ENGINEERING TRIPOS PART IIA

Thursday 7 May 2009 2:30 to 4:00

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: None

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 An infinitely long solid semi-cylindrical body of radius R_0 is perfectly insulated at mid-plane and its surface is kept at temperature T_0 as in Fig. 1. Heat is generated within the body at a constant rate of $\dot{Q}$ per unit volume. The thermal conductivity of the material, k, is constant.

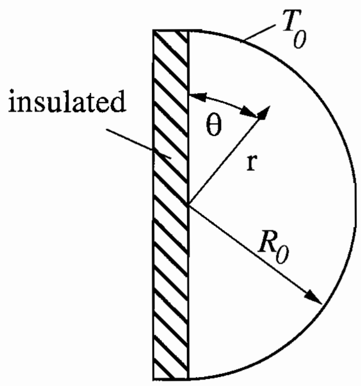

Fig. 1

(a) By performing an energy balance on a differential volume element $r\,d\theta\,dr$ per unit length of the body, show that the temperature variation in the body is governed by

$$\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right)+\frac{1}{r}\left(\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right)+\frac{\dot{Q}}{k}=0.$$

[30%]

(b) Justify $\left(\frac{\partial T}{\partial \theta}\right)=0$ by using the given boundary conditions and the geometry in Fig. 1. [20%]

(c) Solve the resulting equation for $T(r)$ as a function of $\dot{Q}$, R_0 and k. [25%]

(d) Carefully sketch the isotherms and adiabats in a $r-\theta$ plane and indicate their notable features. [25%]

2 A square heater with side l, marked as (1) in Fig. 2 has emissivity of $\varepsilon_1 = 0.9$ and operates at temperature T_1. Its radiation to an infinitely large region (3) on the right to the centre line is maximised by placing a perfectly insulated refelector (2) as in Fig. 2. The ratio l/L is 0.5 and the region (3) is at temperature T_3. Both the heater and the reflector can be assumed to be diffuse grey bodies.

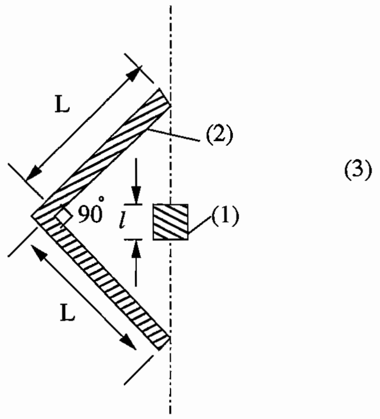

Fig. 2

(a) Sketch an equivalent electrical network for the radiative heat exchange bewteen the heater, the reflector and the hemispherical region for the following two cases: Case A – without the reflector; and Case B – with the reflector. Identify all the nodes and resistances. [15%]

(b) Express the net rate of heat exchange per unit length q_0 between the heater and the region (3) in the Case A using the above parameters and the Stefan–Boltzmann constant, σ. [10%]

(c) By considering an appropriate imaginary surface, determine the net rate of heat received per unit length q_R by the region (3) in the Case B. Express your answer using the above parameters and the Stefan–Boltzmann constant, σ. [45%]

(d) Determine the temperature of the reflector. Express your answer as a function of the above parameters. [25%]

(e) Comment on the ratio of your answers in (b) to (c), including the role of emissivities. [5%]

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3 A solar heater shown in Fig. 3 is fitted to a south facing wall for residential heating. The heater casings are insulated and their inner surfaces absorb the sunlight coming via the outer casing. After some initial transients, the inner surfaces attain a steady temperature of T_s which is larger than the air temperature T_1 entering at the bottom. A natural convective flow, with a mass flow rate per unit depth of $\dot{m}$, results and the air leaves at temperature T_2. The mean density of air inside the heater is ρ. The flow is thermally fully developed from location 2 marked in Fig. 3, and is governed by

$$\nu \frac{d^2 w}{dx^2} = -g\beta(T - T_1) \qquad \text{and} \qquad w\frac{\partial T}{\partial z} \approx \alpha \frac{\partial^2 T}{\partial x^2}$$

where β is the volumetric expansion coefficient of air, ν is its kinematic viscosity and α is its thermal diffusivity. The gravity constant is g.

