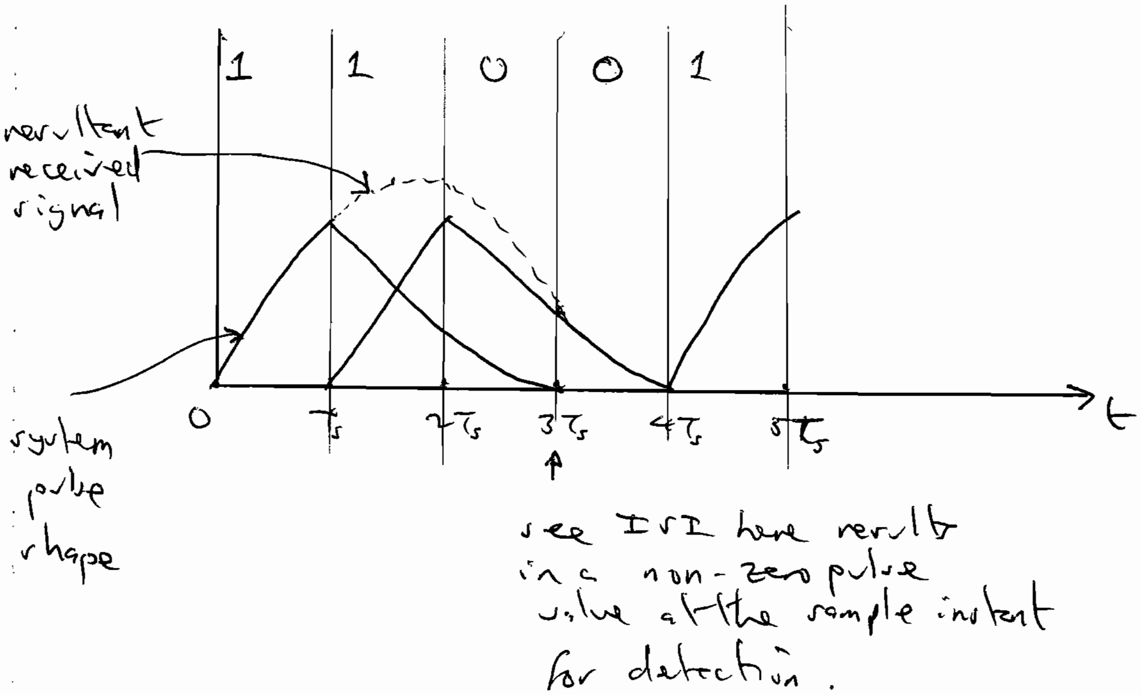
1:

Part IIA Paper 3F4 Data Transmission

30 Apr. 2009. Dr. I.J. Wassell / Prof. N.G. Kingsbury

1.) (a) If the system pulse response extends over more than one symbol period, then symbols become smeared into adjacent symbol periods. If the adjacent symbol values have a non-zero value at the ideal sample instant for the symbol currently being detected, intersymbol interference results.



The presence of ISI will potentially push the signal amplitude closer to a decision threshold increasing the prob. of bit error in the presence of system additive noise.

[4]

(b) $$\sum_{k=-\infty}^{\infty} P_R\left(f - \frac{k}{T_s}\right) = \text{constant}$$

or equivalently,

$$\sum_{k=-\infty}^{\infty} P_R\left(\omega - k\frac{2\pi}{T_s}\right) = \text{constant}$$

[2]

(Iii)

(c) Now,

(i)

$$P_R(\omega) = \frac{1}{2}\left[2\,\mathrm{sinc}\,2T_s\omega + \mathrm{sinc}(2T_s\omega - \pi) + \mathrm{sinc}(2T_s\omega + \pi)\right]$$

We need to plot this function, and then repeat it at multiples of the bit rate.

So, • $\mathrm{sinc}\ 2T_s\omega$ has a value of 1 at $\omega = 0$.

Has zero values when $2T_s\omega = \pm n\pi$, $n = 1, 2, \ldots$

i.e., $\omega = \pm \frac{n\pi}{2T_s}$.

• $\mathrm{sinc}\ (2T_s\omega - \pi)$ has a value of 1 at $2T_s\omega - \pi = 0$

i.e., $\omega = \pi/2T_s$.

