

ENGINEERING TRIPOS PART IIA

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Thursday 30 April 2009 9.00 – 10.30

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Module 3A5

THERMODYNAMICS AND POWER GENERATION

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of the question is indicated in the right margin.*

*There are no attachments.*

STATIONERY REQUIREMENTS

Single-sided script paper.

SPECIAL REQUIREMENTS

Engineering Data Book.

CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) With reference to Fig. 1, a steady-flow of gas passes through a device in a power plant, entering at state 1 and leaving at state 2. The only external heat exchange is with the environment at temperature  $T_0$ . Neglecting changes in kinetic and potential energy between inlet and outlet, derive the 'exergy equation',

$$\dot{m}(e_1 - e_2) = \dot{W}_X + T_0 \Delta \dot{S}_{irrev} + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q}_0$$

where  $\dot{m}$  is the mass flow rate,  $e$  is the specific steady-flow exergy,  $\dot{W}_X$  is the shaft power output,  $\Delta \dot{S}_{irrev}$  is the rate of entropy creation within the device, and the infinitesimal heat transfer rate  $d\dot{Q}_0$  between the device at local temperature  $T$  and the environment is positive in the direction shown. The integral implies summation over the surface of the device. [20 %]

(b) In a closed cycle gas turbine, helium (assumed to be a perfect gas) flows in turn through an adiabatic compressor, a heater, an uncooled adiabatic turbine and a cooler, before re-entering the compressor. In the heater, the helium receives heat from an external source and in the cooler it rejects heat to the environment at a temperature of 290 K. The mass flowrate of helium is  $0.25 \text{ kg s}^{-1}$ . Table 1 gives the values of  $p$ ,  $T$ ,  $h$  and  $s$  around the cycle and these values may be used in the calculations without further validation.

- (i) For the compressor and turbine calculate the value of each term on the right hand side of the exergy equation derived in part (a). [20 %]
- (ii) For the cooler calculate the value of each term on the right hand side of the exergy equation. Derive any equations required and state, giving reasons, whether your analysis would still hold if the gas were (a) semi-perfect or (b) imperfect. [30 %]
- (iii) Demonstrate that the sum of all the power and 'lost power' terms is equal to the rate of exergy supplied to the heater. [15 %]
- (iv) Calculate the thermal and rational efficiencies of the cycle. Calculate also the maximum possible thermal efficiency for a cycle having the same heater inlet and outlet conditions. [15 %]

(cont.)

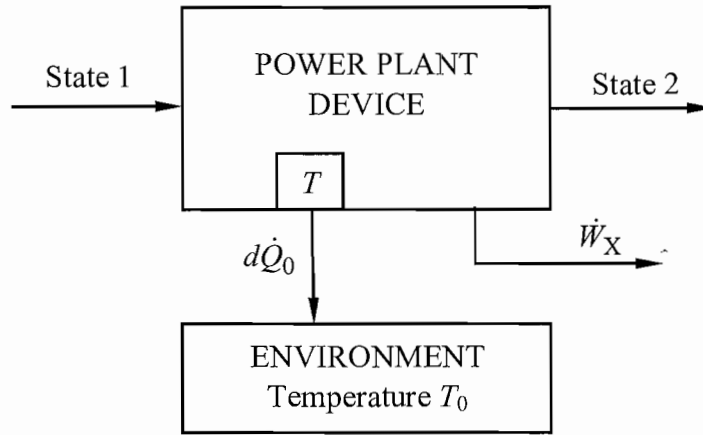


Fig. 1

		$p$ (bar)	$T$ (K)	$h$ (kJ kg <sup>-1</sup> )	$s$ (kJ kg <sup>-1</sup> K <sup>-1</sup> )
State 1	Compressor inlet	1.00	290.0	0.0	0.0
State 2	Heater inlet	12.00	880.0	3062.1	0.5925
State 3	Turbine inlet	12.00	1750.0	7577.4	4.1604
State 4	Cooler inlet	1.05	740.0	2335.5	4.7603

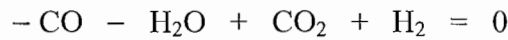
Table 1 Property values for Question 1.

(TURN OVER)

2 Note that parts (a) and (b) of this question are unrelated to each other.

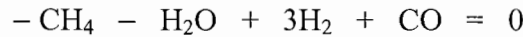
(a) A chemical reformer operating in steady-flow is supplied with methane and steam in the ratio 1 mole of methane to 4 moles of steam. The 'syngas' exiting from the reformer is then raised to a temperature of 1200 K and a pressure of 2 bar. It may be assumed that each component of the syngas behaves as an ideal gas.

