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ENGINEERING TRIPOS PART IIA

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4 May 2009

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Module 3E3 – CR1B

MODELLING RISK

*Answer not more than **two** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

Version: 3

1 The historical data for a specific UK town are as follows:

| Month | Sales (number of homes) | Unemployment rate (%) | Consumer Confidence Index (CCI) |
|---|---|---|---|
| 1 | 20 | 7.2 | 72 |
| 2 | 41 | 4.0 | 89 |
| 3 | 17 | 7.3 | 85 |
| 4 | 35 | 5.5 | 65 |
| 5 | 25 | 6.8 | 80 |
| 6 | 31 | 6.0 | 82 |
| 7 | 38 | 5.4 | 90 |
| 8 | 50 | 3.6 | 90 |
| 9 | 15 | 8.4 | 63 |
| 10 | 19 | 7.0 | 60 |
| 11 | 14 | 9.0 | 65 |

(a)

(i) Calculate the three-month moving average sales for months 3 to 7. Based on this method, what would be your sales forecast for month 8? **[12%]**

The three month moving average for months 3 to 7 is given by:

$m_3 = (17+41+20)/3 = 26$
$m_4 = (35 + 17 + 41)/3 = 31$
$m_5 = (25 + 35 + 17)/3 = 25.67$
$m_6 = (31 + 25 + 35)/3 = 30.33$
$m_7 = (38 + 31 + 25)/3 = 31.33$

The forecast for month 8 is just the moving average for the month before. Therefore, the forecast for month 8 is 31.33 or 32 houses.

(ii) Calculate sales forecast for month 8 using exponential smoothing with a smoothing constant of 0.2. **[14%]**

$M_1 = Y_1 = 20$
$M_2 = 0.2Y_2 + 0.8M_1 = 0.2(41) + 0.8(20) = 24.2$
$M_3 = 0.2Y_3 + 0.8M_2 = 0.2(17) + 0.8(24.2) = 22.76$
$M_4 = 0.2Y_4 + 0.8M_3 = 0.2(35) + 0.8(22.76) = 25.208$
$M_5 = 0.2Y_5 + 0.8M_4 = 0.2(25) + 0.8(25.208) = 25.166$

$M_6 = 0.2Y_6 + 0.8M_5 = 0.2(31) + 0.8(25.166) = 26.333$
$M_7 = 0.2Y_7 + 0.8M_6 = 0.2(38) + 0.8(26.333) = 28.666$

The forecast for month 8 is just the weighted average for month 7. $M_7 = 28.666$ or 29 houses.

(iii) Calculate the regression equation for the sales data with $x$ as unemployment rate and $y$ as sales. ($\sum xy = 1750.8, \sum x^2 = 476.3$) **[16%]**

$\bar{x} = 6.38 \qquad \bar{y} = 27.73$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{1750.8 - 11(6.38)(27.73)}{476.3 - 11(6.38)^2} = -6.84$$

$a = \bar{y} - b\bar{x} = 27.73 - (-6.84)(6.38) = 71.37$
$y = 71.37 - 6.84x$

(iv) Using the regression equation in part (i), predict home sales in town if the unemployment rate is 4.5%. **[8%]**

$y = 71.37 - 6.84(4.5) = 40.59$ (41 house sales)

(v) Without performing any calculations, comment on the role of CCI as a possible sales predictor. **[16%]**

The consumer confidence index can be a sales predictor. Here are three different models each of which uses CCI as a predictor: (1) A simple regression model in which sales is the dependent variable and CCI is the independent variable, (2) A multiple regression model in which sales is the dependent variable and CCI and the unemployment rate are independent variables, (3) A two-stage regression model in which the unemployment rate is used to predict CCI, which is in turn used to predict sales. The third model is potentially better than the second one in particular when there is a multicolinearity problem between the unemployment rate and CCI.