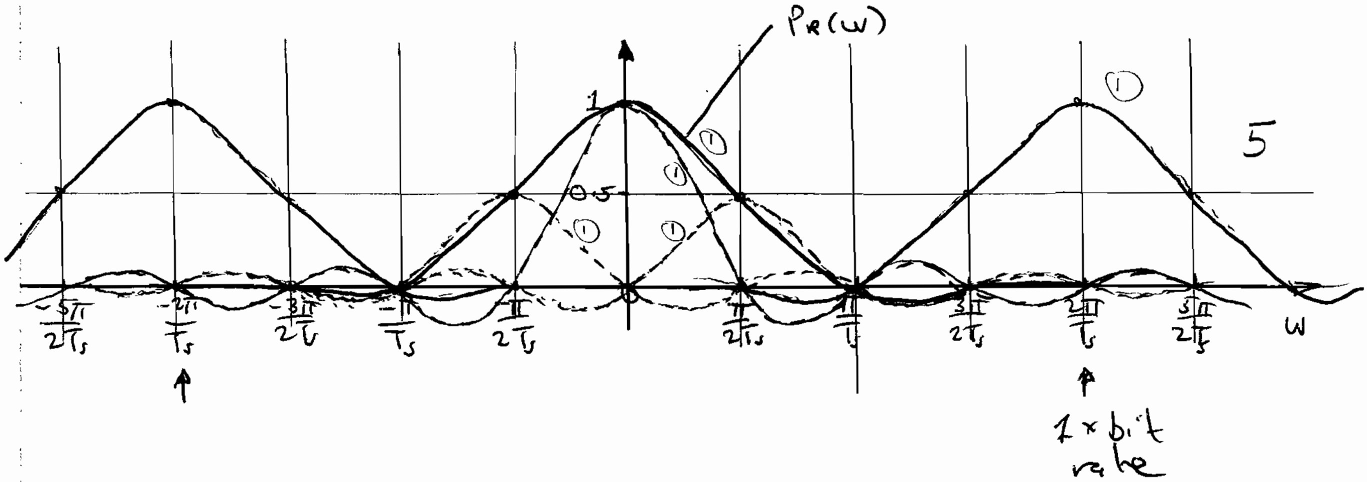
Has zero values when $2T_s\omega - \pi = \pm n\pi$, $n = 1, 2, \ldots$

i.e., $2T_s\omega = \pm n\pi + \pi$

$$\omega = \frac{\pi(\pm n + 1)}{2T_s}$$

which are at multiples of $\pi/2T_s$ (except at $\pi/2T_s$!)

• Similar situation for $\mathrm{sinc}\ (2T_s\omega + \pi)$.



5

See this does not sum to a constant value hence ISI will be present in the time domain pulse.

1

[6]

1 iii

(ii) $P_R(\omega) = \frac{1}{2}\left[2\,\mathrm{sinc}\,2T_s\omega + \mathrm{sinc}(2T_s\omega - \pi) + \mathrm{sinc}(2T_s\omega + \pi)\right]$

$$= \frac{1}{2}\left[\frac{2\sin 2T_s\omega}{2T_s\omega} + \frac{\sin(2T_s\omega - \pi)}{2T_s\omega - \pi} + \frac{\sin(2T_s\omega + \pi)}{2T_s\omega + \pi}\right]$$

$$= \frac{1}{2}\left[\frac{\sin 2T_s\omega}{T_s\omega} + \frac{(2T_s\omega + \pi)(-\sin 2T_s\omega) + (2T_s\omega - \pi)(-\sin 2T_s\omega)}{4T_s^2\omega^2 - \pi^2}\right]$$

$$= \frac{1}{2}\left[\frac{(4T_s^2\omega^2 - \pi^2)\sin 2T_s\omega - (2T_s\omega + \pi)T_s\omega\sin 2T_s\omega - (2T_s\omega - \pi)T_s\omega\sin 2T_s\omega}{T_s\omega(4T_s^2\omega^2 - \pi^2)}\right]$$

$$= \frac{1}{2}\left[\frac{4T_s^2\omega^2 - \pi^2 - 2T_s^2\omega^2 - T_s\omega - 2T_s^2\omega^2 + T_s\omega}{T_s\omega(4T_s^2\omega^2 - \pi^2)}\right]\sin 2T_s\omega$$

$$= \frac{1}{2}\left[\frac{\pi^2}{T_s\omega(\pi^2 - 4T_s^2\omega^2)}\right]\sin 2T_s\omega$$

[3]

So we can see the function decays as $1/\omega^3$.

PS. An alternative way to express $P_R(\omega)$ is,

$$P_R(\omega) = \left(\frac{1}{1 - \frac{4T_s^2\omega^2}{\pi^2}}\right)\mathrm{sinc}\,2T_s\omega.$$

(d) $P_R(\omega) = \frac{1}{2}\left[2\,\mathrm{sinc}\,2T_s\omega + \mathrm{sinc}(2T_s\omega - \pi) + \mathrm{sinc}(2T_s\omega + \pi)\right]$

$$= \frac{1}{2}\left[2\,\mathrm{sinc}\,2T_s\omega + \mathrm{sinc}\,2T_s\left(\omega - \frac{\pi}{2T_s}\right) + \mathrm{sinc}\,2T_s\left(\omega + \frac{\pi}{2T_s}\right)\right]$$

$$= \frac{1}{2}\left[2\,\mathrm{sinc}\,2T_s\omega + \mathrm{sinc}\,2T_s(\omega - \omega_0) + \mathrm{sinc}\,2T_s(\omega + \omega_0)\right] \quad 1$$

where, $\omega_0 = \frac{\pi}{2T_s}$.