- (i) Assuming that the only species present in the syngas are  $\text{H}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{CO}$  and  $\text{CO}_2$  and that they are in equilibrium subject to the reaction,



confirm that the mole fraction of  $\text{H}_2$  is 0.4807 and find the mole fractions of the other three species. [50 %]

- (ii) The methane reforming reaction is,



At 1200 K, the equilibrium constant defined for the reaction as written above is  $K_p = 29.48$ . Estimate the mole fraction of methane in the syngas given that its value is likely to be very low. [20 %]

(b) In a model for the liquid-vapour equilibrium of a pure substance at temperatures well below the critical temperature, it is assumed that the vapour phase behaves as a perfect gas with specific gas constant  $R$  and constant isobaric specific heat capacity  $c_{pg}$ . For the liquid phase it is assumed that the specific enthalpy is independent of pressure, the isobaric specific heat capacity  $c_{pf}$  is constant, and the specific volume is negligible compared to that of the vapour. At temperature  $T = T_0$ , measurements are available of the saturated vapour pressure  $p_s = p_{s0}$  and the specific enthalpy of evaporation  $h_{fg} = h_{fg0}$ . The Clausius-Clapeyron equation (which may be used without proof) is

$$\frac{dp_s}{dT} = \frac{h_{fg}}{v_{fg}T}$$

Show that the saturated vapour pressure as a function of temperature is given by,

$$\ln\left(\frac{p_s}{p_{s0}}\right) = \frac{h_{fg0}}{RT_0}\left(1 - \frac{T_0}{T}\right) + \frac{(c_{pf} - c_{pg})}{R}\left[1 - \frac{T_0}{T} + \ln\left(\frac{T_0}{T}\right)\right] \quad [30 \%$$

3 (a) An open circuit industrial gas turbine operates with a compressor pressure ratio of 20. The pressure and temperature of the air entering the compressor are 1 bar and 25 °C respectively. The compressor is adiabatic and has a *polytropic* efficiency of 0.88. The air leaving the compressor is supplied to an adiabatic combustion chamber to which gaseous fuel is added at 25 °C. The fuel is burned completely with an air-fuel mass ratio of 46.2. The turbine is uncooled and adiabatic with an *isentropic* efficiency of 0.90. The turbine exhaust pressure is 1.22 bar.

Pressure losses in the combustor can be neglected. Assume that air behaves as a perfect gas with  $c_p = 1.005 \text{ kJ kg}^{-1} \text{ K}^{-1}$  and  $\gamma = 1.40$ , and that the products of combustion can also be treated as a perfect gas with  $c_p = 1.100 \text{ kJ kg}^{-1} \text{ K}^{-1}$  and  $\gamma = 1.35$ . For the combustion of the fuel with air,  $\Delta H_{298}^0 = -50.0 \text{ MJ kg}^{-1}$ .

- (i) Show that the temperature of the products of combustion at entry to the turbine is 1700 K. [30 %]
- (ii) Calculate the net work output from the gas turbine per kg of air entering the compressor. [25 %]
- (iii) Determine the overall efficiency of the plant. [5 %]

(b) To improve the performance of the above gas turbine, a recuperator is added to transfer heat from the turbine exhaust gas to the air leaving the compressor. If the heat exchanger effectiveness for the recuperator is 0.85, calculate the new air-fuel mass ratio and determine the overall efficiency of the recuperated power plant. Assume that the inlet and exit conditions for both the turbine and compressor are unchanged from part (a). The pressure losses within the recuperator can be neglected. [30 %]

(c) Briefly explain how the addition of an intercooler affects the overall efficiency and power output of a recuperated gas turbine. [10 %]

(TURN OVER)

4 Where necessary, use the properties of steam given in Table 2 on the following page.

(a) Sketch a graph of temperature versus heat transferred for the heat recovery steam generator (HRSG) of a combined-cycle gas turbine (CCGT) power plant that has a single-pressure steam cycle. With reference to this, explain how the HRSG boiler efficiency, the steam cycle efficiency and the CCGT overall efficiency are affected by the choice of pressure level for the steam. Assume that the turbine exit temperature of the gas turbine, the steam turbine inlet temperature and the pinch-point temperature difference are all fixed. [25 %]

(b) In a CCGT power plant, the exhaust from the gas turbine enters the HRSG at 540 °C. This exhaust gas may be treated as perfect with  $c_p = 1.10 \text{ kJ kg}^{-1} \text{ K}^{-1}$ . The steam cycle has a temperature of 500 °C at entry to the steam turbine and a condenser pressure of 0.04 bar. Wet saturated water from the condenser is compressed by a feed pump to the steam pressure level before entering the HRSG. Only one pressure level is used and there is no reheat. The HRSG pressure losses and the feed pump work can be neglected. The pinch-point temperature difference in the HRSG is 15 °C and it may be assumed that the environment is at the same temperature as the condenser.