(b)

(i) Briefly explain CAPM and the meaning of Beta under CAPM. **[12%]**

The CAPM is an asset pricing model that measures the required relationship between risk and returns from investment for diversified investors. Beta is a measure of the systematic or market risk of a given investment or portfolio of

assets. It describes the correlation between the returns of the asset or portfolio and the returns of the financial market. This risk is non diversifiable.

(ii) Stocks A and B have a correlation of -1. CAPM Betas for these stocks are 0.5 and -2 respectively. Correlations of stocks A and B with the market portfolio are 0.5 and -0.6 in that order. The variance of the market portfolio is 0.25. Find the variance of an equally weighted portfolio composed of A and B. **[22%]**

$$\beta_A = \frac{\sigma_A \sigma_M \rho_{AM}}{\sigma^2_M} = 0.5 \text{ leads to } \sigma_A = 0.5$$

$$\beta_B = \frac{\sigma_B \sigma_M \rho_{BM}}{\sigma^2_M} = -2 \text{ leads to } \sigma_B = 1.67$$

$$\sigma^2_{portf} = w^2_A \sigma^2_A + w^2_B \sigma^2_B - 2 w_A w_B \rho_{AB}$$

The variance of the equally weighted portfolio is 1.26.

2 (a) Simple queuing systems are conventionally labelled by **U / V / s / κ / W**.

(i) Briefly explain the Kendall's notation. **[10%]**

The Kendall's notation is the convention used to specify the characteristics of the service system:

U and V denote the interarrival and service time distributions.

s is the number of servers in the system.

κ is the system capacity (maximum number of customers in service and queue).

W is the queuing discipline (e.g. FIFO or LIFO).

(ii) What are the characteristics of an M/G/s queue? **[6%]**

This service system has the following characteristics:

Exponential Interarrival distribution, Unspecified or general service time distribution, s number of servers, unlimited capacity and FIFO discipline.

(b) The visa section of the French Embassy in London has two phone operators arranging appointments for any potential applicant (customer) wishing to apply for a visa in person at the embassy. Let $n$ denote the number of customers in the service system. The system is in a steady-state condition and the steady state probabilities in the system are $\mathbf{p_0}$=1/13, $\mathbf{p_1}$=4/13, $\mathbf{p_2}$=4/13, $\mathbf{p_3}$=2/13, $\mathbf{p_4}$=1/13, $\mathbf{p_5}$=1/13, $\mathbf{p_n}$=0 if $n$>5.

(i) Calculate the expected number of customers in the system and the expected number of customers in the queue. **[16%]**

$L = \Sigma^{\infty}_{n=0} n\, p_n = 4/13 + 2 \times 4/13 + 3 \times 2/13 + 4 \times 1/13 + 5 \times 1/13$
$= 27/13$ customers in the system.

$L_q = \Sigma^{\infty}_{n=s} (n-s)\, p_n = 2/13 + 2 \times 1/13 + 3 \times 1/13 = 7/13$ customers in the queue.

(ii) Calculate the expected number of customers being served. **[8%]**

$E$ (number of customers served) $= L - L_q = 20/13$

(iii) Suppose the call arrival rate is 3 customers per hour. Calculate the mean time in the system and the mean waiting time in the queue. **[12%]**

Using Little's law:

$W = L/\lambda = 9/13$ hours

$W_q = L_q/\lambda = 7/39$ hours

(iv) Calculate the mean service time per customer and the utilisation factor of the system. **[16%]**

$W = W_q + 1/\mu$

$\mu = 1/(W - W_q) = 39/20$ customers per hour

The utilisation rate is $\rho = \lambda/(s\mu) = 3/(2 \times 39/20) = 10/13$.

(c) At the embassy, only one operator processes the initial screening stage of visa applications. He processes 10 applications every hour on average. This operator leaves his desk for 15 minutes to take a "productivity" break.