Need to find $p_R(t)$. From Data Book,

$$p_R(t) = \frac{1}{2}\left[2g(t) + g(t)e^{j\omega_0 t} + g(t)e^{-j\omega_0 t}\right] \quad 1$$

(1 iv)

where $g(t)$ is the inverse Fourier transform of $\mathrm{sinc}\, 2T_s w$.

$$p_R(t) = \frac{g(t)}{2}\left[2 + e^{jw_0 t} + e^{-jw_0 t}\right]$$

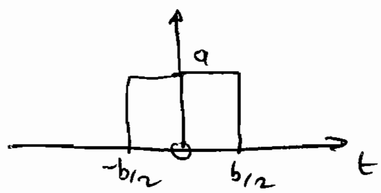
$$= \frac{g(t)}{2}\left[2 + 2\cos w_0 t\right]$$

$$= g(t)\left[1 + \cos w_0 t\right]$$

$$= g(t)\left[1 + \cos \frac{\pi}{2T_s} t\right].$$ 1

Now need to get $g(t)$. From Data Book,

$$ab\ \mathrm{sinc}\left(\frac{wb}{2}\right) \rightleftharpoons$$



So, $\mathrm{sinc}\ 2T_s w = ab\ \mathrm{sinc}\left(\frac{wb}{2}\right)$

gives $2T_s = b/2$, i.e., $b = 4T_s$.

$ab = 1 \quad \therefore a = \frac{1}{b} = \frac{1}{4T_s}$

i.e., a rect function duration $4T_s$ and amplitude $\frac{1}{4T_s}$.

So, $p_R(t) = \frac{1}{4T_s}\left[1 + \cos\frac{\pi}{2T_s} t\right] \quad -2T_s \le t \le 2T_s$.

1

$= 0$ elsewhere.

At $t = T_s$,

$$p_R(t) = \frac{1}{4T_s}\left(1 + \cos\frac{\pi}{2T_s} T_s\right)$$

$$= \frac{1}{4T_s}\left(1 + \cos \pi/2\right)$$

$$= \frac{1}{4T_s}$$ 1

same result for $t = -T_s$. [5]

2:

2.) (a) Block code: Data symbols are grouped into blocks, and each block is separately coded into a single codeword 1

Binary: Each data and code symbol is either 1 or 0. 1

Linear: Any 2 valid codewords when added together using modulo-2 arithmetic, produce a valid codeword. 1

Systematic: Any valid code word contains the corresponding data word as part of the codeword (usually the first part). 1

[4]

(b) (i) For a systematic code,

$$H = [-P^T | I]$$

now $G = [I | P]$ 1

so,
$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$ 1 [2]

(ii) We need to find the minimum weight of the non-zero codewords. 1

$$c = dG$$

d	c	
0 0	0 0 0 0 0	
0 1	0 1 0 1 1	w = 3
1 0	1 0 1 0 1	w = 3
1 1	1 1 1 1 0	w = 4

1

(2ii)

See the minimum weight, i.e., the minimum Hamming distance $= 3$.
The code can detect $d_{min} - 1 = 3 - 1 = \underline{\underline{2 \text{ errors}}}$. 1

The code can correct

$$\left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = \left\lfloor \frac{3-1}{2} \right\rfloor = \underline{\underline{1 \text{ error}}}. \quad 1$$

[4]

(iii) The standard array needs 5 syndromes for all single bit error patterns, plus 1 syndrome for no errors, i.e., 6 syndromes in total.