For a steam pressure level of 40 bar calculate the following:

- (i) the ratio of the mass flow rate of steam to the mass flow rate of gas; [15 %]
- (ii) the temperature of the gas leaving the HRSG; [10 %]
- (iii) the maximum possible work output from the steam cycle per unit mass of gas flowing through the HRSG; [15 %]
- (iv) the maximum possible steam cycle efficiency. [10 %]

(c) With a new steam pressure level of 150 bar, the ratio of the mass flow rate of steam to the mass flow rate of gas is 0.118.

- (i) Calculate the new maximum possible steam cycle efficiency. [10 %]
- (ii) State, giving reasons, which steam pressure level (40 bar or 150 bar) will give the highest overall efficiency for the CCGT power plant. [15 %]

(cont.)

	Specific enthalpy $\text{kJ kg}^{-1}$	Specific entropy $\text{kJ kg}^{-1} \text{K}^{-1}$	Temperature $^{\circ}\text{C}$
Saturated Liquid at 0.04 bar	121.4	0.422	28.96
Saturated Liquid at 40 bar	1087.5	2.797	250.35
40 bar, 500 $^{\circ}\text{C}$	3446.0	7.092	
150 bar, 500 $^{\circ}\text{C}$	3310.8	6.348	

Table 2 Properties of water and steam for Question 4.

**END OF PAPER**

**ENGINEERING TRIPOS PART IIA 2009**

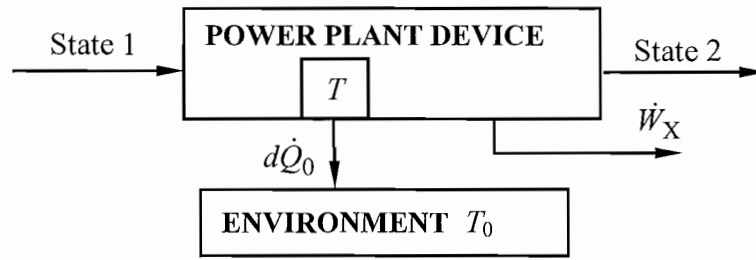
**MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION**

**SOLUTIONS TO TRIPOS QUESTIONS**

**J.B. YOUNG & C.A. HALL**



1. (a)



Neglecting changes in KE and PE between inlet and outlet, the SFEE and SFSE are :

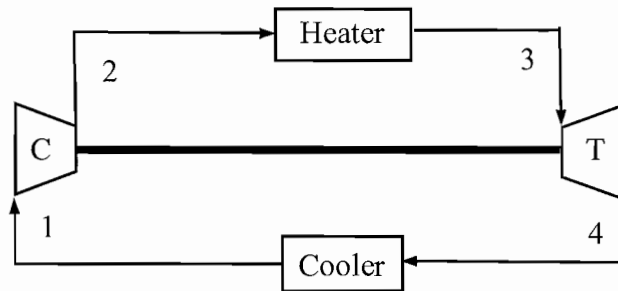
$$\dot{m}(h_2 - h_1) = -\dot{Q}_0 - \dot{W}_X = \int -d\dot{Q}_0 - \dot{W}_X$$

$$\dot{m}(s_2 - s_1) = \int -\frac{d\dot{Q}_0}{T} + \Delta\dot{S}_{irrev}$$

Multiplying the SFSE by  $T_0$  and subtracting the SFEE gives,

$$\dot{m}[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)] = \dot{m}(e_1 - e_2) = \dot{W}_X + T_0 \Delta\dot{S}_{irrev} + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q}_0 \quad [20 \%]$$

(b)



(i) For the compressor (taking property values from Table 1 in the Tripos paper) :

$$[\dot{W}_X]_{comp} = \dot{m}(h_1 - h_2) = 0.25 \times (0.0 - 3062.1) = -765.53 \text{ kW}$$

$$[\dot{W}_{L,Q}]_{comp} = 0 \quad (\text{Compressor is adiabatic})$$

$$[\dot{W}_{L,CR}]_{comp} = \dot{m}T_0(s_2 - s_1) = 0.25 \times 290.0 \times (0.5925 - 0.0) = 42.96 \text{ kW} \quad [10 \%]$$