(i) What is the probability that exactly one applicant will stand in front of the operator's desk during the 15 minute break (assume it is an M/M/1 queue)? **[14%]**

The arrival rate is 10/60 = 0.167 jobs/min.

$$P_1(15) = \frac{(1/6 \times 15)}{1} e^{-0.167 \times 15}$$
$$= 0.205$$

(ii) If the inter-arrival time between applicants is exponentially distributed with a rate $\lambda$, how many customers would you expect to arrive during the 15 minute break?

Expected number of customers = $\lambda$ * t

With an arrival rate $\lambda$ = 0.167, in 15 minutes we should expect 2.5 applicants to arrive.

**[4%]**

(d) Why do we need the balance equation of the birth-and-death process in queueing theory? **[6%]**

The balance equation allows us to find closed form solutions for the performance measures of simple queuing systems.

(e) What are the characteristics of the queueing system described by the Pollaczek-Khitchine formula? **[8%]**

It is an M/G/1 queueing system (exponential interarrival, general service time distribution and 1 server).

3 (a) Briefly explain risk aversion and risk premium. **[12%]**

Risk aversion is a person's attitude towards risk. It represents the reluctance of this person to accept a gamble with uncertain outcomes instead of another gamble with a safer outcome. Other attitudes to risk include risk neutrality and risk seeking. The risk premium is the expected monetary value an investor or decision-maker is willing to give up to avoid a risky investment or decision.

(b) IMB Ltd. is developing a new computer system. The company is trying to decide on the manufacturing and assembly processes to be used. They are considering whether to manufacture the keyboards themselves, buy them from another local company, or buy them from a supplier in a developing country with lower labour costs. The following pay-off table has been calculated for different levels of demand:

| | Sales | | |
|---|---|---|---|
| | Low | Medium | High |
| Manufacture themselves | -15 | 10 | 55 |
| Buy locally | 5 | 20 | 40 |
| Buy abroad | 10 | 30 | 25 |

Determine the best policy on the basis of the maximax, maximin, and minimax regret criteria. **[30%]**

Maximax

| | Sales | | |
|---|---|---|---|
| | Low | Medium | High |
| Manufacture themselves | -15 | 10 | ***55*** |
| Buy locally | 5 | 20 | ***40*** |
| Buy abroad | 10 | ***30*** | 25 |

Maximin

| | Sales | | |
|---|---|---|---|
| | Low | Medium | High |
| Manufacture themselves | ***-15*** | 10 | 55 |
| Buy locally | ***5*** | 20 | 40 |
| Buy abroad | ***10*** | 30 | 25 |

Minimax Regret

| | Sales | | | Max Regret |
|---|---|---|---|---|
| | Low | Medium | High | |
| Manufacture themselves | ***10 – (-15) = 25*** | 30 – 10 = 20 | Best 55 – 55 = 0 | 25 (2) |
| Buy locally | 10 – 5 = 5 | 30 – 20 = 10 | ***55 – 40 = 15*** | ***15*** (1) |
| Buy abroad | *Best 10 – 10 = 0* | Best 30 –30 = 0 | ***55 – 25 = 30*** | 30 (3) |

Maximax would suggest that manufacturing systems in-house will give the highest payoff (£55k) in the best case. Maximin suggests that buying abroad is a low risk strategy giving the highest payoff (£10k) in the worst case, and minimax regret suggests buying locally (maximum regret £15k).

(c) Suppose the price of a stock is £100. The exercise prices of call and put options trading on the stock are also £100. These options mature in 24 months. Both European and American options are traded. The annualized volatility of the rate of return on the stock is 30%. The risk-free rate of return is 8% per annum.

Use the binomial option pricing approach with a time step of twelve months to price European call and put options on the stock. **[30%]**

Discount factor per step = 0.926
Time step, dt = 1 year
Growth factor per step = 1.08
Up step size, u = 1.350
Down step size, d = 0.741
Probability of up move, p = 0.5623

European call option:

Option price = [(100* (1.35^2)-100)*0.5623^2]/$(1.08)^2$ = 22.286

European put option:

Option price = [(100-100* (0.741^2))*0.4377^2]/$(1.08)^2$ = 7.410

(d) You are planning to scale down the operations of your manufacturing plant by

70% sometime in the next six months. Suppose the value of the plant today is £100m. Your plan is to save £70m from scaling down operations. Uncertainty in the operating environment is estimated by an annualized standard deviation of 50% in cash flows. The risk-free rate of return is 8% per annum.