Code C has $2^{n-k} = 2^{5-2} = 2^3 = 8$ syndromes.
So we have 2 syndromes in excess of those required to correct all single bit errors.
We will allocate these to double bit error patterns that yield syndromes that are different to those identifying single bit errors.

					syndrome	
	0 0 0 0 0	0 1 0 1 1	1 0 1 0 1	1 1 1 1 0	0 0 0	0.5
single bit error ptns	1 0 0 0 0	1 1 0 1 1	0 0 1 0 1	0 1 1 1 0	1 0 1	0.5
	0 1 0 0 0	0 0 0 1 1	1 1 1 0 1	1 0 1 1 0	0 1 1	0.5
	0 0 1 0 0	0 1 1 1 1	1 0 0 0 1	1 1 0 1 0	1 0 0	0.5
	0 0 0 1 0	0 1 0 0 1	1 0 1 1 1	1 1 1 0 0	0 1 0	0.
	0 0 0 0 1	0 1 0 1 0	1 0 1 0 0	1 1 1 1 1	0 0 1	0.
OR	1 1 0 0 0	1 0 0 1 1	0 1 1 0 1	0 0 1 1 1	1 1 0	0
	0 0 1 1 0	0 1 1 0 1	1 0 0 1 1	1 1 0 0 0		
OR	1 0 0 1 0	1 1 0 0 1	0 0 1 1 1	0 1 1 0 0	1 1 1	0
	0 1 1 0 0	0 0 1 1 1	1 1 0 0 1	1 0 0 1 0		

where to calculate the syndromes we have used,

$$s = c_r H^T$$

$$c_r = c + e \qquad \text{where } c \text{ is a valid codeword and } e \text{ is an error pattern.}$$

so,
$$s = (c + e) H^T$$
$$= c H^T + e H^T$$
$$= e H^T \qquad \text{since by defn } c H^T = 0.$$

(2iii)

So we can see that the single bit error patterns will pick out single rows of H^T, i.e., columns of H.

For the remaining 2 syndromes we can choose from the following error patterns since they give syndromes different to those corresponding with the single bit error patterns, i.e.,

	double error pattern	syndrome
or	1 1 0 0 0	1 1 0
	0 0 1 1 0	1 1 0
OR	1 0 0 1 0	1 1 1
	0 1 1 0 0	1 1 1

See each syndrome has 2 possible error patterns, either of which we could enter into the SA. 1

[6]

(iv) Syndromes uniquely associated with a single bit error pattern can be used to identify particular error patterns, so the decoder calculates $s = c_r H^T$ to calc the syndrome. This identifies the error pattern (single bit) that can be used to 'correct' the appropriate bit. 1

Syndromes corresponding with the two assigned double bit error patterns do not uniquely identify these patterns. Hence correction cannot be performed. 1

So could,

- Declare that an invalid codeword has been received – knows as incomplete decoding.

(2iv)

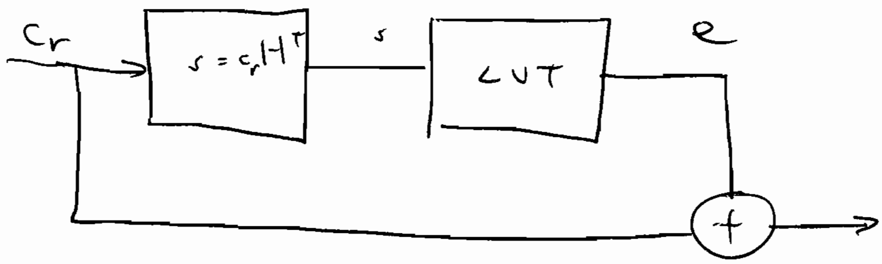
- ~~Have~~ Select one of the possible two error patterns corresponding with the double bit syndrome and use this to perform the correction.

- Instead of declaring an incomplete decoding, just output the data bits (easy to find since the code is systematic) and output these. 2

~~(v) Code C can detect $d_{min} - 1 = 3 - 1 = 2$ errors.~~

~~To use the syndrome decoder as an error detector, just ~~declare~~ note all non-zero syndromes ~~to~~ identify an error and do not attempt correction.~~

Simplified block diagram



(4)

3 (a)

$$s(t) = \mathrm{Re}\left(p(t)\, e^{j\omega_c t}\right)$$

$$= \frac{1}{2}\left(p(t)e^{j\omega_c t} + p^*(t)\, e^{-j\omega_c t}\right)$$

where p^* is complex conjugate of p.

Take Fourier transforms

$$S(\omega) = \frac{1}{2}\left[FT\left(p(t)e^{j\omega_c t}\right) + FT\left(p^*(t)\, e^{j\omega_c t}\right)\right]$$

Now from Data Book: $\quad p(t)\, e^{j\omega_c t} \Leftrightarrow P(\omega - \omega_c)$

if $p(t) \Leftrightarrow P(\omega)$:

$$\cancel{p^*(t) \Leftrightarrow P^*(-\omega)}$$

$$\cancel{p^*(t)e^{-j\omega_c t}}$$

$$\cancel{= \left[p(t)\, e^{j\omega_c t}\right]^* \Leftrightarrow P^*(}$$

$$p^*(t)\, e^{-j\omega_c t} \Leftrightarrow \int_{-\infty}^{\infty} p^*(t)e^{-j\omega_c t}\, e^{-j\omega t}\, dt$$

$$= \int p^*(t)\, e^{-j(\omega+\omega_c)t}\, dt$$

$$= \left[\int p(t)\, e^{j(\omega+\omega_c)t}\, dt\right]^* = P^*\left(-(\omega+\omega_c)\right)$$

$$\therefore\ S(\omega) = \frac{1}{2}\left[P(\omega-\omega_c) + P^*\left(-(\omega+\omega_c)\right)\right]$$