For the turbine :

$$[\dot{W}_X]_{turb} = \dot{m}(h_3 - h_4) = 0.25 \times (7577.4 - 2335.5) = 1310.48 \text{ kW}$$

$$[\dot{W}_{L,Q}]_{turb} = 0 \quad (\text{Turbine is adiabatic})$$

$$[\dot{W}_{L,CR}]_{turb} = \dot{m}T_0(s_4 - s_3) = 0.25 \times 290.0 \times (4.7603 - 4.1604) = 43.49 \text{ kW} \quad [10 \%]$$

(ii) For the cooler, the SFSE can be written (using the SFEE and  $Tds = dh - vdp$ ),

$$\dot{m}(s_1 - s_4) = \int_4^1 -\frac{d\dot{Q}_0}{T} + \Delta\dot{S}_{irrev} = \int_4^1 \frac{\dot{m}dh}{T} + \Delta\dot{S}_{irrev} = \int_4^1 \dot{m} \left( ds + \frac{v}{T} dp \right) + \Delta\dot{S}_{irrev}$$

For a perfect or semi-perfect (but not imperfect) gas  $pv = RT$  and hence,

$$\Delta\dot{S}_{irrev} = -\int_4^1 \dot{m} \frac{v}{T} dp = -\int_4^1 \dot{m} R \frac{dp}{p} = \dot{m} R \ln \left( \frac{p_4}{p_1} \right)$$

For helium  $R = 2.08$  kJ/kg K. Hence,

$$[\dot{W}_{L,CR}]_{cool} = T_0 \Delta\dot{S}_{irrev} = 0.25 \times 2.08 \times 290.0 \times \ln(1.05) = 7.36 \text{ kW}$$

$$[\dot{W}_X]_{cool} = 0 \quad (\text{No shaft power})$$

$$\begin{aligned} [\dot{W}_{L,Q}]_{cool} &= \dot{m}(e_4 - e_1) - T_0 \Delta\dot{S}_{irrev} \\ &= 0.25 \times [(2335.5 - 290 \times 4.7603) - (0)] - 7.36 = 231.39 \text{ kW} \end{aligned} \quad [30 \%]$$

$$\begin{aligned} \text{(iii) Power plus lost power} &= [\dot{W}_X]_{turb} + [\dot{W}_X]_{comp} + \\ &\quad [\dot{W}_{L,CR}]_{turb} + [\dot{W}_{L,CR}]_{comp} + [\dot{W}_{L,CR}]_{cool} + [\dot{W}_{L,Q}]_{cool} \\ &= 1310.48 - 765.53 + 43.49 + 42.96 + 7.36 + 231.39 = 870.15 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Exergy supply rate to heater} &= \dot{m}(e_3 - e_2) \\ &= 0.25 \times [(7577.4 - 290 \times 4.1604) - (3062.1 - 290 \times 0.5925)] = 870.15 \text{ kW} \end{aligned} \quad [15 \%]$$

$$\text{(iv) Rate of heat supply} = \dot{m}(h_3 - h_2) = 0.25 \times (7577.4 - 3062.1) = 1128.83 \text{ kW}$$

$$\text{Thermal efficiency} = \frac{[\dot{W}_X]_{turb} + [\dot{W}_X]_{comp}}{\dot{m}(h_3 - h_2)} = \frac{1310.48 - 765.53}{1128.83} = 0.483 \quad (48.3 \%)$$

$$\text{Rational efficiency} = \frac{[\dot{W}_X]_{turb} + [\dot{W}_X]_{comp}}{\dot{m}(e_3 - e_2)} = \frac{1310.48 - 765.53}{870.15} = 0.626 \quad (62.6 \%)$$

$$\text{Maximum thermal efficiency} = \frac{\dot{m}(e_3 - e_2)}{\dot{m}(h_3 - h_2)} = \frac{870.15}{1128.83} = 0.771 \quad (77.1 \%) \quad [15 \%]$$