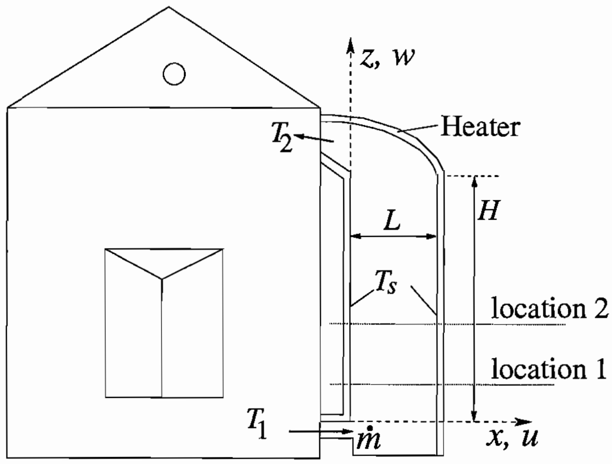

Fig. 3

(a) Carefully sketch the variations of air temperature T and velocity w at locations 1 and 2. [10%]

(b) What are the physical meanings conveyed by the above two equations? [10%]

(c) The distances x and z can be scaled respectively using the thermal thickness δ_T and H. The temperature T can be scaled using $\Delta T = (T_s - T_1)$. Using these scalings and the above momentum equation show that an estimate for w is $g\beta\Delta T\delta_T^2/\nu$. [15%]

(d) Using the scalings and estimate for w in (c) and the energy equation, estimate the time τ required to reach steady state in terms of the above parameters. You may note that $\delta_T \sim \sqrt{\alpha\tau}$. [15%]

(cont.

(e) Obtain $\dot{m}$ in terms of the above parameters by integrating the momentum equation with appropriate boundary conditions. You may take $(T - T_1) \approx (T_s - T_1)$. Also, express the total heat transfer rate per unit depth in the heater using the above parameters.

[50%]

4 A mechanical component made from steel with 0.1% carbon concentration is to be hardened by heating it to 950 K in a carbon rich environment until the carbon content is raised to 0.456% at a depth of 0.5 mm. The surface concentration of carbon is 1%. The carbon concentrations are given in mass basis and the temperature of the component is uniform. The diffusion coefficient of carbon in steel is

$$\mathscr{D} = 2.67 \times 10^{-5} \exp\left(\frac{-17400}{T}\right) \quad \text{m}^2\text{s}^{-1}$$

where T is the absolute temperature in Kelvin. The variation of carbon concentration, C, in the component is given by

$$\frac{C - C_o}{C_{\text{in}} - C_o} = \text{erf}(w)$$

where C_{in} is the initial concentration and C_o is the surface concentration. The symbol w is $y/\sqrt{4\mathscr{D}t}$, where y is the distance from the surface and t is time. The error function $\text{erf}(w)$ is given in Table 1.

(a) Carefully sketch the carbon concentration variation in the component with time. [10%]

(b) Calculate the time, in days, required to harden the component as given above. [15%]

(c) What time is required to achieve 0.456% carbon content at a depth of 1 mm with the conditions given above. [15%]

(d) Calculate the temperature required to halve the hardening time obtained in (b). [50%]

(e) Discuss other possible methods, if there are any, to achieve a reduction in the hardening time obtained in (b). [10%]

(cont.

Table 1: Error function table

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679
0.06	0.06762	0.44	0.46622	1.16	0.89910
0.08	0.09008	0.48	0.50275	1.20	0.91031
0.10	0.11246	0.52	0.53790	1.30	0.93401
0.12	0.13476	0.56	0.57162	1.40	0.95228
0.14	0.15695	0.60	0.60386	1.50	0.96611
0.16	0.17901	0.64	0.63459	1.60	0.97635
0.18	0.20094	0.68	0.66378	1.70	0.98379
0.20	0.22270	0.72	0.69143	1.80	0.98909
0.22	0.24430	0.76	0.71754	1.90	0.99279
0.24	0.26570	0.80	0.74210	2.00	0.99532
0.26	0.28690	0.84	0.76514	2.20	0.99814
0.28	0.30788	0.88	0.78669	2.40	0.99931
0.30	0.32863	0.92	0.80677	2.60	0.99976
0.32	0.34913	0.96	0.82542	2.80	0.99992
0.34	0.36936	1.00	0.84270	3.00	0.99998

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List of Answers

1. (c) $T(r) - T_o = \frac{\dot{Q} R_0^2}{4k} \left[1 - \left(\frac{r}{R_0} \right)^2 \right]$

2. (b) $q_0 = 1.8 l \sigma (T_1^4 - T_3^4)$

   (c) $q_R = 2.514 l \sigma (T_1^4 - T_3^4)$

   (d) $T_2 = (0.603 T_1^4 + 0.397 T_3^4)^{1/4}$

3. (d) $\tau_s \sim \left(\frac{\nu \, H}{g \, \beta \, \Delta T \, \alpha} \right)^{1/2}$

   (e) $\dot{m} = \frac{\rho \, g \, \beta \, \Delta T \, L^3}{12 \, \nu}$ and $\dot{Q} = \dot{m} c_p \, (T_2 - T_1)$

4. (b) $t = 6.78$days

   (c) $t = 27.11$days

   (d) $T = 987.3$K