[4]

3(b)(i) $p_k(t) = [S_{2k} + j\, S_{2k+1}]\, g(t - kT_s)\, e^{j\phi_0}$

Since the even and odd symbol streams are uncorrelated we may consider the S_{2k} & S_{2k+1} components of $p_k(t)$ independently.

Let $p_e(t) = \sum_k S_{2k}\, g(t - kT_s)\, e^{j\phi_0}$

& $p_o(t) = \sum_k S_{2k+1}\, g(t - kT_s) .\, j\, e^{j\phi_0}$

~~ACF of~~ We observe that $p_e(t)$ is a constant phase $e^{j\phi_0}$ multiplied by a ~~pol~~ 4-level data stream S_{2k} in which the data impulses have been filtered or convolved with a filter whose impulse response is $g(t)$. 1

The discrete ACF of the even random symbol stream S_{2k} is: $R_{SS,e}(L) = E\{S_{2k}\, S_{2(k-L)}\}$ 1

Now $R_{SS,e}(0) = \cancel{\tfrac{1}{16}((-3)(-3) + (-1)(-3) + (+1)(-3) + (+3)(-3))}$

$\frac{1}{4}((-3).(-3) + (-1).(-1) + (+1).(+1) + (+3).(+3))$

$= \frac{20}{4} = 5$ 0.5

& $R_{SS,e}(|L| \geq 1) = \frac{1}{16}[(-3).(-3) + (-3).(-1) + (-3).(+1) + (-3).(+3) + (-1).(-3) + \ldots (+3).(-3) + \ldots (+3).(+3)]$

$= 0$ since symbols are uncorrelated. 0.5

(i)
3(b)(cont)

Hence the ACF of the stream of data impulses is:

$$C_{ss,e}(\tau) = \frac{1}{T_s} \sum_{L} R_{ss,e}(L)\, \delta(\tau - LT_s)$$

$$= \frac{5}{T_s}\, \delta(\tau) \qquad 1$$

~~Similarly~~ Similarly $R_{ss,o}(L) = E\{S_{2k+1}\, S_{2(k-L)+1}\}$

$= 5$ when $L = 0$

$= 0$ when $|L| \geq 1$

& $C_{ss,o}(\tau) = \frac{5}{T_s}\, \delta(\tau)$ also. 1 $S_e(\omega)$

Returning to the even symbols, the power spectrum of the stream of even symbol impulses is the Fourier Transform of the ACF:

$$\therefore S_e(\omega) = \int C_{ss,e}(\tau)\, e^{-j\omega\tau}\, d\tau = \frac{5}{T_s} \cdot e^{-j\omega 0} \cdot 1 = \frac{5}{T_s} \qquad 1$$

Since the data symbols are convolved with $g(t)$ & scaled by $e^{j\phi_0}$ in the expression for $p_e(t)$ above, the power spectrum of $p_e(t)$ is: $|e^{j\phi_0}|^2$

$$|P_e(\omega)|^2 = S_e(\omega) \cdot |G(\omega)|^2 \cdot \;/ = \frac{5}{T_s} \cdot |G(\omega)|^2 \cdot 1 \qquad 1$$

similarly $|P_o(\omega)|^2 = S_o(\omega) \cdot |G(\omega)|^2 \cdot |j\, e^{j\phi_0}|^2 = \frac{5}{T_s} \cdot |G(\omega)|^2 \cdot 1$

$\therefore$ The overall power spectrum of $p(t)$ is:

$$|P(\omega)|^2 = |P_e(\omega)|^2 + |P_o(\omega)|^2 = \underline{\underline{\frac{10}{T_s}\, |G(\omega)|^2}}$$

3. (b) (1) (cont) If $g(t)$ is rectangular, & of unit ampl. width duration T_s, from the data book we get:

$$G(\omega) = 1 . T_s \operatorname{sinc}\left(\frac{\omega T_s}{2}\right)$$

$$\therefore |G(\omega)|^2 = T_s^2 \operatorname{sinc}^2\left(\frac{\omega T_s}{2}\right)$$

$$\therefore |P(\omega)|^2 = \underline{\underline{10 T_s \operatorname{sinc}^2\left(\frac{\omega T_s}{2}\right)}}$$ 1