(i) Use the binomial option pricing approach with a time step of three months to value the option to scale down operations. Is there an optimal time for taking such an action? **[20%]**

Scaling down operations is equivalent to exercising a real put option on 70% of your operations. Since this action can be taken now, in three months or in six months, the decision to scale down should be valued as an early exercise (American) put option.

With T the maturity of the option, r the risk free rate, σ the volatility, and t the required time step, we have:
Discount factor per step = $(1+ r)^{-t}$
Growth factor per step = $(1+r)^{t}$
Up step size, u = $e^{\sigma\sqrt{T-t}}$
Down step size, d = $e^{-\sigma\sqrt{T-t}}$
Probability of up move, p = $\dfrac{e^{r(T-t)} - d}{u - d}$

Strike = 70M
Discount factor per step = 0.9802
Time step, dt = 0.25 years
Growth factor per step = 1.0202
Up step size, u = 1.284
Down step size, d = 0.779
Probability of up move, p = 0.478

Valuation:

6 months maturity

Put option value = [(70-70* (0.779^2))*0.522^2]/$(1.02)^2$ = 7.21M

3 months maturity:

Put option value = [(70-70* (0.779))*0.522]/(1.02) = 7.92M

Scaling down operations in three months is the best option.

(ii) Suppose that the decision to scale down operations incurs social costs to your company. What would you recommend doing? **[8%]**

If these social costs are greater than the value of the option to scale down, it's better to keep the plant operating as usual, and maybe delay the downsizing to a later date.

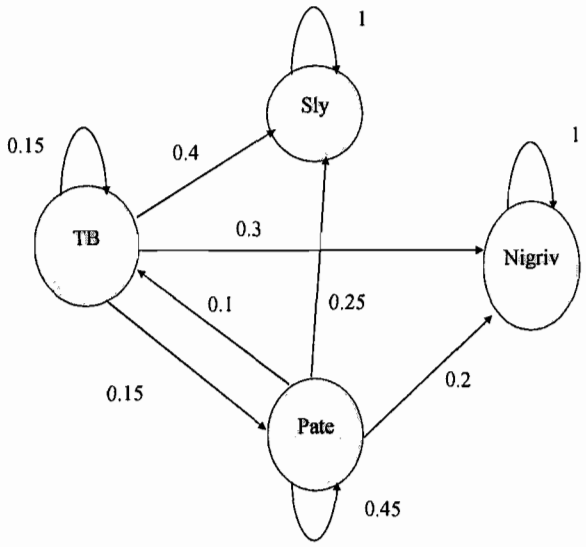
4 (a) Briefly explain the difference between risk and uncertainty. **[16%]**

Risk is defined by the occurrence of a negative event. Uncertainty is the lack of complete certainty about future events. It encompasses the risks and opportunities behind any given commitment. As opposed to risk, uncertainty is not necessarily negative to decision-makers, and in fact can be a source of value.

(b) Suppose country A's broadcasting industry is controlled by four companies: Sly TV, Nigriv Media, TB Vision and Pate. If customers subscribe to either Sly or Nigriv, they will never switch to another broadcaster. If they subscribe to TB Vision this year, the probabilities that they will switch to Sly, Nigriv and Pate next year are 0.4, 0.3 and 0.15, respectively. If they subscribe to Pate, the probabilities that they will switch to Sly, Nigriv and TB next year are 0.25, 0.2 and 0.1, respectively. Each customer subscribes to exactly one broadcaster per year.