2. (a) (i) If the syngas contains only H<sub>2</sub>, H<sub>2</sub>O, CO and CO<sub>2</sub>, the chemical reaction is,



$$\text{Conservation of atomic H} \quad 2a + 2b = 12$$

$$\text{Conservation of atomic C} \quad c + d = 1$$

$$\text{Conservation of atomic O} \quad b + c + 2d = 4$$

The equilibrium equation for Databook Reaction 8,  $-\text{CO} - \text{H}_2\text{O} + \text{CO}_2 + \text{H}_2 = 0$ , is,

$$\left(\frac{P_{\text{CO}}}{P_0}\right)^{-1} \left(\frac{P_{\text{H}_2\text{O}}}{P_0}\right)^{-1} \left(\frac{P_{\text{CO}_2}}{P_0}\right)^{+1} \left(\frac{P_{\text{H}_2}}{P_0}\right)^{+1} = K_{p8}$$

where  $p_0$  is standard pressure (1 bar). If  $n = a + b + c + d$  is the total number of moles and the mixture pressure is  $p$ , the equation for  $K_{p8}$  becomes,

$$\left(\frac{c p}{n p_0}\right)^{-1} \left(\frac{b p}{n p_0}\right)^{-1} \left(\frac{d p}{n p_0}\right)^{+1} \left(\frac{a p}{n p_0}\right)^{+1} = \frac{ad}{bc} = K_{p8}$$

From the Databook for Reaction 8 at 1200 K,  $\ln(K_{p8}) = -0.311$ . Hence,  $K_{p8} = 0.7327$

From the atom balances,  $b = 6 - a$ ;  $b + d = 3$  so  $d = a - 3$ ;  $c = 1 - d = 4 - a$

Hence,  $n = a + b + c + d = 7$

Assuming  $X_{\text{H}_2} = 0.4807$  as given in the question, we find  $a = 0.4807 \times 7 = 3.365$

(Note that the mole fraction  $X_{\text{H}_2}$  is not the same as the number of moles  $a$  in the calculation.)

Then :  $b = 2.635$ ,  $c = 0.635$ ,  $d = 0.365$

Thus :  $K_{p8} = \frac{ad}{bc} = \frac{3.365 \times 0.365}{2.635 \times 0.635} = 0.7340$  (which agrees with the Data Book value)

The mole fractions are therefore,

$$X_{\text{H}_2} = \frac{a}{n} = 0.481, \quad X_{\text{H}_2\text{O}} = \frac{b}{n} = 0.376, \quad X_{\text{CO}} = \frac{c}{n} = 0.091, \quad X_{\text{CO}_2} = \frac{d}{n} = 0.052 \quad [50 \%]$$

(a) (ii) The equilibrium constant for the reforming reaction is given by,

$$\left(\frac{p_{\text{CH}_4}}{p_0}\right)^{-1} \left(\frac{p_{\text{H}_2\text{O}}}{p_0}\right)^{-1} \left(\frac{p_{\text{H}_2}}{p_0}\right)^{+3} \left(\frac{p_{\text{CO}}}{p_0}\right)^{+1} = K_{p(\text{ref})}$$

In terms of the mole fractions,

$$\frac{(X_{\text{H}_2})^3 (X_{\text{CO}})}{(X_{\text{CH}_4})(X_{\text{H}_2\text{O}})} \left(\frac{p}{p_0}\right)^2 = K_{p(\text{ref})}$$

Assuming the mole fraction of  $\text{CH}_4$  is so small that the other mole fractions are unaffected,

$$X_{\text{CH}_4} = \frac{(X_{\text{H}_2})^3 (X_{\text{CO}})}{K_{p(\text{ref})} (X_{\text{H}_2\text{O}})} \left(\frac{p}{p_0}\right)^2 = \frac{0.481^3 \times 0.091 \times 2^2}{29.48 \times 0.376} = 0.0037 \quad [20 \%]$$

(b) The vapour is a perfect gas and so its specific enthalpy is a function only of temperature. The liquid specific enthalpy is independent of pressure and so it also is a function only of temperature. The vapour and liquid isobaric specific heat capacities are constant and so the specific enthalpy of evaporation is given by,

$$\begin{aligned} h_{fg}(T) &= h_g(T) - h_f(T) = [h_g(T) - h_g(T_0)] + [h_g(T_0) - h_f(T_0)] + [h_f(T_0) - h_f(T)] \\ &= h_{fg0} + (c_{pg} - c_{pf})(T - T_0) \end{aligned}$$