[8]

3. b (ii) Using the results from parts (a) & (b)(i), the power spectrum of the modulated signal is:

$$|S(\omega)|^2 = \left| \frac{1}{2}\left(P(\omega - \omega_c) + P^*(-(\omega+\omega_c))\right) \right|^2$$

$$= \frac{1}{4}\left|P(\omega-\omega_c)\right|^2 + \frac{1}{4}\left|P^*(-(\omega+\omega_c))\right|^2$$

$$+ \left(\text{terms like } P(\omega-\omega_c).P^*(-(\omega+\omega_c))\right) \qquad 1$$

If $\omega_c \gg \frac{2\pi}{T_s}$, ω_c will be much larger than the width of all the sidelobes of $P(\omega)$, so $P(\omega-\omega_c)$, which is centred on $\omega=\omega_c$, will not overlap in energy with $P(-(\omega+\omega_c))$, which is centred on $\omega = -\omega_c$. Hence all the cross-product terms like $P(\omega-\omega_c).P^*(-(\omega+\omega_c))$ will be zero. 1

$$\therefore |S(\omega)|^2 = \frac{1}{4}\left[\frac{10}{T_s}\left|G(\omega-\omega_c)\right|^2 + \frac{10}{T_s}\left|G^*(-(\omega+\omega_c))\right|^2\right]$$

$$= \frac{5}{2}T_s\left[\operatorname{sinc}^2\left(\frac{(\omega-\omega_c)T_s}{2}\right) + \operatorname{sinc}^2\left(\frac{(\omega+\omega_c)T_s}{2}\right)\right] \qquad 1$$

[3]

3(c) In 16-QAM, $T_s = 4T_b$ where T_b is the bit period for the composite bit stream, since 4 bits are conveyed ~~of~~ per symbol period T_s (two bits on S_{2k} & two on S_{2k+1}).

With QPSK, ~~only 2~~ $T_s = 2T_b$ & with 64-QAM, $T_s = 6T_b$. Hence 16-QAM needs only half the bandwidth of QPSK, while it needs ~~2~~ $\frac{6}{4} = \frac{3}{2}$ times the bandwidth of 64-QAM. 1

These factors are due to the presence of ~~the~~ T_s ~~term~~ term in the power spectrum, $\operatorname{sinc}^2\left(\frac{\omega T_s}{2}\right)$; so the larger T_s is, the narrower the sinc^2 function ~~f~~ becomes w.r.t. ω. 1

To compensate for this trend, we find that QPSK needs about 4dB less signal power for a given bit error rate, compared to 16-QAM, and that 64-QAM needs about 4dB more signal power for a given bit error rate compared to 64-QAM. 1

This is because the QPSK constellation points are more widely spaced, & the 64-QAM points are less widely spaced than those for 16-QAM. 1

[5]

4. (a) E_b means "Energy per bit" at any given point in the communication path. It is equal to the mean signal power multiplied by the bit period. 1

~~(b)~~ N_0 means "Noise power spectral density" at any given point in the receiver. It is equal to the noise power in a unit bandwidth filter. 1

E_b/N_0 is a dimensionless quantity which represents the signal/noise power ratio in dimensionless units, & allows easy comparison between different systems for demodulating digital signals, independent of the bit rate or bandwidth of the signals. 1

~~(b)~~

[3]

4(b)(i) For QPSK, when $u = \frac{E_b}{N_0}$

$$P_{QPSK} \simeq 2\,Q\left(\sqrt{2u}\right) \qquad \because \; x = \sqrt{2u}$$

$$\simeq \frac{2e^{-2u/2}}{1{\cdot}64\sqrt{2u} + \sqrt{1{\cdot}52u+4}} = \frac{2e^{-u}}{1{\cdot}64\sqrt{2u} + \sqrt{1{\cdot}52u+4}}$$ 1

$$P_{MFSK} \simeq \frac{1}{4}\left(2e^{-u/2}\right)^m$$

$$= \frac{2^m}{4}\,.\,e^{-mu/2}$$ 1

Considering the exponential terms first, as these will tend to dominate, the exponential terms will be the same when $m = 2$, and the MFSK term will decay faster with u when $m \geqslant 3$.

If $m = 2$, $P_{MFSK} = \frac{2^2}{4} e^{-2u/2} = e^{-u}$

Hence $\frac{P_{MFSK}}{P_{QPSK}} = \frac{1}{2}\left(1{\cdot}64\sqrt{2u} + \sqrt{1{\cdot}52u+4}\right) \geqslant 2$ for all $u \geqslant 0$.