(i) Represent this business situation using a state-transition diagram. **[18%]**

Transition Diagram




(ii) If TV subscribers are initially distributed as 40%, 30%, 10% and 20%, for Sly, Nigriv, TB and Pate, respectively, find the market share for each broadcaster after 2 years. **[24%]**

Let

Sly: state 1, Nigriv: state 2, TB: state 3 and Pate: state 4

We have $S_0 = [0.4, 0.3, 0.1, 0.2]$ and are looking for $S_2 = S_0 P^2$

$S_2 = [0.53025, 0.4015, 0.01575, 0.0525]$

(c) After two years, Sly TV considers analysing the effect of poor handling of customers enquiries by their phone service operators on their business. Analysis over the last month has indicated that probabilities of customers switching satisfaction states every time they call Sly customer services are the following:

| | | To Satisfied | Dissatisfied |
|---|---|---|---|
| **From** | **Satisfied** | **0.65** | **0.35** |
| | **Dissatisfied** | **0.45** | **0.55** |

(i) Find the steady state probabilities of satisfaction and dissatisfaction among Sly customers. **[16%]**

State 1: satisfaction, State 2: dissatisfaction and P': above matrix.

$[\pi_1, \pi_2]P' = [\pi_1, \pi_2]$, hence $\pi_1 = 9/16$ and $\pi_2 = 7/16$ respectively.

(ii) Under the current customer service operation policy, a monthly cost of -1 is incurred if customers are dissatisfied with their calls, while a profit of 0.35 is generated under a satisfaction state. Find the average payoff of this policy per month. What long-term proportions of satisfaction and dissatisfaction are required for Sly to break even? **[14%]**

Average payoff = 9/16 * 0.35 – 1* 7/16 = -0.2406.

In order for Sly to break even on this policy, we need to solve the equation average payoff = p * 0.35 – 1* (1-p) = 0, which implies that p = 74.1%, where p is the rate of customer satisfaction.

(d) Briefly explain the expected value of perfect information. **[12%]**

It is the difference between the maximum expected value of undertaking a given commitment with perfect information and the maximum expected monetary value from undertaking the same commitment without perfect information. Rationally, the expected value of perfect information should be the maximum price that one would be willing to pay to gain access to perfect information.

**END**

**Numerical Answers to 3E3 Modelling Risk**

1(a)(i) The forecast for month 8 is 31.33 or 32 houses.
1(a)(ii) The forecast for month 8 is 28.666 or 29 houses.
1(a)(iii) The regression equation is $y = 71.37\text{-}6.84\ x$.
1(a)(iv) y= 40.59 (41 house sales)
1(a)(v) ...

1(b)(i) ...
1(b)(ii) The variance of the equally weighted portfolio is 1.26.

2(a) ...

2(b)(i) L=*27/13*, $L_q$= *7/13*.
2(b)(ii) *E* (number of customers served) = *20/13*.
2(b)(iii) $W$ = *9/13*, $W_q$ = *7/39*.
2(b)(iv) $\mu$ = *39/20*, $\rho$ = *10/13*.

2(c)(i) $P_1(15) = 0.205$.
2(c)(ii) In 15 minutes we should expect 2.5 applicants to arrive.

2(d) ...

2(e) ...

3(a) ...

3(b) Maximax would suggest that manufacturing systems in-house will give the highest payoff (£55k) in the best case. Maximin suggests that buying abroad is a low risk strategy giving the highest payoff (£10k) in the worst case, and minimax regret suggests buying locally (maximum regret £15k).

3(c) European call option: 22.286, European put option: 7.410.

3(d)(i) 7.21M, 7.92M. It's better to scale down operations in 3 months.
3(d)(ii) ...

4(a) ...

4(b)(i) ...
4(b)(ii) $S_2$= [0.53025, 0.4015, 0.01575, 0.0525]

4(c)(i) $\pi_1$= *9/16 and* $\pi_2$ = *7/16*
*4(c)(ii)* Average payoff = -0.2406. 74.1% customer satisfaction is needed to break-even.

4(d) ...

Version: 2