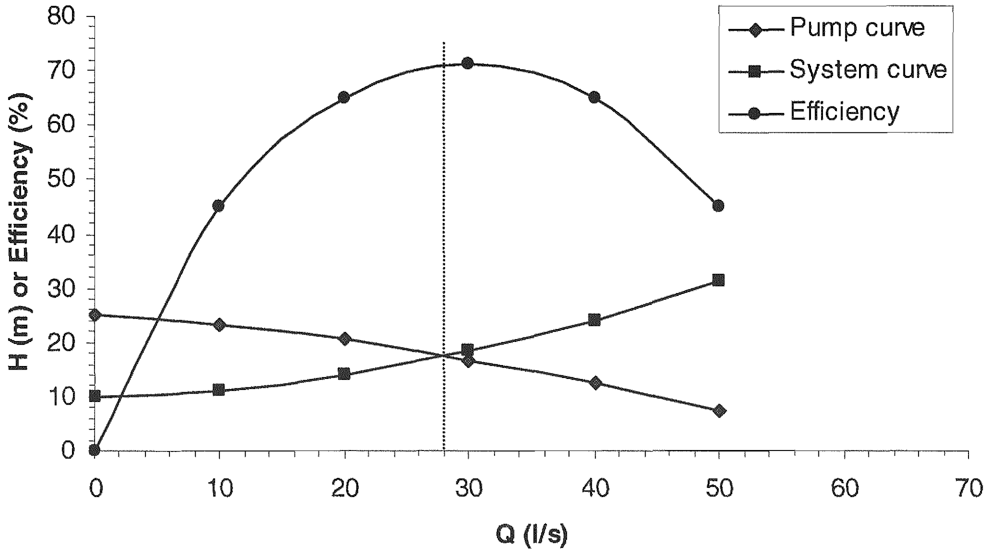


(c)

The head loss in the pipeline is $$H = 10 + 6.2\frac{U^2}{2g} + \lambda\frac{L}{D}\frac{U^2}{2g}$$

λ is dependent on the relative roughness height $\frac{k_s}{D} = 0.00015$ and Reynolds number.

Q (l/s)	10	20	30	40	50
Re ($\times 10^5$)	0.56	1.13	1.10	2.25	2.81
λ	0.021	0.0185	0.0172	0.0165	0.016
Local loss (m)	0.03	0.13	0.29	0.51	0.80
h_f (m)	1.18	3.82	7.99	13.63	20.65
H (m)	11.11	13.95	18.28	24.14	31.45



The intersection point gives the operating conditions: $H = 17.5$ m, $Q = 28.0$ litre/s.
The operating efficiency is 71%. Therefore, power consumption is

$$P_p = \rho g Q_p H_p / \eta = \frac{1000 \times 9.8 \times 0.028 \times 17.5}{0.71} = 6763 \text{ Watts}$$

(d)

Calculate the specific speed, and then decide.
The specific speed of centrifugal pumps is low, while the specific speed of axial pumps is high.

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List of Numerical Answers

Q1. (a) 0 in the first hour, and 7.3 mm in the second hour
(b) 11.7 $m^3\ s^{-1}$ during the fifth hour
(d) 1.44 $m^3\ s^{-1}$

Q2. (a) Subcritical
(b) $h_1 = \sqrt[3]{\dfrac{q^2}{3g}}$
(c) 2.53 hours

Q3. (a) (i) 0.016 m (ii) 0.045 kg $m^{-1}\ s^{-1}$
(b) (i) 0.512 kg m^{-3} (ii) 0.012 kg m^{-3}

Q4. (c) H = 17.5 m, Q = 28.0 l s^{-1}, P = 6.76 kW