Hence $P_{MFSK} \geqslant 2\,P_{QPSK}$ for $u \geqslant 0$ which is not acceptable. 1

If $m=3$, $P_{MFSK} = \frac{2^3}{4} e^{-3u/2} = 2\,.\,e^{-3u/2}$

Hence $\frac{P_{MFSK}}{P_{QPSK}} = e^{-u/2}\left(1{\cdot}64\sqrt{2u} + \sqrt{1{\cdot}52u+4}\right)$ 1

4 (b)(i) (cont)

For relatively large u, the $e^{-u/2}$ term will decay much faster than the terms in $\sqrt{u}$ increase, so $m = 3$ is the minimum m to make 1

$P_{MFSK} < P_{QPSK}$ at large u. [5]

(ii) For QPSK to give $P_{QPSK} = 10^{-6}$,

$$e^{-u} = \tfrac{1}{2} \cdot 10^{-6} \cdot \left(1.64\sqrt{u} + \sqrt{1.52u + 4}\right)$$

$$\geq 10^{-6}$$

$$\therefore \quad u \leq -\ln(10^{-6}) = 13.82$$ 1

Substitute this u into the RHS above gives:

$$e^{-u} \simeq 10^{-6}\,\tfrac{1}{2}\left(1.64 \cdot 5.\cancel{1225}2574 + \sqrt{\cancel{23.94}\,25.01}\right)$$

$$= 6.812 \cdot 10^{-6}$$

$$\therefore \quad u \simeq -\ln(6.812 \cdot 10^{-6}) = 11.90$$ 2

One more iteration gives:

$$e^{-u} \simeq 10^{-6}\,\tfrac{1}{2}\left(1.64 \cdot 4.\cancel{2392}8785 + \sqrt{22.09}\right)$$

$$= \cancel{12.363}\ 6.3504 \cdot 10^{-6}$$

$$\therefore \quad u = -\ln(6.3504 \cdot 10^{-6}) = 11.97$$

(Note this iteration is not really necessary in exam conditions.)

4 (b) (ii) (cont)

For MFSK with $M = 32$:

$m = 5$ since $M = 2^m$ 1

$\therefore$ To get $P_{MFSK} = 10^{-6}$

$$\left(2e^{-u/2}\right)^m = 4.10^{-6}$$

$$\therefore e^{-u/2} = \frac{1}{2}\left(4.10^{-6}\right)^{1/5} = 0.0416$$

$$\therefore u = -2\ln(0.0416) = 6.358 \quad 1$$

$$\therefore \frac{\text{Power of QPSK}}{\text{Power of MFSK}} = \frac{11.\cancel{xx}97}{6.358} = \cancel{xxxx}\ 1.8827$$

Converted to dB this becomes $\cancel{20}\ 10 \log_{10}(\cancel{xxx}\ 1.8827) = \underline{\underline{\cancel{2.50}\ 2.75}}$ dB 1

[6]

4 (c) ~~The~~ If the bit period is T_b seconds, the symbol rate for QPSK is $1/2T_b$ sym/s.

If rectangular symbols of duration $2T_b$ are used, then the power spectrum of QPSK is proportional to $\mathrm{sinc}^2\left(\frac{\omega . 2T_b}{2}\right)$ (from the data book ~~g~~ Fourier Transform page). Hence the first zeros ~~occ~~ occur when

$$\frac{\omega . 2T_b}{2} = \pm \pi$$

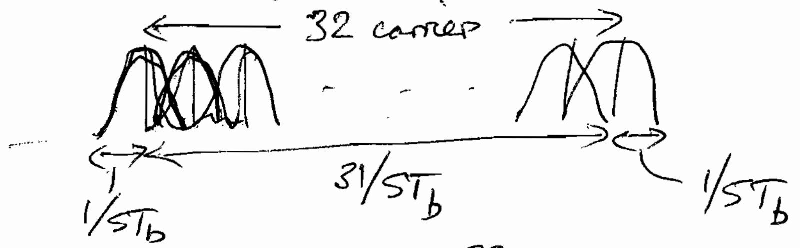
ie when $\omega = \pm \frac{\pi}{T_b}$ rad/s or $f = \pm \frac{1}{2T_b}$ Hz.

Therefore the bandwidth of QPSK between the first spectral zeros is $\underline{\underline{1/T_b}}$ Hz. 1

4(c) (cont)

For MFSK with $M=32$, $m=5$ & so the symbol period is $5T_b$ seconds. We can regard MFSK as M separate on/off amplitude shift keyed signals each with its own carrier frequency. To keep the 32 carriers orthogonal to each other (as in the DFT), over the symbol period, the spacing between them must be $\frac{1}{5T_b}$ Hz. Each carrier will have a power spectrum proportional to $\mathrm{sinc}^2\left(\frac{\omega . 5T_b}{2}\right)$, since they are being modulated on/off by rectangular pulses of duration $5T_b$, so the bandwidth required will be $\frac{1}{5T_b}$ Hz at each end of the spectrum due to the modulation rate, plus $\frac{32-1}{5T_b}$ Hz to account for the spacing between the first and last carrier frequencies.