Neglecting  $v_f$  compared to  $v_g$  and using  $p_s v_g = RT$ , the Clausius-Clapeyron equation is,

$$\frac{dp_s}{dT} = \frac{h_{fg} p_s}{RT^2}$$

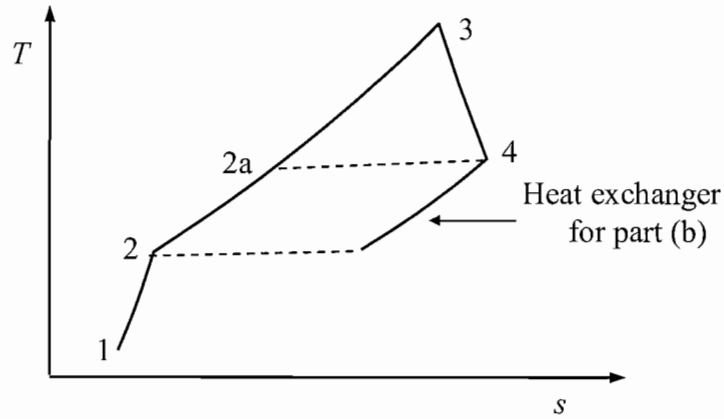
Substituting the expression for  $h_{fg}$  and integrating from  $T_0$  to  $T$  we have,

$$\int_{p_{s0}}^{p_s} \frac{dp'_s}{p'_s} = \int_{T_0}^T \frac{h_{fg0} + (c_{pg} - c_{pf})(T' - T_0)}{RT'^2} dT'$$

Hence,

$$\begin{aligned} \ln\left(\frac{p_s}{p_{s0}}\right) &= \frac{h_{fg0}}{R} \left(\frac{1}{T_0} - \frac{1}{T}\right) + \frac{(c_{pg} - c_{pf})}{R} \ln\left(\frac{T}{T_0}\right) - \frac{(c_{pg} - c_{pf})T_0}{R} \left(\frac{1}{T_0} - \frac{1}{T}\right) \\ &= \frac{h_{fg0}}{RT_0} \left(1 - \frac{T_0}{T}\right) + \frac{(c_{pf} - c_{pg})}{R} \left[1 - \frac{T_0}{T} + \ln\left(\frac{T_0}{T}\right)\right] \end{aligned} \quad [30 \%]$$

3.



(a) (i) For the compressor,

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{(\gamma_a - 1)/(\gamma_a \eta_c)} = 298.15 \times 20^{0.4/(1.4 \times 0.88)} = 788.58 \text{ K}$$

SFEE for the adiabatic combustor ( $A$  = air-fuel mass ratio) :

$$(A+1)h_p(T_3) - Ah_a(T_2) - h_f(T_0) = 0$$

From the definition of  $\Delta H_{298}^0$  (note that the excess air makes no difference) :

$$(A+1)h_p(T_0) - Ah_a(T_0) - h_f(T_0) = \Delta H_{298}^0$$

Subtracting the second equation from the first gives,

$$(A+1)[h_p(T_3) - h_p(T_0)] - A[h_a(T_2) - h_a(T_0)] = -\Delta H_{298}^0$$

$$(A+1)c_{pp}(T_3 - T_0) - Ac_{pa}(T_2 - T_0) = -\Delta H_{298}^0$$

$$T_3 = T_0 + \left[ \frac{-\Delta H_{298}^0 + Ac_{pa}(T_2 - T_0)}{(A+1)c_{pp}} \right]$$

$$= 298.15 + \left[ \frac{50 \times 10^3 + 46.2 \times 1.005 \times (788.58 - 298.15)}{47.2 \times 1.100} \right] = 1699.8 \text{ K} \quad [30 \%$$

(ii) For the turbine (taking  $T_3 = 1700 \text{ K}$ ) :

$$T_{4s} = T_3 \left( \frac{p_4}{p_3} \right)^{(\gamma_p - 1)/\gamma_p} = 1700 \times \left( \frac{1.22}{20} \right)^{0.35/1.35} = 823.3 \text{ K}$$

$$T_4 = T_3 - \eta_t (T_3 - T_{4s}) = 1700 - 0.90 \times (1700 - 823.3) = 910.97 \text{ K}$$

Net work output per kg of air entering compressor :

$$w_{net} = \left( \frac{A+1}{A} \right) c_{pp} (T_3 - T_4) - c_{pa} (T_2 - T_1)$$

$$= \left( \frac{47.2}{46.2} \right) \times 1.100 \times (1700 - 910.97) - 1.005 \times (788.58 - 298.15) = 393.84 \text{ kJ/kg air} \quad [25 \%]$$

(iii) Overall efficiency,

$$\eta_{ov} = \frac{w_{net}}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{393.84}{(1/46.2) \times 50 \times 10^3} = 0.364 \quad [5 \%]$$

(b) The heat exchanger effectiveness is defined by,  $\varepsilon_{HX} = \frac{T_{2a} - T_2}{T_4 - T_2}$ .

$$T_{2a} = T_2 + \varepsilon_{HX} (T_4 - T_2) = 788.58 + 0.85 \times (910.97 - 788.58) = 892.61 \text{ K}$$