Hence bandwidth of 32-FSK $= \frac{1}{5T_b} + \frac{31}{5T_b} + \frac{1}{5T_b} = \underline{\underline{\frac{33}{5T_b}}}$ Hz



Therefore 32 FSK requires $\frac{33}{5} = 6.6$ times as much bandwidth as QPSK for a given data rate, but it does require 2.75 dB less signal power to achieve 10^{-6} bit error rate. It also allows non-coherent detection (but this is only mentioned very briefly in the lecture course).

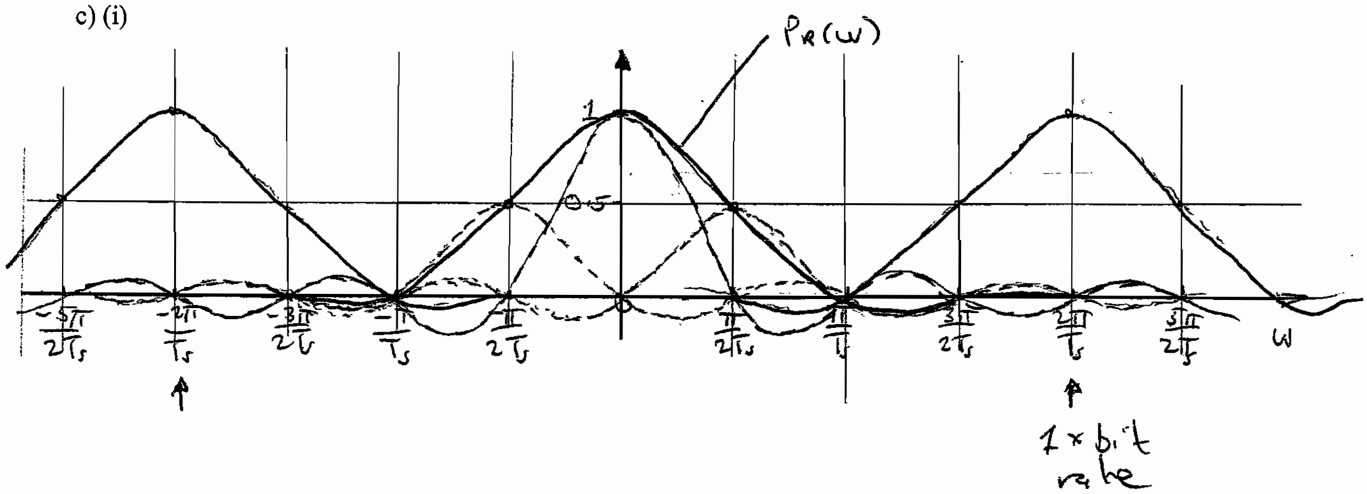
[6]

Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2009 - Answers

1.
a) See notes.

b) $\sum_{k=-\infty}^{\infty} P_R\left(f - \frac{k}{T_s}\right) = \text{constant}$

c) (i)



(ii) $\frac{1}{2}\left(\frac{\pi^2}{T_s\omega(\pi^2 - 4T_s^2\omega^2)}\right)\sin 2T_s\omega$

d) $\frac{1}{4T_s}$

2.
a) See notes.

b) (i) $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

(ii) Detect 2 errors. Correct 1 error.
(iii) See notes.
(iv) See notes.

3.
a) $S(\omega) = \frac{1}{2}\left(P(\omega - \omega_c) + P^*(-(\omega + \omega_c))\right)$

b) (i) $|P(\omega)|^2 = 10T_s \operatorname{sinc}^2\left(\frac{\omega T_s}{2}\right)$

(ii) $|S(\omega)|^2 = \frac{5}{2}T_s\left[\operatorname{sinc}^2\left(\frac{(\omega - \omega_c)T_s}{2}\right) + \operatorname{sinc}^2\left(\frac{(\omega + \omega_c)T_s}{2}\right)\right]$

c) See notes

4.
a) See notes.

b) (i) $m = 3$.

(ii) For QPSK $u = 3$. For 32 MFSK $u = 6.358$.

$$\frac{\text{Power of QPSK}}{\text{Power of MFSK}} = \frac{11.97}{6.358} = 1.883 \approx 2.75\text{dB}$$

c) QPSK bandwidth = $\dfrac{1}{T_b}$ Hz. MFSK bandwidth = $\dfrac{33}{5T_b}$ Hz.