From the SFEE for the combustor,

$$(A+1)c_{pp}(T_3 - T_0) - A c_{pa}(T_{2a} - T_0) = -\Delta H_{298}^0$$

$$A = \left[ \frac{-\Delta H_{298}^0 - c_{pp}(T_3 - T_0)}{c_{pp}(T_3 - T_0) - c_{pa}(T_{2a} - T_0)} \right]$$

$$= \left[ \frac{50 \times 10^3 - 1.100 \times (1700 - 298.15)}{1.100 \times (1700 - 298.15) - 1.005 \times (892.61 - 298.15)} \right] = 51.30$$

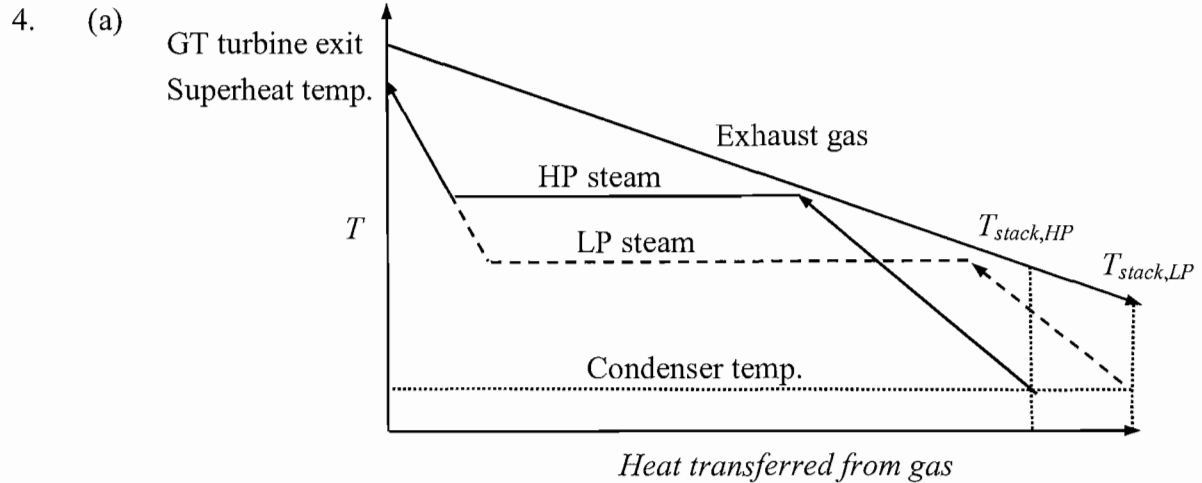
Net work output per kg of air entering compressor :

$$w_{net} = \left( \frac{52.3}{51.3} \right) \times 1.100 \times (1700 - 910.97) - 1.005 \times (788.58 - 298.15) = 391.97 \text{ kJ/kg air}$$

Overall efficiency,

$$\eta_{ov} = \frac{w_{net}}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{391.97}{(1/51.3) \times 50 \times 10^3} = 0.402 \quad [30 \%]$$

(c) Adding an intercooler to a recuperated GT reduces the work input to the compressor without changing the fuel flow rate or the turbine work output. Hence both the overall efficiency and the net work output both increase. [10 %]



As the steam pressure is increased, less heat is transferred from the gas turbine exhaust. The HRSG boiler efficiency, defined by,

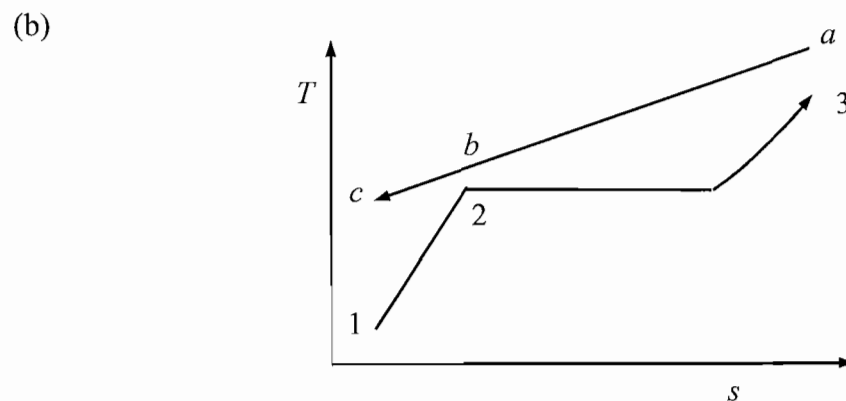
$$\eta_B = \frac{\text{Actual heat transferred}}{\text{Maximum heat transferred (when } T_{stack} = T_0)}$$

therefore decreases. However, the steam cycle efficiency  $\eta_{ST}$  increases because the mean temperature of heat reception for the steam cycle is higher. The CCGT overall efficiency is given by,

$$\eta_{CC} = \eta_{GT} + \eta_B \eta_{ST} (1 - \eta_{GT})$$

so it is not possible to say whether or not  $\eta_{CC}$  increases without further calculation.

[25 %]



(i) The SFEE applied to the HRSG down to the pinch point gives,

$$\dot{m}_s (h_3 - h_2) = \dot{m}_g c_{pg} (T_a - T_b)$$

$$\frac{\dot{m}_s}{\dot{m}_g} = \frac{c_{pg} (T_a - T_b)}{(h_3 - h_2)} = \frac{1.10 \times [540.0 - (250.35 + 15.0)]}{(3446.0 - 1087.5)} = 0.1281$$

[15 %]

(ii) The SFEE applied to the whole HRSG gives,

$$\dot{m}_s (h_3 - h_1) = \dot{m}_g c_{pg} (T_a - T_c)$$

$$T_c = T_a - \frac{\dot{m}_s (h_3 - h_1)}{\dot{m}_g c_{pg}} = 540.0 - \frac{0.1281 \times (3446.0 - 121.4)}{1.10} = 152.8 \text{ }^\circ\text{C} \quad [10 \text{ \%}]$$

(iii) Max. work from steam cycle per unit mass of gas ( $T_0 = 28.96 + 273.15 = 302.11 \text{ K}$ ) :

$$\begin{aligned} w_{st,max} &= \frac{\dot{m}_s}{\dot{m}_g} (e_3 - e_1) = \frac{\dot{m}_s}{\dot{m}_g} [(h_3 - T_0 s_3) - (h_1 - T_0 s_1)] \\ &= 0.1281 \times [(3446.0 - 302.11 \times 7.092) - (121.4 - 302.11 \times 0.422)] \\ &= 167.75 \text{ kJ/kg gas} \end{aligned} \quad [15 \text{ \%}]$$

(iv) Max. steam cycle efficiency =  $\frac{w_{st,max}}{(\dot{m}_s / \dot{m}_g)(h_3 - h_1)}$

$$= \frac{167.75}{0.1281 \times (3446.0 - 121.4)} = 0.394 \text{ (39.4 \%)} \quad [10 \text{ \%}]$$

(c) (i) For the 150 bar steam pressure level the maximum steam cycle work is,

$$\begin{aligned} w_{st,max} &= 0.1180 \times [(3310.8 - 302.11 \times 6.348) - (121.4 - 302.11 \times 0.422)] \\ &= 165.09 \text{ kJ/kg gas} \end{aligned}$$

$$\text{Max. steam cycle efficiency} = \frac{165.09}{0.1180 \times (3310.8 - 121.4)} = 0.439 \text{ (43.9 \%)} \quad [10 \text{ \%}]$$

(ii) The fuel input and the GT work output per unit mass of gas passing through the HRSG will be the same for both cycles. Hence, the cycle with the highest CCGT overall efficiency will be the one with the larger value of  $w_{st,max}$ . This is the cycle with the 40 bar steam pressure level (despite the fact that the cycle with the 150 bar pressure level has the higher steam cycle efficiency). [15 %]



**ENGINEERING TRIPOS PART IIA 2009**  
**MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION**

**ANSWERS**

1. (b) (i) Compressor :     $-765.5$  kW,     $43.0$  kW,     $0.0$   
          Turbine :         $1310.5$  kW,     $43.5$  kW,     $0.0$   
      (ii) Cooler :         $0.0$ ,         $7.4$  kW,     $231.4$  kW  
      (iii) Heater :  $870.2$  kW  
      (iv)  $48.3$  %,     $62.6$  %,     $77.1$  %
2. (a) (i)  $X_{H_2} = 0.481$ ,     $X_{H_2O} = 0.376$ ,     $X_{CO} = 0.091$ ,     $X_{CO_2} = 0.052$   
      (ii)  $X_{CH_4} = 0.0037$
3. (a) (ii)  $393.84$  kJ/kg air, (iii)  $0.364$   
      (b)  $51.3$ ,     $0.402$
4. (b) (i)  $0.1281$   
      (ii)  $152.8$  °C  
      (iii)  $167.8$  kJ/kg of gas  
      (iv)  $39.4$  %  
      (c)  $43.9$  %,    The 40 bar pressure level cycle.